

STUDIES AND EXERCISES
IN
FORMAL LOGIC

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IN
FORMAL LOGIC

INCLUDING A GENERALIZATION OF LOGICAL PROCESSES
IN THEIR APPLICATION TO COMPLEX
INFERENCES

BY

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PREFACE TO THE THIRD EDITION.

THIS edition has been in great part re-written and the book is again considerably enlarged.

In Part I the mutual relations between the extension and the intension of names are examined from a new point of view, and the distinction between real and verbal propositions is treated more fully than in the two earlier editions. In Part II more attention is paid to tables of equivalent propositions, certain developments of Euler's and Lambert's diagrams are introduced, the interpretation of propositions in extension and intension is discussed in more detail, and a brief explanation is given of the nature of logical equations. The chapters on the existential import of propositions and on conditional, hypothetical, and disjunctive (or, as I now prefer to call them, alternative) propositions have also been expanded, and the position which I take on the various questions raised in these chapters is I hope more clearly explained. In Parts III and IV there is less absolutely new matter, but the minor modifications are numerous. An appendix is added containing a brief account of the doctrine of division.

In the preface to earlier editions I was glad to have the opportunity of acknowledging my indebtedness to Professor Caldecott, to Mr W. E. Johnson, to Professor Henry Laurie, to Dr Venn, and to Mrs Ward. In the present edition my indebtedness to Mr Johnson is again very great. Many new developments are due to his suggestion, and in every important discussion in the book I have been most materially helped by his criticism and advice.

J. N. KEYNES.



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REFERENCE LIST OF INITIAL LETTERS SHEWING THE AUTHOR- SHIP OR SOURCE OF QUESTIONS AND PROBLEMS.

B = Professor J. I. Beare, Trinity College, Dublin ;
 C = University of Cambridge ;
 J = W. E. Johnson, King's College, Cambridge ;
 K = J. N. Keynes, Pembroke College, Cambridge ;
 L = University of London ;
 M = University of Melbourne ;
 N = Professor J. S. Nicholson, University of Edinburgh ;
 O = University of Oxford ;
 O'S = C. A. O'Sullivan, Trinity College, Dublin ;
 R = the late Professor G. Croom Robertson ;
 T = F. A. Tarleton, Trinity College, Dublin ;
 V = J. Venn, Gonville and Caius College, Cambridge ;
 W = J. Ward, Trinity College, Cambridge.

Note. A few problems have been selected from the published writings of Boole, De Morgan, Jevons, Solly, Venn, and Whately, from the Port Royal Logic, and from the Johns Hopkins Studies in Logic. In these cases the source of the problem is appended in full.

STUDIES AND EXERCISES IN FORMAL LOGIC.

INTRODUCTION.

1. *Definition of Formal Logic.*—Formal logic may be defined as the science which investigates those regulative principles of thought that have universal validity whatever may be the particular objects about which we are thinking. It is a science which is concerned with the form as distinguished from the matter of thought.

In a proposition some kind of relation is affirmed between certain objects of thought. By the *matter* of the proposition we mean the particular things thus related; by its *form* we mean the mode of their relation. For example, in the propositions, "All men are mortal," "All crystals are solid," the form is the same while the matter is different; in the propositions, "All crystals are solid," "Some rich men are not happy," the form as well as the matter is different. If we express the *matter* of propositions by symbols to which any signification we please may be assigned, then attention is concentrated on their *form*; e.g., *All S is P, Some S is not P*. The employment of non-significant symbols of this kind is accordingly advisable in dealing with most of the problems which fall within the scope of formal logic.

When we speak of a reasoning as being *formally valid*, we mean that its validity is determined solely by its form and is in no way dependent upon the particular subject-matter to which

it relates. The cogency of a formally valid argument will therefore be unaffected, if for the particular terms involved others are substituted. The following is an example: *All whales are mammals; Some water animals are whales; therefore, Some water animals are mammals.*

In formal inference the conclusion is always implicitly contained in the premisses; and mere consistency compels assent to the conclusion if the premisses are once admitted. Formal logic is accordingly sometimes spoken of as the logic of mere consistency; and it follows that the observance of the laws which formal logic investigates will not do more than secure freedom from self-contradiction and inconsistency. Absolute truth cannot be guaranteed by formal logic. At the same time, to draw out correctly all that is essentially involved in any given statement or set of statements is a function of fundamental importance; and the performance of this function alone may in many cases lead to knowledge that is to all intents and purposes new.

Whether formal logic properly constitutes the whole of logic is a disputed question that we need not here attempt to decide. Accepting the above definition of formal logic, it is at any rate open to us to recognise also another branch of the science, in which we take account of the matter of thought and are concerned with all the methods of reasoning and research by the aid of which it is possible to advance in the attainment of truth.

2. *Logic and Language.*—Some logicians, in their treatment of the problems of formal logic, endeavour to abstract not merely from the matter of thought but also from the language which is the instrument of thought; they seek to deal exclusively with the thought-products as they exist in the mind, not with these same products as expressed in language. This method of treatment is not adopted in the following pages. In adopting the alternative method, it is not necessary to maintain that thought is altogether impossible without language. It is enough that all thought-processes of any degree of complexity are as a matter of fact carried on by the aid of language. That language is in this sense the universal instrument of thought

will not be denied by any one; and it seems a fair corollary that the principles by which valid thought is regulated, and more especially the application of these principles, cannot be adequately discussed, unless some account is taken of the way in which this instrument actually performs its functions.

Language is full of ambiguities, and it is impossible to proceed far in logic until a precise interpretation has been placed upon certain forms of words as representing thought. It frequently happens that in everyday discourse the same propositional form may admit of different interpretations, according to the context or to the subject-matter of the statement. But of context and subject-matter formal logic has no cognizance. It is, therefore, necessary to determine definitely which of these interpretations is in our further investigations to be adopted. In ordinary discourse, to take a simple example, the word *some* may or may not be used in a sense in which it is exclusive of *all*; it may, in other words, mean *not-all* as well as *not-none*, or its full meaning may be *not-none*; the logician must determine at the outset in which of these senses he will employ the word. Again, a disjunctive statement in ordinary speech may be understood to imply that the different alternatives are mutually exclusive, or it may not; the logician must fix his meaning. Now if thought were considered exclusively in itself, such questions as these could not arise; they have to do with the expression of thought in language. The fact that they do arise and cannot help arising shews that actually to eliminate all consideration of language from logic is an impossibility.

Moreover, all the thoughts with which logic is concerned are expressed in language, reasonings being combinations of propositions, and propositions combinations of terms. To analyse the import of terms and propositions ought, therefore, to be recognised as included amongst the functions of the science. To investigate the form of conclusions obtainable from premisses of a given form is indeed the main object of formal logic. For this alone a discussion of the import of propositions is indispensable.

It may be added that in the case of some writers who

attempt to rise above mere considerations of language, the only result is needless prolixity and dogmatism in regard to what are really verbal questions, though they are not recognised as such.

The method of treating logic here advocated is sometimes called *nominalist*, and the opposed method *conceptualist*. A word or two of explanation is, however, desirable in order that this use of terms may not prove misleading. Nominalism and Conceptualism usually denote certain doctrines concerning the nature of general notions. Nominalism is understood to involve the assertion that generality belongs to language alone and that there is nothing general in thought. But a so-called nominalist treatment of logic does not involve this. It involves no more than a clear recognition of the importance of language as the instrument of thought; and this is a circumstance upon which modern advocates of conceptualism have themselves insisted.

It is perhaps necessary to add that on the view here taken logic in no way becomes a mere branch of grammar, nor does it cease to be a mental science. Whatever may be the aid derived from language, it remains true that the validity of formal reasonings depends ultimately on laws of thought. Formal logic, therefore, is still concerned primarily with thought, and only secondarily with language as the instrument of thought.

3. *Logic and Psychology*.—Since the laws regulating the processes of formal reasoning are laws which depend upon the constitution of our minds, they fall within the cognizance of psychology as well as of logic. But they are regarded from different points of view by these two sciences. Psychology deals with them as laws in the sense of uniformities, that is, as laws in accordance with which men are found by experience normally to think and reason; psychology investigates also their genesis and origin. Logic, on the other hand, deals with them purely as regulative and authoritative, as affording criteria by the aid of which valid and invalid reasonings may be discriminated, and as determining the formal relations in which different products of thought stand to one another.

Looking at the relations between psychology and logic from

a slightly different standpoint, it may be said that the former is concerned with the actual, the latter with the ideal. Logic does not, like psychology, treat of all the ways in which men actually reach conclusions, or of all the various modes in which, through the association of ideas or otherwise, one belief actually generates another. It is concerned with reasonings only as regards their cogency, and with the dependence of one judgment upon another only in so far as it is a dependence in respect of proof.

Logic has thus a unique character of its own, and is not a mere branch of psychology. Psychological and logical discussions are no doubt apt to overlap one another at certain points, in connexion, for example, with theories of conception and judgment. In the following pages, however, the psychological side of logic is comparatively but little touched upon. The metaphysical questions also to which logic tends to give rise are as far as possible avoided.

PART I.

TERMS.

CHAPTER I.

GENERAL AND SINGULAR NAMES.

4. *The Logic of Terms.*—A *name* is defined by Hobbes as “a word taken at pleasure to serve for a mark which may raise in our minds a thought like to some thought we had before, and which, being disposed in speech and pronounced to others, may be to them a sign of what thought the speaker had or had not before in his mind.”

In this definition the words “taken at pleasure” have rightly been criticised on the ground that they suggest an arbitrary and capricious selection, which is not in accordance with any generally accepted theory of the origin and growth of language. It is true that when an astronomer discovers a new planet and names it, or when a florist raises a seedling dahlia and gives it a distinctive title, or when an inventor names a new invention, the choice made is purely voluntary. But there are not many names in common use whose origin can thus be accounted for. It is agreed that the formation of names has for the most part been a natural and spontaneous process, involving nothing of the nature of deliberate invention and selection.

Hobbes’s definition has been further criticised on the ground that it is not wide enough to cover a many-worded name. Not

all names consist of a single word, *e.g.*, Prime Minister, Lord Chief Justice of England.

Accepting these criticisms, we may substitute for Hobbes's definition the following: A *name* is a word or set of words serving as a mark to raise in our minds a given idea, and also to indicate to others what idea is before the mind of the speaker.

A *term* is a name regarded as the logical subject or predicate of a proposition.

Since every name is the mark of something of which an affirmation or negation can be made, it follows that any given name is capable of being used as a term in some proposition or other. It is in their character as terms that names are of importance to the logician, and it will be found that we cannot in general fully determine the logical characteristics of a given name without explicit reference to its employment as a term.

In dealing with distinctions between names, it is particularly difficult for the logician who follows at all on the traditional lines to avoid discussing problems that belong more appropriately to psychology, metaphysics, or grammar; and in the case of some of the questions which arise it is impossible to give a completely satisfactory answer from the purely logical point of view. This remark applies especially to the distinction between *abstract* and *concrete* terms, a distinction, moreover, which is of little further logical utility or significance. It is introduced in the following pages in accordance with custom; but adequately to discriminate between things and their attributes is the function of metaphysics rather than of logic. The distinction to which by far the greatest importance attaches from the logical standpoint is that between *connotation* and *denotation*.

A *concept* is defined by Sir William Hamilton as "the cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide." In other words, a concept is the mental equivalent of a general name.

With those logicians who seek to exclude from their science all consideration of language a Logic of Concepts takes the place of a Logic of Terms; and in their treatment of this part

of the subject they discuss problems of a markedly psychological character, as, for example, the mode of formation of concepts and the controversy between conceptualism and nominalism. Apart, however, from the fact that the so-called conceptualist school do not draw so clear a line of distinction between logic and psychology, the difference between the two schools is to a large extent merely one of phraseology. Practically the same points, for example, are raised whether we discuss the extension and intension of concepts or the denotation and connotation of names.

5. *Categorematic and Syncategorematic Words.*—A *categorematic* word is one which can by itself be used as a name; a *syncategorematic* word is one which cannot by itself be used as a name, but only in combination with one or more other words.

Any noun substantive in the nominative case, or any other part of speech employed as equivalent to a noun substantive, may be used categorically.

It is a disputed question whether adjectives can be regarded as categorically *per se*, or whether they can be used categorically only by a grammatical ellipsis. No doubt adjectives sometimes stand alone as the predicates or (less frequently) as the subjects of propositions. But, on the other hand, adjectives *per se* and apart from any context are not complete names, since they have an essentially dependent character and must always be understood to qualify some substantive, expressed or unexpressed. For example, in treating *round* and *equiangular* as names we must regard them as equivalent to some such expressions as *round object* and *equiangular figure* respectively. Thus, in such a proposition as “the rich are not to be envied,” “the rich” is equivalent to “rich persons” or “those who are rich,” and the real subject of the proposition must be considered to be a substantive, although—by a grammatical ellipsis—it is expressed by an adjective. Again, when an adjective stands alone as the predicate of a proposition, we cannot discuss its characteristics as a name except by considering it to qualify either the substantive which appears as the subject or else the name of some wider class which includes the subject¹.

¹ Compare De Morgan, *Formal Logic*, p. 42.

Any part of speech, or the inflected cases of nouns substantive, may be used categorematically by what has been termed a *suppositio materialis*, that is, by making a statement about the mere word itself regarded as a group of certain letters or as representing a certain sound; e.g., "*John's* is a possessive case," "*Rich* is an adjective," "*With* is an English word."

6. *General and Singular Names.*—A general name is a name which is actually or potentially predicable in the same sense of each of an indefinite number of things; a singular or individual name is a name which is understood in the particular circumstances under which it is employed to denote some one determinate thing only.

The nature and logical importance of this distinction may be illustrated by considering names as the subjects of propositions. A general name is the name of a divisible class, and predication is possible in respect of the whole or a part of the class; a singular name is the name of a unit indivisible. Hence we may take as the test or criterion of a general name, the possibility of prefixing *all* or *some* to it with any meaning.

Thus, *prime minister of England* is a general name, since it is applicable to more than one individual, and statements may be made which are true of all prime ministers of England or only of some. The name *God* is singular to a monotheist as the name of the Deity, general to a polytheist, or as the name of any object of worship. *Universe* is general in so far as we distinguish different kinds of universes, e.g., the material universe, the terrestrial universe, &c.; it is singular if we mean the totality of all things. *Space* is general if we mean a particular portion of space, singular if we mean space in the aggregate. *Water* is general. Professor Bain takes a different view here; he says, "Names of Material—earth, stone, salt, mercury, water, flame—are singular. They each denote the entire collection of one species of material" (*Logic, Deduction*, pp. 48, 49). But when we predicate anything of these terms it is generally of *any portion* (or of some particular portion) of the material in question, and not of the entire collection of it *considered as one aggregate*; thus, if we say, "Water is composed

of oxygen and hydrogen," we mean any and every particle of water, and the name has all the distinctive characters of the general name. Again, we can distinguish *this* water from *that* water, and we can say, "*some* water is not fit to drink"; but the word *some* cannot, as we have seen above, be attached to a really singular name. Similarly with regard to the other terms in question. It is also to be observed that we distinguish between different kinds of stone, salt, &c.¹

A name is to be regarded as general if it may be *potentially* predicated of more than one object, although it accidentally happens that as a matter of fact it can be actually affirmed of only one, *e.g.*, *an English sovereign six times married*. A really singular name is not even potentially applicable to more than one individual; *e.g.*, *the last of the Mohicans*, *the eldest son of King Edward the First*.

Any general name may be transformed into a singular name by means of an individualising prefix, such as a demonstrative pronoun (*e.g.*, *this book*), or by the use of the definite article, which usually indicates a restriction to some one determinate person or thing (*e.g.*, *the Queen*, *the pole star*). Such restriction by means of the definite article may sometimes need to be interpreted by the context, *e.g.*, *the garden*, *the river*; in other cases some limitation of place or time or circumstance is introduced which unequivocally defines the individual reference, *e.g.*, *the first man*, *the present Lord Chancellor*, *the author of Paradise Lost*. A special class of singular names which have sometimes been erroneously supposed to be the only true singular names will be discussed in the next section.

7. *Proper Names*.—A proper name is a name assigned as a mark to distinguish an individual person or thing from others, but having no intrinsic significance beyond the fact of denoting the individual in question. Proper names form a sub-class of singular names, being distinguished from the singular names

¹ Terms of the kind here under discussion are called by Jevons *substantial terms*. (See *Principles of Science*, chap. 2, § 4.) Their peculiarity is that, although they are concrete, the things denoted by them possess a peculiar homogeneity or uniformity of structure; also we do not as a rule use the indefinite article with them as we do with other general names.

discussed in the preceding section in that they denote individual objects without at the same time conveying any information as to the special attributes possessed by those objects.

Many proper names, *e.g.*, *John*, *Victoria*, are as a matter of fact assigned to more than one individual; but they are not therefore general names, since on each particular occasion of their use, with the exception noted below, there is an understood reference to some one determinate individual only. There is, moreover, no implication that different individuals who may happen to be called by the same proper name have this name assigned to them on account of properties which they possess in common¹. The exception above referred to is when we speak of the class composed of those who bear the name, and who are constituted a distinct class by this common feature alone; *e.g.*, "All Victorias are honoured in their name," "Some Johns are not of Anglo-Saxon origin, but are negroes." The subjects of such propositions as these must, however, be regarded as elliptical; written out more fully, they become *all persons called Victoria, some individuals named John*.

8. *Collective Names*.—A collective name is one which is applied to a group of similar things regarded as constituting a single whole; *e.g.*, *regiment*, *nation*, *army*. A *non-collective* name, *e.g.*, *stone*, may also be the name of something which is composed of a number of precisely similar parts, but this is not in the same way present to the mind in the use of the name².

A collective name may be singular or general. It is the name of a group or collection of things, and so far as it is capable of being correctly affirmed in the same sense of only one such group, it is singular; *e.g.*, *the 29th regiment of foot*, *the English nation*, *the Bodleian Library*. But if it is capable

¹ Professor Bain brings out this distinction very clearly in his definition of a general name: "A general name is applicable to a number of things in virtue of their being similar, or having something in common."

² To *collective* name as above defined there is no distinctive antithetical term in ordinary use. Mr Johnson, however, suggests the word *unitary*. Thus a *unitary* name would be defined as one denoting an object which we regard as being itself the only unit with which we are immediately concerned. The antithesis between the *collective* and *distributive* use of names relates, as we shall see, to predication only.

of being correctly affirmed in the same sense of each of several such groups it is to be regarded as general; *e.g.*, *regiment*, *nation*, *library*¹.

Some logicians imply an antithesis between collective and general names, either regarding collectives as a sub-class of singulars, or else recognising a threefold division into singular, collective, and general. There is, properly speaking, no such antithesis; and both of the above alternatives must be regarded as misleading, if not actually erroneous; for the class of collective names overlaps, as we have just seen, each of the other classes.

The correct and really important logical antithesis is between the *collective* and *distributive* use of names. A collective name such as *nation*, or any name in the plural number, is the name of a collection or group of similar things. These we may regard as one whole, and something may be predicated of them that is true of them only as a whole; in this case the name is used *collectively*. On the other hand, the group may be regarded as a series of units, and something may be predicated of these which is true of them taken individually; in this case the name is used *distributively*².

The above distinction may be illustrated by the propositions, "All the angles of a triangle are equal to two right angles," "All the angles of a triangle are less than two right angles." In the first case the predication is true only of the angles all

¹ It is pointed out by Dr Venn that certain proper names may be regarded as collective, though such names are not common. "One instance of them is exhibited in the case of geographical groups. For instance, the Seychelles, and the Pyrenees, are distinctly, in their present usage, proper names, denoting respectively two groups of things. They simply denote these groups, and give us no information whatever about any of their characteristics" (*Empirical Logic*, p. 172).

² It is held by Dr Venn (*Empirical Logic*, p. 170) that *substantial terms* are always used collectively when they appear as subjects of general propositions. If, however, we take such a proposition as "Oil is lighter than water" it seems clear that the subject is used not collectively, but distributively; for the assertion is made of each and every portion of oil, whereas if we used the term collectively our assertion would apply only to all the portions taken together. The same is clearly true in other instances; for example, in the propositions, "Water is composed of oxygen and hydrogen," "Ice melts when the temperature rises above 32° Fahr."

taken together, while in the second it is true only of each of them taken separately; in the first case, therefore, the term is used collectively, in the second distributively. Compare again the propositions, "The people filled the church," "The people all fell on their knees."¹

¹ When in an argument we pass from the collective to the distributive use of a term, or *vice versâ*, we have what are technically called *fallacies of division* and *of composition* respectively. The following are examples: The people who attended Great St Mary's contributed more than those who attended Little St Mary's, therefore, *A* (who attended the former) gave more than *B* (who attended the latter); All the angles of a triangle are less than two right angles, therefore *A*, *B*, and *C*, which are all the angles of a triangle, are together less than two right angles.

CHAPTER II.

CONCRETE AND ABSTRACT NAMES.

9. *Nature of the distinction between Concrete and Abstract Names*¹.—The distinction between concrete and abstract names may be most briefly expressed by saying that a *concrete* name is the name of a *thing*, whilst an *abstract* name is the name of an *attribute*. The question, however, at once arises as to what is meant by a *thing* as distinguished from an *attribute*; and the only answer to be given is that by a thing we mean whatever is regarded as possessing attributes. Our definitions may, therefore, be made rather more explicit by saying that a *concrete name* is the name of anything which is regarded as possessing attributes, *i.e.*, as a *subject of attributes*; an *abstract name* is the name of anything which is regarded as an attribute of something else, *i.e.*, as an *attribute of subjects*².

¹ The account of the distinction between concrete and abstract names given in this section differs from that given in previous editions; but the substantial difference of view is not so great as might at first sight appear. The objection may perhaps be raised that the treatment of abstract and concrete names now adopted practically reduces the distinction between them to insignificance. No importance need, however, be attached to this objection, seeing that the distinction cannot in any case be regarded as having much logical value.

² The distinction is sometimes expressed by saying that an abstract name is the name of an attribute, a concrete name the name of a *substance*. If by *substance* is merely meant whatever possesses attributes, then this distinction is equivalent to that given in the text; but if, as would ordinarily be the case, a fuller meaning is given to the term, then the division of names into abstract and concrete is no longer an exhaustive one. Take such names as *astronomy*, *proposition*, *triangle*; these names certainly do not denote attributes; but, on the other hand, it seems paradoxical to regard them as names of substances. On the whole, therefore, it is best altogether to avoid the term *substance* in this connexion.

This distinction is in most cases easy of application; for example, *triangle* is the name of all figures that possess the attribute of being bounded by three straight lines, and is a concrete name; *triangularity* is the name of this distinctive attribute of triangles, and is an abstract name. Similarly, *man*, *living being*, *generous* are concretes; *humanity*, *life*, *generosity* are the corresponding abstracts¹.

Abstract and concrete names usually go in pairs as in the above illustrations. A concrete general name is the name of a class of things grouped together in virtue of some quality or set of qualities which they possess in common; the name given to the quality or qualities themselves apart from the individuals to which they belong is the corresponding abstract². Using the terms *connote* and *denote* in their technical sense, as defined in the following chapter, an abstract name denotes the qualities which are connoted by the corresponding concrete name. This relation between concretes and the corresponding abstracts is the one point in connexion with abstract and concrete names that is of real logical importance, and it may be observed that it does not in itself give rise to the somewhat fruitless subtleties with which the distinction is apt to be associated. For when two names are given which are thus

¹ It will be observed that, according to the above definitions, a name is not called abstract, simply because the corresponding idea is the result of abstraction, *i.e.*, attending to some qualities of a thing or class of things to the exclusion as far as possible of others. In this sense all general names, such as *man*, *living being*, &c., would be abstract. At the same time, it is of course true that an abstract name involves a higher degree of abstraction than the corresponding concrete name. There is so much risk of confusion here that the terms *abstract* and *concrete* must be considered not well chosen for marking the distinction had in view.

² Thus, in the case of every general concrete name there is or may be constructed a corresponding abstract. But this is not true of proper names or other singular names regarded strictly as such. We may indeed have such abstracts as *Cesarism* and *Bismarckism*. These names, however, do not denote all the differentiating attributes of Cesar and Bismarck respectively, but only certain qualities supposed to be specially characteristic of these individuals. In forming the above abstracts we generalise, and contemplate a certain type of character and conduct that may possibly be common to a whole class. Compare the last paragraph but one of section 15.

related, there will never be any difficulty in determining which is concrete and which is abstract in relation to the other.

But whilst the distinction is absolute and unmistakeable when names are thus given in pairs, it is by no means always easy of application when we consider names in themselves and not in this definite relation to other names. We shall find the simplest solution of the various problems which arise by admitting at the outset that the division of names into abstract and concrete is not an exclusive one in the sense that every name can once and for all be assigned exclusively to one or other of the two categories.

We are at any rate driven to this if we once admit that attributes may themselves be the subjects of attributes, and it is difficult to see how this admission can be avoided. If, for example, we say that "unpunctuality is irritating," we ascribe the attribute of being irritating to unpunctuality, which is itself an attribute. *Unpunctuality*, therefore, which is primarily an abstract name can also be used in such a way that it is, according to our definition, concrete.

Similarly when we consider that an attribute may appear in different forms or in different degrees, we must regard it as something which can itself be modified by the addition of a further attribute; as, for example, when we distinguish physical courage from moral courage, or the whiteness of snow from the whiteness of smoke, or when we observe that the beauty of a diamond differs in its characteristics from the beauty of a landscape.

We arrive then at the conclusion that while some names are concrete and never anything but concrete, names which are primarily formed as abstracts and continue to be used as such are apt also to be used as concretes, that is to say, they are names of attributes which can themselves be regarded as possessing attributes. They are abstract names when viewed in one relation, concrete when viewed in another¹.

¹ The use of the same term as both abstract and concrete in the manner above described must be distinguished from the not unfrequent case of quite another kind in which a name originally abstract changes its meaning and comes to be used in the sense of the corresponding concrete; as, for example,

10. *Can the distinction between Generals and Singulars be applied to Abstract Names?*—The question whether any abstract names can be considered general has given rise to much difference of opinion amongst logicians. On the one hand, it is argued that all abstract names must necessarily be singular, since an attribute considered purely as such and apart from its concrete manifestations is one and indivisible, and cannot admit of numerical distinction¹. On the other hand, it is urged that some abstracts must certainly be considered general since they are names of attributes of which there are various kinds or subdivisions; and in confirmation of this view it is pointed out that we frequently write abstracts in the plural number, as when we say, “Redness and yellowness are *colours*,” “Patience and meekness are *virtues*.”²

when we talk of the *Deity* meaning thereby God, not the qualities of God. Jevons (*Elementary Lessons in Logic*, pp. 21, 22) gives other examples: “*Relation* properly is the abstract name for the position of two people or things to each other, and those people are properly called *relatives*. But we constantly speak now of *relations*, meaning the persons themselves; and when we want to indicate the abstract relation they have to each other we have to invent a new abstract name *relationship*. *Nation* has long been a concrete term, though from its form it was probably abstract at first; but so far does the abuse of language now go, especially in newspaper writing, that we hear of a *nationality*, meaning a nation, although of course if nation is the concrete, nationality ought to be the abstract, meaning the quality of being a nation.”

¹ This represents the view taken by Jevons. “Abstract terms are strongly distinguished from general terms by possessing only one kind of meaning; for as they denote qualities there is nothing which they can in addition imply. The adjective ‘red’ is the name of red objects, but it implies the possession by them of the quality *redness*; but this latter term has one single meaning—the quality alone. Thus it arises that abstract terms are incapable of number or plurality. Red objects are numerically distinct each from each, and there are a multitude of such objects; but redness is a single existence which runs through all those objects, and is the same in one as it is in another. It is true that we may speak of *rednesses*, meaning different kinds or tints of redness, just as we may speak of *colours*, meaning different kinds of colours. But in distinguishing kinds, degrees, or other differences, we render the terms so far concrete. In that they are merely red there is but one single nature in red objects, and so far as things are merely coloured, colour is a single indivisible quality. Redness, so far as it is redness merely, is one and the same everywhere, and possesses absolute oneness or unity” (*Principles of Science*, ch. 2, § 3).

² Thus Mill writes:—“Some abstract names are certainly general. I mean those which are names not of one single and definite attribute, but of a class of

A mode of reconciling these opposing views is to be found in the recognition that the same name may be regarded as either abstract or concrete according to the point of view taken.

The name of an attribute can be described as general only in so far as the attribute is regarded as exhibiting characteristics which vary in different instances, only in so far, that is to say, as it is itself a subject of attributes. But from this point of view the name, according to the definitions given in the preceding section, is concrete. Thus in the proposition, "Some colours are painfully vivid," we are predicating an attribute of a subject; similarly in the propositions, "All yellows are agreeable to me," "Some courage is the result of ignorance"; we must, therefore, regard the subjects of these propositions as being here used in a concrete sense.

We arrive then at the conclusions, *first*, that an abstract name considered strictly as such, that is to say, as the name of an attribute possessed by a certain class of objects, and without regard to any differences in the manner in which the attribute manifests itself, cannot properly be described as general; but, *secondly*, that names which are primarily names of attributes can also be used in a concrete sense, and may then become at the same time general¹.

11. *The Logical Characteristics of Adjectives.*—The question whether adjectives can be regarded as names *per se*, or

attributes. Such is the word *colour*, which is a name common to whiteness, redness, &c. Such is even the word *whiteness*, in respect of the various shades of whiteness to which it is applied in common; the word *magnitude*, in respect of the various degrees of magnitude and the various dimensions of space; the word *weight*, in respect of the various degrees of weight. Such also is the word *attribute* itself, the common name of all particular attributes. But when only one attribute, neither variable in degree nor in kind, is designated by the name; as visibleness; tangibleness; equality; squareness; milk-whiteness; then the name can hardly be considered general; for though it denotes an attribute of many different objects, the attribute itself is always conceived as one, not many" (*Logic*, i. ch. 2, § 4).

¹ But like other general names they may be individualised by means of an individualising prefix, as in the propositions, "My health is not good," "Your happiness is my only object," "His honesty was not proof against such a temptation."

only by a grammatical ellipsis, has been already considered. Whichever view may be taken on this point, it seems clear that if adjectives are classified as names at all, they are concrete and general. For an adjective is essentially a name which can be applied to whatever possesses a certain attribute; *great*, for instance, is the name of whatever possesses the attribute of greatness. But a name which applies to an indeterminate number of objects is *general*, and the name of a subject of attributes is *concrete*. Hence the conclusion above indicated follows immediately from our definitions of concrete and general names.

No qualification is necessary in order to meet the case of an adjective appearing as the predicate in such a proposition as *Unpunctuality is irritating*. Here *unpunctuality*, though an attribute, is regarded as itself the subject of a further attribute. It is, therefore, regarded from the point of view from which it is a concrete name, and the predicate of the proposition is accordingly concrete also.

An adjective may, however, form part of an abstract name, *e.g.*, we may speak of *great beauty* or of *great strength*; it may also form part of a singular term, *e.g.*, we may speak of *Alexander the Great* or of *the great Goliath*. In cases like these, therefore, combinations of names containing adjectives may be abstract or singular. But it does not follow that adjectives considered by themselves need be regarded as abstract or singular, any more than that such a term as *man* is itself singular because it forms part of the singular term *the first man*.

CHAPTER III.

CONNOTATION AND DENOTATION.

12. *The Extension and Intension of Names.*—Every concrete general name is the name of a real or imaginary class of objects which possess in common certain attributes; and there are, therefore, two aspects under which it may be regarded. We may consider the name (i) in relation to the objects which are called by it; or (ii) in relation to the qualities belonging to those objects. It is desirable to have terms by which to refer to this broad distinction without regard to further refinements of meaning; and the terms *extension* and *intension* will accordingly be employed to express in the most general way these two aspects of general names respectively¹.

Thus, by the extension of *plane triangle* we mean a certain class of geometrical figures, and by its intension certain properties belonging to such figures. Similarly, by the extension of *man* is meant a certain class of material objects, and by its intension the properties of rationality, animality, &c., belonging to these objects.

13. *Connotation, Subjective Intension, and Comprehension.*—The term *intension* has been used in the preceding section to express in the most general way that aspect of general names under which we consider not the objects called by the names but the qualities belonging to those objects. Taking any general name, however, there are at least three different points

¹ It is usual to employ the terms *comprehension* and *connotation* as simply synonymous with *intension*, and *denotation* as synonymous with *extension*. We shall, however, presently find it convenient to differentiate their meanings.

of view from which the properties of the corresponding class may be regarded; and it is to a want of discrimination between these points of view that we may largely attribute the controversies and misunderstandings to which the problem of the connotation of names has given rise in such abundance.

(1) There are those properties which are essential to the class in the sense that the name implies them in its definition. Were any of this set of properties absent the name would not be applicable; and any individual thing lacking them would accordingly not be regarded as a member of the class. The standpoint here taken may be said to be conventional, since we are concerned with the set of characteristics which are supposed to have been conventionally agreed upon as determining the application of the name.

(2) There are those properties which in the mind of any given individual are associated with the name in such a way that they are normally called up in idea when the name is used. These properties will include the marks by which the individual in question usually recognises or identifies an object as belonging to the class. They may not exhaust the essential qualities of the class in the sense indicated in the preceding paragraph, but on the other hand they will probably include some that are not essential to it. The standpoint here taken is subjective and relative. Even when there is agreement as to the actual meaning of a name, the qualities that we naturally think of in connexion with it may vary both from individual to individual, and, in the case of any given individual, from time to time.

We may consider as a special case under this head the complete group of attributes *known* at any given time to belong to the class. All these attributes can be called up in idea by any person whose knowledge of the class is fully up to date; and this group may, therefore, be regarded as constituting the most scientific form of intension from the subjective point of view.

(3) There is the sum-total of properties actually possessed in common by every member of the class. These will include all the qualities included under the two preceding

heads¹, and usually many others in addition. The standpoint here taken is *objective*².

In seeking to give a precise meaning to *connotation*, we may start from the above classification. It suggests three distinct senses in which the term might possibly be used, and as a matter of fact all three have been selected by different logicians, sometimes without any clear recognition of divergence from the usage of other writers. It is of the greatest importance that we should be quite clear in our own minds in which sense we intend to employ the term.

(i) According to Mill's usage, which will be adopted in the following pages, the conventional standpoint is taken when we speak of the connotation of a name. On this view, we do not mean by the connotation of a class-name all the properties possessed in common by the class; nor do we necessarily mean those particular properties which may be mentally associated with the name; but we mean just those properties on account of the possession of which any individual is placed in the class, or called by the name. In other words, we include in the connotation of a class-name only those attributes upon which the classification is based, and in the absence of any of which the name would not be regarded as applicable. For example, although all equilateral triangles are equiangular, equiangularity is not on this view included in the connotation of equilateral triangle, since it is not a property upon which the classification of triangles into equilateral and non-equilateral is based; although all kangaroos may happen to be *Australian* kangaroos, this is not part of what is necessarily implied by the use of the name, for an animal subsequently found in the interior of New Guinea, but otherwise possessing all the properties of kangaroos, would not have the name kangaroo denied to it; although all ruminant animals are cloven-hoofed, we cannot regard cloven-

¹ It is here assumed, as regards the qualities mentally associated with the name, that our knowledge of the class, so far as it extends, is correct.

² When the objective standpoint is taken, there is an implied reference to some particular universe of discourse, within which the class denoted by the name is supposed to be included. The force of this remark will be made clearer at a subsequent stage.

hoofed as part of the meaning of ruminant, and we may say with Mill that were an animal to be discovered which chewed the cud, but had its feet undivided, it would certainly still be called ruminant.

(ii) Some writers who regard proper names as connotative appear to include in the connotation of a name all those attributes which the name suggests to the mind, whether or not they are actually implied by it. And it is here to be observed that a name may in the mind of any given individual be closely associated with properties which even the same individual would in no way regard as implied in the meaning of the name, as for instance, "Trinity undergraduate" with a blue gown. This interpretation of connotation, therefore, is clearly to be distinguished from that given in the preceding paragraph.

We may further distinguish the view, apparently taken by some writers, according to which the connotation of a class-name at any given time would include all the properties *known* at that time to belong to the class.

(iii) Other writers use the term in still another sense and would include in the connotation of a class-name all the properties, known and unknown, which are possessed in common by all members of the class. Thus, Mr E. C. Benecke writes—"Just as the word 'man' denotes every creature, or class of creatures, having the attributes of humanity, whether we know him or not, so does the word properly connote the *whole* of the properties common to the class, whether we know them or not. Many of the facts, known to physiologists and anatomists about the constitution of man's brain, for example, are not involved in most men's idea of the brain; the possession of a brain precisely so constituted does not, therefore, form any part of their meaning of the word 'man.' Yet surely this is properly connoted by the word.... We have thus the denotation of the concrete name on the one side and its connotation on the other, occupying perfectly analogous positions. Given the connotation,—the denotation is all the objects that possess the whole of the properties so connoted. Given the denotation,—the connotation is the whole of the properties possessed in

common by all the objects so denoted" (*Mind*, 1881, p. 532). Professor Jevons also uses the term in the same sense. "A term taken in intent (connotation) has for its meaning the whole infinite series of qualities and circumstances which a thing possesses. Of these qualities or circumstances some may be known and form the description or definition of the meaning; the infinite remainder are unknown" (*Pure Logic*, p. 6)¹.

While rejecting the use of the term *connotation* in any but the first of the above mentioned senses, it will be found desirable also to have terms which can be used with the other meanings which have been indicated. The three terms *connotation*, *intension*, and *comprehension* are commonly employed almost synonymously, and there will certainly be a gain in endeavouring to differentiate their meanings. *Intension*, as already suggested, may be used to indicate in the most general way what may be called the implicational aspect of names; the complex terms *conventional intension*, *subjective intension*, and *objective intension* will then explain themselves. *Connotation* may be used as equivalent to *conventional intension*; and *comprehension* as equivalent to *objective intension*. *Subjective intension* is less important, and we need not seek to invent a single term to be used as its equivalent².

Conventional intension or *connotation* will then include only those attributes which constitute the meaning of a name; *subjective intension* will include those that are mentally associated with it, whether or not they are actually signified by it³; *objective intension* or *comprehension* will include all the attributes

¹ Professor Bain appears to use the term in an intermediate sense, including in the connotation of a class-name not *all* the attributes common to the class but all the *independent* attributes, that is, all that cannot be derived or inferred from others.

² In the second edition what is here called *subjective intension* was called simply *intension*. It is, however, important to have a term which can be used in the more general sense, and for this purpose no other term seems as suitable as *intension* (with its correlative *extension*).

³ It is clear that subjective intension is likely to vary with each individual. How far connotation also may vary with each individual will be discussed in a subsequent section.

possessed in common by all members of the class denoted by the name. We might perhaps speak more strictly of the *connotation* of the name itself, the *subjective intension* of the notion which is the mental equivalent of the name, and the *comprehension* of the class which is denoted by the name¹.

14. *Connotative Names*.—Mill's use of the word *connotative*, which is that generally adopted in modern works on logic, is as follows: "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute" (*Logic*, I. ch. 2, § 5). According to this definition, a connotative name must not only possess extension, but must also have a conventional intension assigned to it.

The following kinds of names are connotative in the above sense:—(1) All concrete general names. (2) Some singular names. For example, *city* is a general name, and as such no one would deny it to be connotative. Now if we say *the largest city in the world*, we have individualised the name, but it does not thereby cease to be connotative. Proper names are, however, according to Mill, non-connotative, since they merely denote a subject and do not imply any attributes. To this point, which is a disputed one, we shall return in the following section. (3) While admitting that most abstract names are non-connotative, since they merely signify an attribute and do not denote a subject, Mill maintains that some abstracts may justly be "considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes" (*Logic*, I. p. 33). To this point also we shall return in a later section.

15. *Are proper names connotative?*—To this question absolutely contradictory answers are given by ordinarily clear thinkers as being obviously correct. To some extent, however,

¹ The distinctions of meaning indicated in this section will be found absolutely essential for clearness of view in discussing certain questions to which we shall pass on immediately; in particular, the questions whether proper names are connotative, and whether connotation and denotation necessarily vary inversely.

the divergence is merely verbal, the term connotation being used in different senses.

Mill speaks decisively, "The only names of objects which connote nothing are proper names; and these have, strictly speaking, no signification" (*Logic*, I. p. 36). The opposite view is taken by Jevons, Mr Bradley, and others.

It is necessary at the outset to guard against a misconception which quite obscures the point really at issue. Thus, with reference to Mill, Jevons says, "Logicians have erroneously asserted, as it seems to me, that singular terms are devoid of meaning in intension, the fact being that they exceed all other terms in that kind of meaning" (*Principles of Science*, 2nd ed. p. 27, with a reference to Mill in the foot-note). But Mill distinctly says that some singular names are connotative, e.g., *the sun*¹, *the first emperor of Rome* (*Logic*, I. pp. 34, 35). Again, Jevons says, "There would be an impossible breach of continuity in supposing that after narrowing the extension of 'thing' successively down to animal, vertebrate, mammalian, man, Englishman, educated at Cambridge, mathematician, great logician, and so forth, thus increasing the intension all the time, the single remaining step of adding Augustus de Morgan, Professor in University College, London, could remove all the connotation, instead of increasing it to the utmost point" (*Studies in Deductive Logic*, pp. 2, 3). But every one would allow that we may narrow down the extension of a term till it becomes individualised without destroying its connotation; "the present Professor of Pure Mathematics in University College, London" is a singular term—its extension cannot be further diminished—but it is certainly connotative.

It must then be clearly understood that only one class of singular names, namely, proper names, are affirmed to be non-

¹ The question has been asked on what grounds *the sun* can be regarded as connotative, while *John* is considered non-connotative; compare T. H. Green, *Philosophical Works*, vol. 2, p. 204. The answer is that *sun* is a general name with a definite signification which determines its application, and that it does not lose its connotation when individualised by the prefix *the*; while *John*, on the other hand, is a name given to an object merely as a mark for purposes of future reference, and without signifying the possession by that object of any special attributes.

connotative; and in regard to these, as defined in section 7, it would seem to be hardly more than a verbal statement to say that they are so. Perhaps, however, those who regard proper names as connotative may criticise the definition referred to as question-begging; and, in any case, there remains a source of ambiguity connected with the meaning of the term connotation. If we differentiate our terms and use them in the senses indicated in section 13, then we must say that while proper names have no connotation, they nevertheless have both subjective intension and comprehension. An individual object can be recognised only through its attributes; and a proper name when understood by me to be a mark of a certain individual undoubtedly suggests to my mind certain qualities¹. The qualities thus suggested by the name constitute its subjective intension. The comprehension of the name will include a good deal more than its subjective intension, namely, the whole of the properties which belong to the individual denoted.

If then by the connotation of a name we meant all the attributes possessed by the individuals denoted by it, or if we meant the attributes suggested by it, Jevons's view would be correct. One or other of these alternatives does appear to be what Jevons himself means, but it is distinctly *not* what Mill means; he means only those attributes which are in the strictest sense signified by the name. Jevons puts his case as follows:—"Any proper name, such as John Smith, is almost without meaning until we know the John Smith in question. It is true that the name alone connotes the fact that he is a Teuton, and is a male; but, so soon as we know the exact individual it denotes, the name surely implies, also, the peculiar features, form, and character, of that individual. In fact, as it

¹ A proper name may have suggestive force even for those who are not actually acquainted with the person or thing denoted by it. Thus *William Stanley Jevons* may suggest any or all of the following to one who never heard the name before: an organised being, a human being, a male, an Anglo-Saxon, having some relative named Stanley, having parents named Jevons. But at the same time, the name cannot be said necessarily to signify any of these things, in the sense that if they were wanting it would be misapplied. Consider, for example, such a name as *Victoria Nyanza*. Some further remarks bearing on this point will be found later on in this section.

is only by the peculiar qualities, features, or circumstances of a thing, that we can ever recognise it, no name could have any fixed meaning unless we attached to it, mentally at least, such a definition of the kind of thing denoted by it, that we should know whether any given thing was denoted by it or not. If the name John Smith does not suggest to my mind the qualities of John Smith, how shall I know him when I meet him? For he certainly does not bear his name written upon his brow" (*Elementary Lessons in Logic*, p. 43). A wrong criterion of connotation in Mill's sense is here taken. The connotation of a name is not the quality or qualities by which I or any one else may happen to recognise the class which it denotes. For example, I may recognise an Englishman abroad by the cut of his clothes, or a Frenchman by his pronunciation, or a proctor by his bands, or a barrister by his wig; but I do not *mean* any of these things by these names, nor do they (in Mill's sense) form any part of the connotation of the names. Compare two such names as *John Duke Coleridge* and *the Lord Chief Justice of England*. They denote the same individual, and I should recognise John Duke Coleridge and the Lord Chief Justice of England by the same attributes; but the names are not equivalent—the one is given to a certain individual as a mere mark to distinguish him from others, and it has no further signification; the other is given because of the performance of certain functions, on the cessation of which the name would cease to apply. Surely there is a distinction here, and one which it is important that we should not overlook.

Nor is it true that such a name as "John Smith" connotes "Teuton, male, &c." John Smith might be a dahlia, or a race-horse, or a negro, or the pseudonym of a woman, as in the case of George Eliot. In none of these cases could the name be said to be misapplied as it would be if a dahlia or a horse were called a man, or a negro a Teuton, or a woman a male.

Still, it may fairly be said that many, if not most, proper names do signify something, in the sense that they were chosen in the first instance for a special reason. For example, *Strongi'th'arm*, *Smith*, *Jungfrau*. But such names even if in a certain sense connotative when first imposed soon cease to

be connotative in the way in which other names are connotative. Their application is in no way dependent on the continuance of the attribute with reference to which they were originally given. As Mill puts it, *the name once given is independent of the reason.* In other words, we ought carefully to distinguish between the connotation of a name, and its history. Thus, a man may in his youth have been strong, but we should not continue to call him strong in his dotage; whilst the name *Strength's arm* once given would not be taken from him. Again, the name *Smith* may in the first instance have been given because a man plied a certain handicraft, but he would still be called by the same name if he changed his trade, and his descendants continue to be called *Smith* whatever their occupations may be¹.

Proper names of course become connotative when they are used to designate a certain type of person; for example, a Diogenes, a Thomas, a Don Quixote, a Paul Pry, a Benedick, a Socrates. But, when so used, such names have really ceased to be proper names at all; they have come to possess all the characteristics of general names.

Before leaving the subject of proper names, attention may be called to a class of singular names, such as *Miss Smith, Captain Jones, President Cleveland, the Lake of Lucerne, the Falls of Niagara*, which may be said to be partially but only partially connotative. Their peculiarity is that they contain as elements terms having a general and permanent signification, and that some change in the object denoted might consequently render them no longer applicable, as, for example, if Captain Jones received promotion and were made a major; while, at the same time, such connotation as they possess is by itself insufficient to determine their application. They occupy an intermediate position, therefore, between connotative singular names, such as *the first man*, and strictly proper names.

16. *Are any abstract names connotative?*—A connotative

¹ It cannot, however, be said that the name necessarily implies ancestors of the same name. As Dr Venn remarks, "he who changes his family name may grossly deceive genealogists, but he does not tell a falsehood" (*Empirical Logic*, p. 185).

name is one which denotes a subject and implies an attribute; and, according to our definition of a concrete name, every name given to a class of things in virtue of some quality or set of qualities which they have in common is concrete. It follows immediately that all connotative names are concrete; and hence, if we regard the names abstract and concrete as strictly contradictories, the above question may at once be answered in the negative.

According, however, to the definitions adopted in the preceding chapter, the terms abstract and concrete, as applied to names, are not mutually exclusive. In so far as attributes can be considered as themselves possessing attributes, attribute-names may be described either as abstract or as concrete, according to the point of view from which they are regarded, and it cannot, therefore, be said that no attribute-names are connotative. Thus *virtue* considered in relation to *virtuous* is to be described as abstract; it denotes the attribute which *virtuous* connotes, that is to say, the attribute on account of the possession of which we describe a person or an action as virtuous. But considered in relation to the various virtues (*courage*, *temperance*, &c.) which constitute its sub-classes, *virtue* is to be described as concrete; it may then be said to denote these various virtues, and to connote the common property by reason of which they are classed together.

Our conclusion then is that names of attributes—names, therefore, which are primarily abstract—may be regarded as connotative; but only in so far as they take on at the same time a concrete character.

17. *Extension and Denotation.*—The terms *extension* and *denotation* are usually employed as synonymous, but there will be some advantage in drawing a certain distinction between them. We shall find that when names are regarded as the subjects of propositions there is usually an implied reference to some more or less limited universe of discourse. For example, we should naturally understand such propositions as *all men are mortal*, *no men are perfect*, to refer to all men who have actually existed on the earth, or are now existing, or will exist hereafter, but we should not understand them to refer to all

fictitious persons or all beings possessing the essential characteristics of men whom we are able to conceive or imagine. The meaning of *universe of discourse* will be further illustrated subsequently. The only reason for introducing the conception at this point is that we propose to use the term *denotation* rather than the term *extension* when there is an explicit or implicit limitation to the objects actually to be found in some restricted universe. By the *extension* of a general name, on the other hand, we shall understand the whole range of objects real or imaginary to which the name can be correctly applied, the only limitation being that of logical conceivability. Every name, therefore, which can be used in an intelligible sense will have a positive extension, but its denotation in a universe which is in some way restricted by time, place, or circumstance may be zero¹.

In the sense here indicated, *denotation* is in certain respects the correlative of *comprehension* rather than of *connotation*.

¹ The value of the above distinction may be illustrated by reference to the divergence of view indicated in the following quotation from Mr Monck, who uses the terms *denotation* and *extension* as synonymous: "It is a matter of accident whether a general name will have any extension (or denotation) or not. *Unicorn, griffin, and dragon* are general names because they have a meaning, and we can suppose another world in which such beings exist; but the terms have no extension, because there are no such animals in this world. Some logicians speak of these terms as having an extension, because we can *suppose* individuals corresponding to them. In this way every general term would have an extension which might be either real or imaginary. It is, however, more convenient to use the word *extension* for a real extension (past, present, or future) only" (*Introduction to Logic*, p. 10). It should be added, in order to prevent possible misapprehension, that by *universe of discourse*, as used in the text, we do not necessarily mean the material universe; we may, for example, mean the universe of fairy-land, or of heraldry, and in such a case, *unicorn, griffin, and dragon* may have denotation (in our special sense) as well as extension greater than zero. What is the particular universe of reference in any given proposition will generally be determined by the context. For logical purposes we must assume that it is conventionally understood and agreed upon, and that it remains the same throughout the course of any given argument. As Dr Venn remarks, "We might include amongst the assumptions of Logic that the speaker and hearer should be in agreement, not only as to the meaning of the words they use, but also as to the conventional limitations under which they apply them in the circumstances of the case" (*Empirical Logic*, p. 180).

For in speaking of denotation we are, 'as in the case of comprehension, taking an objective standpoint; and there is, moreover, in the case of comprehension, as in that of denotation, a tacit reference to some defined universe of discourse. We shall, however, find that, in one very important respect, connotation and denotation are still correlatives.

18. *Dependence of Extension and Intension upon one another*¹.—Taking any class-name X , let us first suppose that there has been a conventional agreement to use it wherever a certain arbitrarily selected set of properties, $P_1, P_2, \dots P_m$, are present. This set of properties will constitute the *connotation* of X , and will, with reference to a given universe of discourse², determine the *denotation* of the name, say, $Q_1, Q_2, \dots Q_y$; that is, $Q_1, Q_2, \dots Q_y$ are all the individuals possessing in common the properties $P_1, P_2, \dots P_m$.

These properties may not, and almost certainly will not, exhaust the properties common to $Q_1, Q_2, \dots Q_y$. Let all the common properties be $P_1, P_2, \dots P_x$; they will include $P_1, P_2, \dots P_m$, and in all probability more besides, and will constitute the *comprehension* of the class-name.

Now it will always be possible in one or more ways to make out of $Q_1, Q_2, \dots Q_y$, a selection $Q_1, Q_2, \dots Q_n$, which shall be precisely typical of the whole class³; that is to say, $Q_1, Q_2, \dots Q_n$ will possess in common those attributes and only those attributes (namely, $P_1, P_2, \dots P_x$) which are possessed in common by $Q_1, Q_2, \dots Q_y$ ⁴. $Q_1, Q_2, \dots Q_n$ may be called the *exemplification* or *ex-*

¹ This section may be omitted on a first reading.

² It will be assumed in the remainder of this section that we are throughout speaking with reference to a given universe of discourse.

³ It may chance to be necessary to make $Q_1, Q_2, \dots Q_n$ coincide with $Q_1, Q_2, \dots Q_y$. But this must be regarded as the limiting case; usually a smaller number of individuals will be sufficient.

⁴ Mr Johnson points out to me that in pursuing this line of argument certain restrictions of a somewhat subtle kind are necessary in regard to what may be called our "universe of attributes." The "universe of objects," which is what we mean by the "universe of discourse," implies *individuality of object* and *limitation of range of objects*; and if we are to work out a thoroughgoing reciprocity between attributes and objects, we must recognise in our "universe of attributes" restrictions analogous to the above, namely, *simplicity of attribute* and *limitation of range of attributes*. By "simplicity of attribute" is meant

tensive definition of X . The reason for selecting the name *extensive definition* will appear in a moment. It will sometimes be convenient correspondingly to speak of the connotation of a name as its *intensive definition*.

We have then, with reference to X ,

- (1) *Connotation*: $P_1 \dots P_m$;
- (2) *Denotation*: $Q_1 \dots Q_n \dots Q_y$;
- (3) *Comprehension*: $P_1 \dots P_m \dots P_x$;
- (4) *Exemplification*: $Q_1 \dots Q_n$.

Of these, either the connotation or the exemplification will suffice to mark out or completely identify the class, although they do not exhaust either all its common properties or all the individuals contained in it. In other words, whether

that the universe of attributes must not contain any attribute which is a *logical function* of any other attribute or set of attributes. Thus, if A, B are two attributes recognised in our universe, we must not admit such attributes as $X (=A \text{ and } B)$, or $Y (=A \text{ or } B)$, or $Z (=not-A)$. We may indeed have a negatively defined attribute, but it must not be the formal contradictory of another or formally involve the contradictory of another. The following example will shew the necessity of this restriction. Let P_1, P_2, P_3 be selected as typical of the whole class $P_1, P_2, P_3, P_4, P_5, P_6$; and let A_1 be an attribute possessed by P_1 alone, A_2 an attribute possessed by P_2 alone, and so on. Then if we recognise A_1 or A_2 or A_3 as a distinct attribute, it is at once clear that P_1, P_2, P_3 will no longer be typical of the whole class; and the same result follows if *not- A_4* is recognised as a distinct attribute. Similarly, without the restriction in question *any* selection (short of the whole) would necessarily fail to be typical of the whole class. As a concrete example, suppose that we select from the class of *professional men* a set of examples that have in common no attribute except those that are common to the whole class. It may turn out that our examples are all *barristers* or *doctors* but none of them *solicitors*. Now if we recognise as a distinct attribute being "either a barrister or a doctor," our selected group will thereby have an extra common attribute not possessed by every professional man. The same result will follow if we recognise the attribute "non-solicitor." Not much need be added as regards the necessity of some limitation in the range of attributes which are recognised. The mere fact of our having selected a certain group would indeed constitute an additional attribute, which would at once cause the selection to fail in its purpose, unless this were excluded as inessential. Similarly, such attributes as position in space or in time &c. must in general be regarded as inessential. For example, I might draw on a sheet of paper a number of triangles sufficiently typical of the whole class of triangles, but for this it would be necessary to reject as inessential the common property which they would possess of all being drawn on a particular sheet of paper.

we start from the connotation or from the exemplification everything else will come out the same¹.

For a concrete illustration of the above the term *metal* may be taken. From the chemical point of view a metal may be defined as an element which can replace hydrogen in an acid and thus form a salt. This then is the *connotation* of the name. Its *denotation* consists of the complete list of elements fulfilling the above condition now known to chemists, and possibly of others not yet discovered². The whole class thus constituted are, however, found to possess other properties in common besides those contained in the definition of the name, for example, fusibility, the characteristic lustre termed metallic, a high degree of opacity, and the property of being good conductors of heat and electricity. The complete list of these properties forms the *comprehension* of the name. Now a chemist would no doubt be able from the full denotation of metal to make a selection of a limited number of metals which would be precisely typical of the whole class³; that is to say, his selected list would possess in common only such properties as are common to the whole class. This selected class would constitute the *exemplification* of the name.

We have so far assumed that (1) connotation or *intensive definition* has first been arbitrarily fixed, and that this has successively determined (2) denotation, (3) comprehension, and—with a certain range of choice—(4) exemplification. But it is clear that theoretically we might start by arbitrarily fixing (i) the exemplification or *extensive definition*; and that this would successively determine (ii) comprehension, (iii) denotation, and then—again with a certain range of choice⁴—(iv) connotation.

¹ It will be observed that connotation and exemplification are distinguished from comprehension and denotation in that they are *selective*, while the latter pair are *exhaustive*. In making our selection our aim will usually be to find the minimum list which will suffice for our purpose.

² It is necessary to distinguish between the *known* extension of a term and its full denotation, just as we distinguish between the *known* intension of a term and its full comprehension.

³ He would take metals as different from one another as possible, such as aluminium, antimony, copper, gold, iron, sodium, zinc.

⁴ It is ordinarily said that "of the denotation and connotation of a term one

It is important from a theoretical point of view to note the possibility of this second order of procedure; and it may, moreover, be said to represent what actually occurs—at any rate in the first instance—in certain cases, as, for example, in the case of natural groups in the animal, vegetable, and mineral kingdoms. Men form classes out of vaguely recognised resemblances long before they are able to give an intensive definition of the class-name, and in such a case if they are asked to explain their use of the name, their reply will be to enumerate typical examples of the class. This would no doubt ordinarily be done in an unscientific manner, but it would be possible to work it out scientifically. The extensive definition of a name will take the form: *X is the name of the class of which $Q_1, Q_2, \dots Q_n$ are typical*. This primitive form of definition may also be called *definition by type*¹.

Undoubtedly, however, it is far more usual, as well as really simpler, to start from an intensive definition, and this in nearly every case corresponds with the ultimate procedure of science.

may, both cannot, be arbitrary," and this is broadly true. It is possible, however, to make the statement rather more exact. Given either intensive or extensive definition, then both denotation and comprehension are, with reference to any assigned universe of discourse, absolutely fixed. But different intensive definitions, and also different extensive definitions, may sometimes yield the same results; and it is therefore possible that, everything else being given, connotation or exemplification may still be *within certain limits indeterminate*. For example, given the class of *parallel straight lines*, the connotation may be determined in two or three different ways; or, given the class of *equilateral equiangular triangles*, we may select as connotation either having three equal sides or having three equal angles. Again, given the connotation of *metal*, it would no doubt be possible to select in more ways than one a limited number of metals possessing in common only those attributes which are also possessed by the whole class.

¹ It is not of course meant that when we start from an extensive definition, we are classing things together at random without any guiding principle of selection. No doubt we shall be guided by a resemblance between the objects which we place in the same class, and in this sense intension may be said always to have the priority. But the resemblance may be unanalysed, so that we may be far more familiar with the application of the class-name than with its implication; and even when a connotation has been assigned to the name, it may be extensively controlled, and constantly subject to modification, just because we are much more concerned to keep the denotation fixed than the connotation.

For logical purposes, it is accordingly best to assume this order of procedure, unless an explicit statement is made to the contrary¹.

19. *Inverse Variation of Extension and Intension*².—In general, as intension is increased or diminished, extension is diminished or increased accordingly, and *vice versâ*. If, for example, *rational* is added to the *connotation* of *animal*, the *denotation* is diminished, since all irrational animals are now excluded, whereas they were previously included. On the other hand, if the *denotation* of *animal* is to be extended so as to include the vegetable kingdom, it can only be by omitting *sensitive* from the *connotation*. Hence the following law has been formulated: "In a series of common terms standing to one another in a relation of subordination³ the extension and intension vary inversely." Is this law to be accepted? It must be observed at the outset that the notion of inverse variation is at any rate not to be interpreted in any strict mathematical sense. It is certainly not true that whenever the number of attributes included in the intension is altered in any manner, then the number of individuals included in the extension will be altered in some assigned numerical proportion. If, for example, to the *connotation* of a given name different single attributes are added, the *denotation* will be affected in very different degrees in different cases. Thus, the addition of *resident* to the *connotation* of *member of the Senate of the University of Cambridge* will reduce its *denotation* in a much greater degree than the addition of *non-resident*. There is in short no regular law of variation. The statement must not then be understood to mean more than that any increase or diminution of the intension of a name will necessarily be accompanied by *some*

¹ It is worth noticing that in practice an intensive definition is often followed by an enumeration of typical examples, which, if well selected, may themselves almost amount to an extensive definition. In this case, we may be said to have the two kinds of definition supplementing one another.

² This section may be omitted on a first reading.

³ As in the *Tree of Porphyry*: Substance, Corporeal Substance (Body), Animate Body (Living Being), Sensitive Living Being (Animal), Rational Animal (Man). In this series of terms the intension is at each step increased, and the extension diminished.

diminution or increase of the extension as the case may be, and *vice versa*. We will discuss the alleged law in this form, considering, first, connotation and denotation, exemplification and comprehension; and, secondly, denotation and comprehension.

A. (1) Let connotation be supposed arbitrarily fixed, and used to determine denotation in some assigned universe of discourse. Then it will not be true that connotation and denotation will necessarily vary inversely. For suppose the connotation of a name, *i.e.*, the attributes signified by it, to be *a, b, c*. It may happen that in fact wherever the attributes *a* and *b* are present, the attributes *c* and *d* are also present. In this case, if *c* is dropped from the connotation, or *d* added to it, the denotation of the name will remain unaffected. We have concrete examples of this, if we suppose *equiangularity* added to the connotation of *equilateral triangle*, or *cloven-hoofed* to that of *ruminant*, or *having jaws opening up and down* to that of *vertebrate*, or if we suppose *invalid* dropped from the connotation of *invalid syllogism with undistributed middle*. It is clear, however, that if any alteration in denotation takes place when connotation is altered, it must necessarily be in the opposite direction. Some individuals possessing the attributes *a* and *b* may lack the attributes *c* or *d*; but no individuals possessing the attributes *a, b, c*, or *a, b, c, d* can fail to possess the attributes *a, b*, or *a, b, c*. For example, if to the connotation of *metal* we add *fusible*, it makes no difference to the denotation; but if we add *having great weight*, we exclude potassium, sodium, &c.

The *law of variation of denotation with connotation* may then be stated as follows:—If the connotation of a term is arbitrarily enlarged or restricted, the denotation in an assigned universe of discourse will either remain unaltered or will change in the opposite direction¹.

¹ Since reference is here made to the actual denotation of a term in some more or less restricted universe of discourse, the above law may be said to turn partly on material, and not on purely formal, considerations. It should, therefore, be added that although an alteration in the connotation of a term will not always alter its actual denotation in an assigned universe of discourse, it will always affect potentially its extension. If, for example, the connotation of a term *X* is *a, b, c*, and we add *d*; then the (real or imaginary) class of *X*'s that

(2) Let exemplification be supposed arbitrarily fixed, and used to determine comprehension. It is unnecessary to shew in detail that a corresponding *law of variation of comprehension with exemplification* will hold good, namely:—If the exemplification (extensive definition) of a term is arbitrarily enlarged or restricted, the comprehension in an assigned universe of discourse will either remain unaltered or will change in the opposite direction.

B. We may now consider the relation between the *comprehension* and the *denotation* of a term. Let $P_1, P_2, \dots P_x$ be the totality of attributes possessed by the class X , and $Q_1, Q_2, \dots Q_y$ the totality of objects included in the class X . Both these groups are objectively, not arbitrarily¹, determined; and the relation between them is reciprocal. $P_1, P_2, \dots P_x$ are the only attributes possessed in common by the objects $Q_1, Q_2, \dots Q_y$; and $Q_1, Q_2, \dots Q_y$ are the only objects possessing all the attributes $P_1, P_2, \dots P_x$.

We cannot suppose any direct arbitrary alteration either in comprehension or in denotation. We can, however, establish the following law of inverse variation, namely, that *any arbitrary alteration in either intensive definition or extensive definition which results in an alteration of either denotation or comprehension will also result in an alteration in the opposite direction of the other.*

Let X and Y be two terms which are so related that the definition (either intensive or extensive, as the case may be) of Y includes all that is included in the definition of X and more besides. We have to shew that either the denotations and comprehensions of X and Y will be identical or if the denotation of one includes more than the denotation of the other then its comprehension will include less and *vice versa*.

(a) Let X and Y be determined by connotation or intensive

are not d is necessarily excluded, while it was previously included, in the extension of the term X . Hence, if the connotation of a term is arbitrarily enlarged or restricted, the extension will be *potentially* restricted or enlarged accordingly. Cf. Jevons, *Principles of Science*, ch. xxx. § 13.

¹ What is arbitrary is the intensive definition ($P_1, P_2, \dots P_m$) or the extensive definition ($Q_1, Q_2, \dots Q_n$) which determines them both.

definition. Thus, let X be determined by the set of properties $P_1...P_m$, and Y by the set $P_1...P_{m+1}$, which includes the additional property P_{m+1} .

Then P_{m+1} either does or does not always accompany $P_1...P_m$.

If the former, no object included in the denotation of X is excluded from that of Y , so that the denotations of X and Y are the same; and it follows that the comprehensions of X and Y are also the same.

If the latter, then the denotation of Y is less than that of X by all those objects that possess $P_1...P_m$ without also possessing P_{m+1} . At the same time, the comprehension of Y includes at least P_{m+1} in addition to the properties included in the comprehension of X .

(b) Let X and Y be determined by exemplification or extensive definition. Thus, let X be determined by the set of examples $Q_1...Q_n$, and Y by the set $Q_1...Q_{n+1}$, which includes the additional object Q_{n+1} .

Then Q_{n+1} either does or does not possess all the properties common to $Q_1...Q_n$.

If the former, no property included in the comprehension of X is excluded from that of Y , so that the comprehensions of X and Y are the same; and it follows that the denotations of X and Y are also the same.

If the latter, then the comprehension of Y is less than that of X by all those properties that belong to $Q_1...Q_n$ without also belonging to Q_{n+1} . At the same time, the denotation of Y includes at least Q_{n+1} in addition to the objects included in the denotation of X .

All cases have now been considered, and it has been shewn that the law above formulated holds good universally. This law and the laws given on pages 37 and 38 must together be substituted for the law of inverse relation between extension and intension in its usual form if full precision of statement is desired.

It should be observed that in speaking of variations in comprehension or denotation, no reference is intended to changes in things or in our knowledge of them. The variation is always

supposed to have originated in some arbitrary alteration in the intensive or extensive definition of a given term, or in passing from the consideration of one term to that of another with a different extensive or intensive definition. Thus fresh things may be discovered to belong to a class, and the comprehension of the class-name may not thereby be affected. But in this case the denotation has not itself varied; only our knowledge of it has varied. Or we may discover fresh attributes previously overlooked; in which case similar remarks will apply. Again, new things may be brought into existence coming under the denotation of the name, and still its comprehension may remain the same. Or possibly new qualities may be developed by the whole of the class. In these cases, however, there is no *arbitrary* alteration in the application or implication of the name, and hence no real exception to what has been above laid down.

20. *Formal and Material treatment of Connotation.*—In speaking of the connotation of a name we may have in view either the signification that the name bears in common acceptance, or some special meaning that a given individual may choose to assign to it. It has to be borne in mind that as a matter of fact different people may by the same name intend to signify different things, that is to say, they would include different attributes in the connotation of the name; and not unfrequently some of us may be unable to say precisely what is the meaning that we ourselves attach to the words we use. In formal logic, however, it is necessary to work on the assumption that every name has a fixed and definite connotation. In other words, we assume that every name employed is either used in its ordinary sense and that this is precisely determined, or else that, being used with a special meaning, this meaning is adhered to consistently and without equivocation. Formal logic is indifferent to what particular connotation is attached to any given term; but it prescribes absolute consistency.

Mill in his treatment of connotation goes beyond this. He discusses the principles in accordance with which the connotation of names should be determined¹. This is the treatment of

¹ *Logic*, Book i. chapter 8; Book iv. chapter 4.

the subject proper to material or applied logic¹. When we define names already in use our object is to give them, that which formal logic assumes them to have, a fixed and definite connotation. It may be observed that in the case of an ideal language properly employed every name would have the same fixed and precise meaning for everyone.

EXERCISES.

21. Enquire whether the following names are respectively connotative or non-connotative: *Cæsar, Czar, Lord Beaconsfield, the highest mountain in Europe, Mont Blanc, the Weisshorn, Greenland, the Claimant, the pole star, Homer, a Daniel come to judgment.* [K.]

22. "A proper name at least connotes that the object called by the name is identical with that to which it has previously been assigned." Examine this statement. [J.]

23. P means AB ; and it is found that all A 's are B , but not all B 's are A . Determine the relations between P and A in regard to connotation and denotation, and between the common attributes of A and of B . [J.]

¹ If we are to advance in accurate knowledge, all sources of ambiguity must be cleared out of the way. One of the means, moreover, whereby we effect progress in science is by making our conceptions more clear, and our classifications more appropriate. For these ends we must aim at constructing precise definitions, which, however, we must be prepared to modify from time to time as the occasion arises. Applied logic, therefore, since it is concerned with all the means whereby we make progress in science, must discuss the principles in accordance with which scientific definitions should be constructed and employed.

CHAPTER IV.

REAL, VERBAL, AND FORMAL PROPOSITIONS.

24. *Real (Synthetic), Verbal (Analytic or Synonymous), and Formal Propositions.*—(1) A real proposition is one which gives information of something more than the meaning or application of names; as when a proposition predicates of a connotative subject some attribute not included in its connotation, or when a connotative term is predicated of a non-connotative subject. For example, *all bodies have weight, the angles of any triangle are together equal to three right angles, negative propositions distribute their predicates, Wordsworth is a great poet.*

Real propositions are also described as *synthetic, ampliative, accidental.*

(2) A verbal proposition is one which states only what is implied in the meaning of the terms involved, or which gives information only with regard to the application of names.

Two classes of verbal propositions are to be distinguished, which may be called respectively *analytic* and *synonymous*. In the former the predicate gives a partial or complete analysis of the connotation of the subject; *e.g., bodies are extended, an equilateral triangle is a triangle having three equal sides, a negative proposition has a negative copula*¹. Definitions are included under this division of verbal propositions; and the importance of definitions is so great, that it is clearly erroneous to speak of verbal propositions as being in all cases trivial. In

¹ Since we do not here really advance beyond an analysis of the subject-notion, Dr Bain describes the verbal proposition as the "notion under the guise of the proposition." Hence the appropriateness of treating verbal propositions under the general head of Terms.

general they are trivial only in so far as their true nature is misunderstood; when, for example, people waste time in pretending to prove what has been already assumed in the meaning assigned to the terms employed¹.

Besides propositions giving a more or less complete analysis of the connotation of names, the following—which we may speak of as *synonymous* propositions—are to be included under the head of verbal propositions: (a) where the subject and predicate are both proper names, *e.g.*, *Tully is Cicero*; (b) where they are dictionary synonyms, *e.g.*, *wealth is riches*, *a story is a tale*, *charity is love*. In both these cases information is given only with regard to the application of names.

Analytic propositions are also described as *explicative* and as *essential*. Very nearly the same distinction, therefore, as that between *verbal* and *real* propositions is expressed by the pairs of terms—*analytic* and *synthetic*, *explicative* and *ampliative*, *essential* and *accidental*. These terms do not, however, cover quite the same ground as verbal and real, since they leave out of account *synonymous* propositions, which cannot, for example, be properly described as either analytic or synthetic².

The distinction between real and verbal propositions as above given assumes that the use of terms is fixed by their connotation and that this connotation is determinate³. Whether

¹ By a *verbal dispute* is meant a dispute that turns on the meaning of words. Dr Venn observes that purely verbal disputes are very rare, since “a different usage of words almost necessarily entails different convictions as to facts” (*Empirical Logic*, p. 296). This is true and important; it ought indeed always to be borne in mind that the problem of scientific definition is not a mere question of words, but a question of things. At the same time, disputes which are partly verbal are exceedingly common, and it is also very common for their true character in this respect to be unrecognised. When this is the case, the controversy is more likely than not to be fruitless. The questions whether proper names are connotative and whether every syllogism involves a *petitio principii* may be taken as examples. We certainly go a long way towards the solution of these questions by clearly differentiating between different meanings which may be attached to the terms employed.

² Thus, Mansel calls attention to “a class of propositions which are not, in the strict sense of the word, analytical, *viz.*, those in which the predicate is a single term synonymous with the subject” (Mansel’s *Aldrich*, p. 170).

³ We can, however, adapt the distinction to the case in which the use of terms is fixed by extensive definition. We may say that whilst a proposition

any given proposition is as a matter of fact verbal or real will depend on the meaning which we attach to our terms; and since it is not the function of formal logic to decide this question, this science cannot attempt to determine under which category any given proposition should be placed. Still, while we cannot with certainty distinguish a real proposition by its form, it may be observed that the attachment of a sign of quantity, such as *all*, *every*, *some*, &c., to the subject of a proposition may in general be regarded as an indication that in the view of the person laying down the proposition a fact is being stated and not merely a term explained. Verbal propositions, on the other hand, are usually unquantified or indesignate (see section 41). For example, in order to give a partially correct idea of the meaning of such a name as *square*, we should not say "all squares are four-sided figures," or "every square is a four-sided figure," but "a square is a four-sided figure."

(3) There are propositions usually classed with verbal propositions which should more correctly be placed in a class by themselves, namely, those which are true whatever may be the meaning of the terms involved; *e.g.*, *all A is A*, *No A is not-A*, *All A is either B or not-B*, *If all A is B then no not-B is A*, *If all A is B and all B is C then all A is C*. These may be called formal propositions, since their validity is determined by their bare form¹.

Formal propositions are the only propositions whose truth is

(expressed affirmatively and with a copula of inclusion) is *intensively verbal* when the connotation of the predicate is a part or the whole of the connotation of the subject, it is *extensively verbal* when the subject taken in extension is a part or the whole of the extensive definition of the predicate. Thus, if the use of the term *metal* is fixed by an extensive definition, that is to say, by the enumeration of certain typical metals, of which we may suppose *iron* to be one, then it is a verbal proposition to say that *iron is a metal*. If, however, *tin* is not included amongst the typical metals, then it is a real proposition to say that *tin is a metal*.

¹ Propositions which are in appearance purely formal have sometimes an epigrammatic force and are used for rhetorical purposes, *e.g.*, *A man's a man (for a' that)*. In such cases, however, there is usually an implication which gives the proposition the character of a real proposition; thus, in the above instance the true force of the proposition is that *Every man is as such entitled to respect*.

examined and guaranteed by formal logic itself irrespective of other sources of knowledge. In a sense they seem almost to coincide with the scope of formal logic; for any formally valid reasoning can be expressed by a formal hypothetical proposition as in the last two of the examples given above.

We have then three classes of propositions—*formal*, *verbal*, and *real*—the validity or invalidity of which is determined respectively by their bare form, by the mere meaning or application of the terms involved, by questions of fact concerning the things denoted by these terms¹.

25. *Nature of the Analysis involved in Analytic Propositions*².—Confusion is not unfrequently introduced into discussions relating to analytic propositions by a want of clearness in regard to the nature of the analysis involved. As above identified with a division of the verbal proposition, an analytic proposition gives an analysis, partial or complete, of the *connotation* of the subject-term. Some writers, however, appear to have in view an analysis of the *subjective intension* of the subject-term. There is of course nothing absolutely incorrect in this interpretation, if consistently adhered to, but it makes the distinction between analytic and synthetic propositions logically valueless and for all practical purposes nugatory. "Both intension and extension," says Mr Bradley, "are relative to our knowledge. And the perception of this truth is fatal to a well-known Kantian distinction. A judgment is not fixed as 'synthetic' or 'analytic': its character varies with the knowledge possessed by various persons and at different times. If the

¹ Real propositions are divided into true and false according as they do or do not accurately correspond with facts. By verbal and formal propositions we usually mean propositions which from the point of view taken are valid. A proposition which from either of these points of view is invalid is spoken of as a contradiction in terms. Properly speaking we ought to distinguish between a verbal contradiction in terms and a formal contradiction in terms, the contradiction depending in the first case upon the force of the terms employed and in the second case upon the mere form of the proposition; e.g., *some men are not animals*, *A is not-A*. Any purely formal fallacy may be said to resolve itself into a formal contradiction in terms. It should be added that a mere term, if it is complex, may involve a contradiction in terms; e.g., *Roman Catholic* (if the separate terms are interpreted literally), *A not-A*.

² This section may be omitted on a first reading.

meaning of a word were confined to that attribute or group of attributes with which it set out, we could distinguish those judgments which assert within the whole one part of its contents from those which add an element from outside; and the distinction thus made would remain valid for ever. But in actual practice the meaning itself is enlarged by synthesis. What is added to-day is implied to-morrow. We may even say that a synthetic judgment, so soon as it is made, is at once analytic."¹

If by intension is meant subjective intension, and by an analytic judgment one which analyses the intension of the subject, the above statements are certainly unimpeachable. It is indeed so obviously true that in this sense synthetic judgments are only analytic judgments in the making, that to dwell upon the distinction itself at any length would be only waste of time. It is, however, misleading, to say the least of it, to identify subjective intension with *meaning*"; and this is especially the case in the present connexion, since it may with a certain degree of plausibility be maintained that *some* synthetic judgments are only analytic judgments in the making, even when by an analytic judgment is meant one which analyses the *connotation* of the subject. For undoubtedly the connotation of names is not in practice unalterably fixed. As our knowledge progresses, many of our definitions are modified, and hence a

¹ *Principles of Logic*, p. 172. Professor Veitch expresses himself somewhat similarly. "Logically all judgments are analytic, for judgment is an assertion by the person judging of what he knows of the subject spoken of. To the person addressed, real or imaginary, the judgment may contain a predicate new—a new knowledge. But the person making the judgment speaks analytically, and analytically only; for he sets forth a part of what he knows belongs to the subject spoken of. In fact, it is impossible anyone can judge otherwise. We must judge by our real or supposed knowledge of the thing already in the mind" (*Institutes of Logic*, p. 237).

² Compare the following criticism of Mill's distinction between real and verbal propositions: "If every proposition is merely verbal which asserts something of a thing under a name that already presupposes what is about to be asserted, then every statement by a scientific man is *for him* merely verbal" (T. H. Green, *Works*, vol. ii. p. 233). This criticism seems to lose its force if we clearly grasp the distinction between connotation and subjective intension.

form of words which is synthetic at one period may become analytic at another.

But, in the first place, it is very far indeed from being a universal rule that newly-discovered properties of a class are taken ultimately into the connotation or intensive definition of the class-name. Dr Bain (*Logic, Deduction*, pp. 69 to 73) seems to imply the contrary; but his doctrine on this point is not defensible either on the ground of logical expediency or of what actually occurs. As to logical expediency, it is a generally recognised principle of definition that we ought to aim at including in a definition the minimum number of properties necessary for identification rather than the maximum which it is possible to include¹. And as to what actually occurs, it is easy to find cases where we are able to say with confidence that certain common properties of a class never will as a matter of fact be included in the definition of the class-name; for example, *equiangularity* will never be included in the definition of *equilateral triangle*, or *having cloven hoofs* in the definition of *ruminant animal*.

In the second place, even when freshly discovered properties of things come ultimately to be included in the connotation of their names, the process is at any rate gradual, and it is, therefore, incorrect to say—in the sense in which we are now using the terms—that a synthetic judgment becomes in the very process of its formation analytic. On the other hand, it may reasonably be assumed that in any given discussion the meaning of our terms is fixed, and the distinction between analytic and synthetic propositions then becomes highly significant and important. It may be added that when a name changes its meaning, any proposition in which it occurs does not strictly speaking remain the same proposition as before. We ought

¹ If we include in the definition of a class-name all the common properties of the class, how are we to make any universal statement of fact about the class at all? Given that the property *P* belongs to the whole of the class *S*, then by hypothesis *P* becomes part of the meaning of *S*, and the proposition *all S is P* merely makes this verbal statement, and is no assertion of any matter of fact at all. We are, therefore, involved in a kind of vicious circle.

rather to say that the same form of words now expresses a different proposition¹.

EXERCISES.

26. Enquire whether the following propositions are real or verbal: (a) Homer wrote the *Iliad*, (b) Milton wrote *Paradise Lost*. [c.]

27. If all x is y , and some x is z , and p is the name of those z 's which are x ; is it a verbal proposition to say that all p is y ? [v.]

¹ This point is brought out by Mr Monck in the admirable discussion of the above question contained in his *Introduction to Logic*, pp. 130 to 134.

CHAPTER V.

FURTHER DIVISIONS OF NAMES.

28. *Contradictory Terms*.—A pair of terms are called contradictories if they are so related that between them they exhaust the entire universe to which reference is made, whilst in that universe there is no individual of which both can be at the same time affirmed. The nature of this relation is expressed in the two laws of Contradiction and Excluded Middle:—*Nothing is at the same time both X and not-X; Everything is X or not-X.*

Dr Venn (*Empirical Logic*, p. 191) distinguishes between formal contradictories and material contradictories, according as the relation in which the pair of terms stand to one another is or is not apparent from their mere form. Thus *X* and *not-X* are formal contradictories; so are *human* and *non-human*. Material contradictories, on the other hand, are not constructed “for the express purpose of indicating their mutual relation.” No formal contradiction, for example, is apparent between *British* and *Foreign*, or between *British* and *Alien*; and yet “within their range of appropriate application—which in the latter case includes persons only, and in the former case is extended to produce of most kinds—these two pairs of terms fulfil tolerably well the conditions of mutual exclusion and collective exhaustion.”

In the case of material contradictories, the contradictory relation usually holds good only in a fixed and very restricted universe of discourse. There are, however, some cases, as Dr Venn points out, where the range covered is very wide; e.g., *male* and *female*, *material* and *spiritual*. In the case of formal contradictories, the universe of discourse must be determined

by the context, and may sometimes coincide with the whole universe of nameable things.

Properly speaking, formal logic is concerned with formal contradictories only; at any rate the relation between terms which are formally contradictory is from the logical standpoint by far the most important question discussed in the present connexion. It should be added that in order formally to construct the contradictory of a given term it is best to use the negative prefix "not" or "non." Terms formed with other negative prefixes, *e.g.*, *unpleasant*, *impious*, are often not the true contradictories of the corresponding positive terms. This will be further brought out in the following section.

29. *Contrary Terms*.—The *contrary*¹ of a term is usually defined as the term denoting that which, in some particular universe, is furthest removed from that which is denoted by the original term; *e.g.*, *black* and *white*, *wise* and *foolish*. Contraries differ from contradictories in that they admit of a mean, and therefore do not between them exhaust the entire universe of discourse. It follows that although two contraries cannot both be true of the same thing at the same time, they may both be false. Such pairs of terms as *pleasant* and *unpleasant*, *pious* and *impious*, are—withstanding the negative prefix—contraries, not contradictories; for they admit of a mean. Thus, the word *unpleasant*, as Mill observes, does not connote the mere absence of pleasantness, but a less degree of what is signified by the word *painful*; intermediate between *pleasant* and *unpleasant* may be that which is *indifferent*.

By some writers, the term *contrary* is used in a wider sense than the above, contrariety being identified with simple incompatibility; thus, *blue* and *yellow*, equally with *black*, would in this sense be called *contraries* of *white*². Other writers use the term *repugnant* to express the mere relation of incompatibility; thus *red*, *blue*, *yellow* are in this sense *repugnant* to one another³.

¹ De Morgan uses the terms *contrary* and *contradictory* as equivalent, his definition of them corresponding to that given in the preceding section.

² Compare the discussion of *contrary* propositions in section 55.

³ So long as we are confined to simple terms the relations of *contrariety* and

30. *Positive and Negative Names.*—From the purely formal standpoint the distinction between positive and negative names may be expressed by saying that of two formal contradictories the one which is without the negative prefix is *positive* while that which has the negative prefix is *negative*; e.g., *X* and *not-X* are respectively positive and negative.

The distinction, however, which is ordinarily drawn between positive and negative names is not purely formal. It may be expressed as follows:—a positive name implies the *presence* of certain definite attributes, or, if non-connotative, denotes a particular person or thing—e.g., *man*, *Socrates*; a negative name implies the *absence* of one or other of certain definite attributes, or denotes everything with the exception of some particular person or thing—e.g., *not-man*, *not-Socrates*. If it is asked which of two contradictory names is on this view positive and which negative, the answer is that the distinction lies in the manner in which the denotation of the name is determined. A strictly negative name has its denotation determined indirectly. It denotes an indefinite and unknown class outside a definite and limited class. In other words, we first mark off the class denoted by the positive name, and then the negative name denotes what is left. The fact that its denotation is thus determined is the distinctive characteristic of the negative name¹.

This distinction will usually coincide with the formal distinction first drawn; thus in either sense such terms as *not-man*, *not-white* will be negative. When, however, symbols are used, it is impossible to say which of two such terms as *X* and *not-X* really partakes of the indefinite character above ascribed to negative names, since, for example, there is nothing to

repugnancy cannot be expressed *formally* or in mere symbols. But it is otherwise when we pass on to the consideration of complex terms. Thus, while *X* and *not-X* are formal contradictories, *XY* and *X not-Y* may be said to be formal repugnants, *XY* and *not-X not-Y* formal contraries.

¹ It may be observed that in the case of material contradictories, it is often difficult to say which, if either, should be regarded as negative. For example, in the universe of property, *personal* and *real* may be considered contradictories; but if we are to call one positive and the other negative it is not clear which should be which.

prevent our having originally written *X* for *not-white*, in which case *white* becomes *not-X*, and *X* is the really *negative* term. It is clear, therefore, that our second distinction does not depend on purely formal considerations. We may perhaps avoid confusion between the two distinctions by speaking of terms as *formally negative* or *materially negative* as the case may be.

Mill observes that "names which are positive in form are often negative in reality, and others are really positive though their form is negative." The fact that a positive term may be negative in form results from the circumstance that the negative prefix is sometimes given to the contrary of a term, although the contrary of a positive term is itself as a rule positive also. For instance, to repeat an example already quoted from Mill in a slightly different connexion, "the word *unpleasant*, notwithstanding its negative form, does not connote the mere absence of pleasantness, but a less degree of what is signified by the word *painful*, which, it is hardly necessary to say, is positive." On the other hand, some names positive in form may be, with reference to a limited universe of discourse, negative in force, *e.g., alien, foreign*¹.

From the standpoint of formal logic, however, it is a matter of indifference whether any given term is materially positive or negative. What the formal logician is really concerned with is the relation between contradictory terms. *Not-X* is the contradictory of *X*, and *X* is the contradictory of *not-X*, whichever of the terms may be more strictly the positive and the negative respectively².

¹ The term *Turanian*, as employed in the science of language, is another example. This term is used to denote groups lying outside the Aryan and Semitic groups, but not distinguished by any positive characteristics which they possess in common.

² To the distinction between positive and negative names, Mill adds a class of names called *privative*. "A privative name is equivalent in its signification to a positive and a negative name taken together; being the name of something which has once had a particular attribute, or for some other reason might have been expected to have it, but which has it not. Such is the word *blind*, which is not equivalent to *not seeing*, or to *not capable of seeing*, for it would not, except by a poetical or rhetorical figure, be applied to stocks and stones" (*Logic*, i. p. 44). Perhaps also *idle*, which Mill gives as a negative name, should

31. *Infinite or Indefinite Names.*—*Infinite* and *indefinite* are designations applied to names having a thoroughgoing negative character; to such a name, for example, as *not-white*, understood as denoting the whole infinite or indefinite class of things of which *white* cannot truly be affirmed, including such entities as virtue, a dream, Time, a soliloquy, New Guinea, the Seven Ages of Man.

Some writers hold that no significant term can be really infinite or indefinite in the sense above indicated. They say that if a term like *not-white* is to have any significance at all it must be understood as denoting, not all things whatsoever except white things, but only things that are black, red, green, yellow, &c., that is, all *coloured* things except such as are white. In other words, the universe of discourse which any pair of contradictory terms *X* and *not-X* between them exhaust is considered to be necessarily limited to the proximate genus of which *X* is a species; as, for example, in the case of *white* and *not-white*, the universe of colour.

This view must be regarded as erroneous. It is based on the argument that it is an utterly impossible feat to hold together a chaotic mass of the most different things in any one idea¹. But the answer to this argument is that we do not profess to hold together the things denoted by a negative name by reference to any positive elements which they may have in common; they are held together simply by the fact that they all lack some one or other of certain determinate elements. In other words, the argument only shews that a *negative* name has no *positive* concept corresponding to it².

rather be regarded as privative. It does not mean merely "not-working," but "not-working where there is the capacity to work." Sometimes indeed by a figure of speech we refer to inanimate objects as "lying idle"; but in the strict sense of the term we should hardly speak of a stone as "idle." It cannot be said that the separate recognition of a class of *privative* names as above defined is of logical importance; and it may be added that by some logicians the term *privative* is used as simply equivalent to *negative*.

¹ Compare Lotze, *Logic*, § 40.

² Mrs Ladd Franklin states very well the counter-argument. "The intent of the positive term and of the negative term are extremely different; the one involves a *combination* of quality-elements, the other an *alternation* of absences of quality-elements. When, therefore, Lotze says that it remains a for ever

To say that *not-X* is unmeaning if it is interpreted as embracing everything in the universe except *X*, is to say what is in reality self-contradictory; for in this very statement a meaning is assigned to *not-X* in the sense under discussion. If, moreover, we are unable to denote by *not-X* all things whatsoever except *X*, it is difficult to see in what way we shall be able to denote these things when we have occasion to refer to them¹.

From our present point of view it is important again to call attention to the distinction between such forms as *unholy*, *inhuman*, *discourteous* and such forms as *non-holy*, *non-human*, *non-courteous*. The latter may be used with reference to any universe of discourse which may be selected, however extensive it may be. But not so the former; in their case there is undoubtedly a restriction to some particular universe of discourse which is more or less limited in its range. We can, for example, speak of a *table* as *non-human*, although we cannot speak of it as *inhuman*. A want of clear recognition of this distinction may perhaps be partly responsible for the denial that any terms can properly be described as infinite or indefinite.

32. Relative Names.—A name is *relative*, when, over and above the object which it denotes, it implies in its signification

insoluble task to abstract the qualities of the *not-man*, he says what is true but unimportant. *Not-man* is not destitute of import, as Lotze says it is, but its intent consists in an alternation of deficiencies of some one, at least, of the elements of the intent of man" (*Mind*, January, 1892, pp. 130, 1).

¹ Writers who take the view which we are here criticizing must in consistency deny the universal validity of the process of immediate inference called obversion (cf. section 67). Thus Lotze, rightly on his own view, will not allow us to pass from *spirit is not matter* to *spirit is not-matter*; in fact he rejects altogether the form of judgment *S is not-P* (*Logic*, § 40). Some writers, who follow Lotze on the general question here raised, appear to go a good deal further than he does, not merely disallowing such a proposition as *virtue is not-blue* but also such a proposition as *virtue is not blue*, on the ground that if we say 'virtue is not blue', there is no real predication, since the notion of colour is absolutely foreign to an unextended and abstract concept such as 'virtue'. Lotze, however, expressly draws a distinction between the two forms *S is non-Q* and *S is not Q*, and tells us that "everything which it is wished to secure by the affirmative predicate *non-Q* is secured by the intelligible negation of *Q*". (*Logic*, § 72; cf. § 40). On the more extreme view it is wrong to say that *Virtue is either blue or it is not blue*; but Lotze himself does not thus deny the universality of the law of excluded middle.

another object, to which in explaining its meaning reference must be made. The name of this other object is called the correlative of the first. Non-relative names are sometimes called absolute.

Jevons considers that in certain respects all names are relative. "The fact is that everything must really have relations to something else, the water to the elements of which it is composed, the gas to the coal from which it is manufactured, the tree to the soil in which it is rooted" (*Elementary Lessons in Logic*, p. 26). Again, by the law of relativity, consciousness is possible only under circumstances of change. We cannot think of any object except as distinguished from something else. Every term, therefore, implies its negative as an object of thought. Take the term *man*. It is an ambiguous term, and in many of its meanings is clearly relative—for example, as opposed to master, to officer, to wife. If in any sense it is absolute it is when opposed to not-man; but even in this case it may be said to be relative to not-man. To avoid this difficulty, Jevons remarks, "Logicians have been content to consider as relative terms those only which imply some peculiar and striking kind of relation arising from position in time or space, from connexion of cause and effect, &c.; and it is in this special sense, therefore, that the student must use the distinction."

A more satisfactory solution of the difficulty may be found by calling attention to the distinction already drawn between the point of view of connotation and the subjective and objective points of view respectively. From the subjective point of view all notions are relative by the law of relativity above referred to. Again, from the objective point of view all things, at any rate in the phenomenal world, are relative in the sense that they could not exist without the existence of something else; *e.g.*, man without oxygen, or a tree without soil. But when we say that a *name* is relative, we do not mean that what it denotes cannot exist or be thought about without something else also existing or being thought about; we mean that its signification cannot be explained without reference to something else which is called by a correlative name, *e.g.*, *husband*,

parent. It cannot be said that in this sense all names are *relative*.

The fact or facts constituting the ground of both correlative names is called the *fundamentum relationis*. For example, in the case of partner, the fact of partnership; in the case of husband and wife, the facts which constitute the marriage tie; in the case of shepherd and sheep, the acts of tending and watching which the former exercises over the latter.

Sometimes the relation which each correlative bears to the other is the same; for example, in the case of partner, where the correlative name is the same name over again. Sometimes it is not the same; for example, father and son, slave-owner and slave.

The consideration of relative names is not of importance except in connexion with the logic of relatives, to which further reference will be made subsequently.

33. Simple Terms and Complex Terms.—A *simple term* may be defined as one which is represented by a single symbol; e.g., S, P, Q . The combination of simple terms yields a *complex term*. Complex terms will be discussed in detail in Part IV; but it is desirable to call attention at once to the two fundamental ways in which simple terms may be combined so as to yield complex terms.

In the first place, terms may be *combined conjunctively*. Thus, the complex term PQ is formed by the conjunctive combination of the simple terms P and Q , and it denotes whatever belongs both to the class P and to the class Q .

In the second place, terms may be *combined alternatively* (or, as it is more usually expressed, *disjunctively*). Thus, the complex term P or Q is formed by the alternative combination of the simple terms P and Q , and it denotes whatever belongs either to the class P or to the class Q or to both these classes.

The elements of a complex term formed by conjunctive combination may be spoken of as *determinants*; the elements of a complex term formed by alternative combination as *alternants*. Thus in the term AB , A and B are determinants; in the term A or B , they are alternants.

Complex terms may of course themselves be combined in

the above ways; and hence terms of the second or any higher degree of complexity may involve both conjunctive and alternative combination, e.g., *PQ* or *QR*.

EXERCISES.

34. Give one example of each of the following—(i) a collective general name, (ii) a singular abstract name, (iii) a connotative singular name, (iv) a connotative abstract name. Add reasons justifying your example in each case. [K.]

35. Discuss the logical characteristics of the following names:—*beauty, fault, Mrs Grundy, immortal, nobility, slave, sovereign, the Times, truth, ungenerous*. [K.]

[In discussing the character of any name it is necessary first of all to determine whether it is *univocal*, that is, used in one definite sense only, or *equivocal* (or *ambiguous*), that is, used in more senses than one. In the latter case, its logical characteristics may of course vary according to the sense in which it is used.]

36. It has been maintained that the doctrine of terms is extra-logical. Justify or controvert this position. [J.]

PART II.

PROPOSITIONS.

CHAPTER I.

PROPOSITIONS AND THEIR PRINCIPAL SUBDIVISIONS.

37. *Kinds of Propositions.*—A proposition may be defined as a sentence indicative or declaratory (as distinguished, for example, from sentences imperative or interrogative); in other words, a proposition is a sentence making an affirmation or denial, as—*All S is P, No vicious man is happy. It is the verbal expression of a judgment.*

Kant classified judgments according to four different principles (*Quantity, Quality, Relation, and Modality*) each yielding three subdivisions, as follows:

- | | |
|----------------------|-------------------------------------------------------|
| (1) <i>Quantity.</i> | (i) <i>Singular.....This S is P.</i> |
| | (ii) <i>Particular.....Some S is P.</i> |
| | (iii) <i>Universal.....All S is P.</i> |
| (2) <i>Quality.</i> | (i) <i>Affirmative.....All S is P.</i> |
| | (ii) <i>Negative.....No S is P.</i> |
| | (iii) <i>Infinite.....All S is not-P.</i> |
| (3) <i>Relation.</i> | (i) <i>Categorical.....S is P.</i> |
| | (ii) <i>Hypothetical...If S is P then Q is R.</i> |
| | (iii) <i>Disjunctive.....Either S is P or Q is R.</i> |

- (4) *Modality.* (i) *Problematic ...S may be P.*
 (ii) *Assertoric.....S is P.*
 (iii) *Apodeictic.....S must be P.*

The above arrangement is open to certain criticisms, but from its symmetry it will serve as a useful starting point in our discussion of the various kinds of propositions.

38. *Categorical, Hypothetical, and Disjunctive Propositions.*—The usual division of propositions according to *relation* is into *categorical*, *hypothetical*, and *disjunctive*. A proposition is *categorical* if the affirmation or denial which it contains is absolute, as—*All S is P; Some rich men are not to be envied.* It is *hypothetical* (or *conditional*) if the affirmation or denial is made under a condition, as—*If S is P, Q is R; Where ignorance is bliss, 'tis folly to be wise.* It is *disjunctive* if the affirmation or denial is made with an alternative, as—*Either S is P or Q is R; He is either a knave or a fool*¹.

¹ In lieu of the above threefold division, some logicians commence with a twofold division, the second member of which is again subdivided, the term *hypothetical* being employed sometimes in a wider and sometimes in a narrower sense. To prevent confusion, it may be helpful to give the following table of the usage of one or two modern logicians with regard to this division.

Whately, Mill, and Bain :—

1. Categorical.
2. Hypothetical,
or Compound, { (1) Conditional.
or Complex. { (2) Disjunctive.

Hamilton and Thomson :—

1. Categorical.
2. Conditional. { (1) Hypothetical.
{ (2) Disjunctive.

Fowler (following Boethius) :—

1. Categorical.
2. Conditional { (1) Conjunctive.
or Hypothetical. { (2) Disjunctive.

Mansel gives at once the threefold division :—

1. Categorical.
2. Hypothetical or Conditional.
3. Disjunctive.

A distinction between *conditionals* and *hypotheticals*, differing from all the above, will be suggested later on. The term *alternative* will also be used as synonymous with *disjunctive*. See chapters 8 and 9.

For the present we shall concern ourselves entirely with categorical propositions, deferring the consideration of the import of hypotheticals and disjunctives, and also the question whether the different forms are or are not mutually convertible.

39. *An analysis of the Categorical Proposition.*—A categorical proposition consists of two names (which are respectively the subject and the predicate), united by a copula, and usually preceded by a sign of quantity. It thus contains four elements, two of which—the subject and the predicate—constitute its *matter*, while the remaining two—the copula and the sign of quantity—constitute its *form*¹.

The subject is that name about which affirmation or denial is made. The predicate is that name which is affirmed or denied of the subject.

When propositions are brought into strictly logical form it is desirable that the subject should precede the predicate; but in ordinary discourse this order is sometimes inverted for the sake of literary effect, *e.g.*, in the proposition—*Sweet are the uses of adversity*. The means of discriminating between subject and predicate in doubtful cases will be discussed subsequently.

The sign of quantity attached to the subject indicates the extent to which the individuals denoted by the subject-term are referred to². Thus, in the proposition *All S is P* the sign of

¹ The *logical* analysis of a proposition must be distinguished from its *grammatical* analysis. Grammatically only two elements are recognised, namely, the subject and the predicate. Logically we further analyse the grammatical subject into quantity and logical subject, and the grammatical predicate into copula and logical predicate.

² Miss Jones (*General Logic*, p. 10) distinguishes between *term* and *term-name*. Thus, in the proposition "Some mistakes are irremediable," *some mistakes* is said to be a term, *mistake* a term-name. This usage may sometimes be convenient, but on the whole it seems better to adhere to the traditional use of the word *term* and not include in its signification the sign of quantity attached to the subject of a proposition. This of course necessitates our regarding quantity as a distinct element of a proposition, as in the text. We should accordingly hold that in the propositions *All S is P*, *Some S is P*, the terms are the same, while the quantity of the propositions differs. Miss Jones, on the other hand, would consider the *term-names* the same, but the *terms* themselves different. It may be observed that with this usage we can no longer say that every syllogism contains three and only three terms.

quantity is *all*, and we accordingly understand the affirmation to be made of each and every individual denoted by the term *S*.

The *copula* is the link of connexion between the subject and the predicate, and indicates whether the latter is *affirmed* or *denied* of the former.

The different logical elements of the proposition are by no means always separately expressed in the propositions of ordinary discourse; but by analysis and expansion they may be made to appear without any change of meaning. Some grammatical change of form is, therefore, often necessary before propositions can be dealt with logically. Thus, in such a proposition as "All that love virtue love angling" the copula is not separately expressed. The proposition may, however, be written—

sign of quantity	subject	copula	predicate
<i>All</i>	<i>lovers of virtue</i>	<i>are</i>	<i>lovers of angling;</i>

and in this form the four different logical elements are made distinct. The older logicians distinguished between propositions *secundi adjacentis* and propositions *terti adjacentis*. In the former, the copula and the predicate are not separated, *e.g.*, The man runs, All that love virtue love angling; in the latter, they are made distinct, *e.g.*, The man is running, All lovers of virtue are lovers of angling.

40. *The Quantity and Quality of Propositions.*—Propositions are divided into *universal* and *particular*, according as the predication is made of the whole or of a part of the subject. This division of propositions is said to be according to their *quantity*¹.

Propositions are also divided into *affirmative* and *negative*, according as the predicate is affirmed or denied of the subject. This division of propositions is said to be according to their *quality*².

The combination of these two principles of division yields four fundamental forms of proposition as follows:—

¹ Other ways of dividing propositions according to their quantity, including Kant's threefold division, will be referred to subsequently.

² Kant's threefold division according to quality will be considered in section 47.

(1) the universal affirmative—*All S is P* (or *Every S is P*, or *Any S is P*, or *All S's are P's*)—usually denoted by the symbol A;

(2) the particular affirmative—*Some S is P* (or *Some S's are P's*)—usually denoted by the symbol I;

(3) the universal negative—*No S is P* (or *No S's are P's*)—usually denoted by the symbol E;

(4) the particular negative—*Some S is not P* (or *Not all S is P*, or *Some S's are not P's*, or *Not all S's are P's*)—usually denoted by the symbol O.

These symbols A, I, E, O, are taken from the Latin words *affirmo* and *nego*, the affirmative symbols being the first two vowels of the former, and the negative symbols the two vowels of the latter.

Besides these symbols, it will also be found convenient sometimes to use the following—

$SaP = \text{All } S \text{ is } P;$

$SiP = \text{Some } S \text{ is } P;$

$SeP = \text{No } S \text{ is } P;$

$SoP = \text{Some } S \text{ is not } P.$

The above are useful when it is desired that the symbol which is used to denote the proposition as a whole should also indicate what symbols have been chosen for the subject and the predicate respectively. Thus,

$MaP = \text{All } M \text{ is } P;$

$PoQ = \text{Some } P \text{ is not } Q.$

It will further be found convenient sometimes to denote *not-S* by S' , *not-P* by P' , and so on. Thus we shall have

$S'aP' = \text{All not-}S \text{ is not-}P;$

$PiQ' = \text{Some } P \text{ is not-}Q.$

The universal negative should not be written in the form All S is not P¹; for this form is ambiguous and would usually be interpreted as merely particular, the *not* being taken to qualify the *all*, so that we have *All S is not P* = *Not-all S is P*. Thus, "All that glitters is not gold" is intended for an O

¹ Similar remarks apply to the form *Every S is not P*.

proposition, and is equivalent to "Some things that glitter are not gold."

41. *Indefinite Propositions*.—According to quantity, propositions have sometimes been divided into (1) Universal, (2) Particular, (3) Singular, (4) Indefinite¹. Singular propositions will be discussed in the following section.

By an *indefinite proposition* is meant one "in which the quantity is not explicitly declared by one of the designatory terms *all, every, some, many, &c.*"; e.g., *S is P, Cretans are liars*. We may perhaps say with Hamilton that *indesignate* or *preindesignate* would be a better term to employ. There can be no doubt that, as Mansel remarks, "the true indefinite proposition is in fact the particular, the statement *some A is B* being applicable to an uncertain number of instances, from the whole class down to any portion of it. For this reason particular propositions were called indefinite by Theophrastus" (*Aldrich*, p. 49).

When a proposition is given in the *indesignate* form, we can generally tell from our knowledge of the subject-matter or from the context whether it is meant to be universal or particular. Probably in the majority of cases *indesignate propositions* are intended to be understood as universals, e.g., "Comets are subject to the law of gravitation"; but if we are really in doubt with regard to the quantity of the proposition, it must logically be regarded as particular².

42. *Singular Propositions*.—By a *singular or individual proposition* is meant a proposition in which the affirmation or

¹ This is a further expansion of Kant's threefold division into universal, particular, and singular.

² In the *Port Royal Logic* a distinction is drawn between *metaphysical universality* and *moral universality*. "We call metaphysical universality that which is perfect and without exception; and moral universality that which admits of some exception, since in moral things it is sufficient that things are generally such" (*Port Royal Logic*, Professor Baynes's translation, p. 150). The following are given as examples of moral universals: *All women love to talk; All young people are inconstant; All old people praise past times*. Indesignate propositions may almost without exception be regarded as universals either metaphysical or moral. But it seems clear that moral universals have in reality no valid claim to be called universals at all. Logically they ought not to be treated as more than particulars, or at any rate pluratives (see section 44).

denial is made of a single individual only; for example, *Brutus is an honourable man*; *Much Ado about Nothing is a play of Shakespeare's*; *My boat is on the shore*.

Singular propositions may usually be regarded as forming a sub-class of universals, since in every singular proposition the affirmation or denial is of the whole of the subject¹. Such propositions have, however, certain peculiarities of their own, as will be pointed out subsequently; e.g., they have not like other universal propositions a contrary distinct from their contradictory.

Hamilton distinguishes between universal and singular propositions, the predication being in the former case of a *whole undivided*, and in the latter case of a *unit indivisible*. The distinction here indicated is sometimes useful; but it can with advantage be expressed somewhat differently. A singular proposition may generally without risk of confusion be denoted by one of the symbols **A** or **E**; and in syllogistic inferences, a singular may ordinarily be treated as equivalent to a universal proposition. The use of independent symbols for singular propositions (affirmative and negative) would introduce considerable additional complexity into the treatment of the syllogism; and for this reason it seems desirable as a rule to include singulars under universals. Universal propositions may, however, be divided into *general*² and *singular*, and there will then be terms

¹ It is argued by Father Clarke that singulars ought to be included under particulars, on the ground that when a predicate is asserted of one member only of a class, it is asserted of a portion only of the class. "Now if I say, *This Hottentot is a great rascal*, my assertion has reference to a smaller portion of the Hottentot nation than the proposition *Some Hottentots are great rascals*. The same is the case even if the subject be a proper name. *London is a large city* must necessarily be a more restricted proposition than *Some cities are large cities*; and if the latter should be reckoned under particulars, much more the former" (*Logic*, p. 274). This view fails to recognise that what is really characteristic of the particular proposition is not its restricted character—for, as we shall find, the particular is not inconsistent with the universal—but its *indefinite character*.

² Lotze (*Logic*, § 68) distinguishes between *general* and *universal* judgments. In the former the predication is of the whole of an indefinite class, including both examined and unexamined cases. In the latter we have merely a summation of what is found to be true in every individual instance of the subject. "The universal judgment is only a collection of many singular judg-

whereby to call attention to the distinction wherever it may be necessary or useful to do so.

There is also a certain class of propositions which, while *singular* inasmuch as they relate but to a single individual, possess also the indefinite character which belongs to the *particular* proposition: for example, *A certain man had two sons; A great statesman was present; An English officer was killed.* Having two such propositions in the same discourse we cannot, apart from the context, be sure that the same individual is referred to in both cases. Carrying the distinction indicated in the preceding paragraph a little further, we have a fourfold division of propositions:—*general definite*, “All *S* is *P*”; *general indefinite*, “Some *S* is *P*”; *singular definite*, “This *S* is *P*”; *singular indefinite*, “A certain *S* is *P*.” This classification admits of our working with the ordinary twofold distinction into universal and particular—or, as it is here expressed, definite and indefinite—wherever this is adequate, as in the traditional doctrine of the syllogism; while at the same time it introduces a further distinction which may in certain connexions be of importance.

43. *Multiple Quantification.*—The application of a predicate to a subject is sometimes limited with reference to times or conditions, and this may be treated as yielding a *secondary* quantification of the proposition; for example, *All men are sometimes unhappy, In some countries all foreigners are unpopular.* This differentiation may be carried further so as to

ments, the sum of whose subjects does as a matter of fact fill up the whole extent of the universal concept;.....the universal proposition, ‘all men are mortal,’ leaves it still an open question whether, strictly speaking, they *might* not all live for ever, and whether it is not merely a remarkable concatenation of circumstances, different in every different case, which finally results in the fact that no one remains alive. The general judgment on the other hand, ‘man is mortal,’ asserts by its form that it lies in the character of mankind that mortality is inseparable from every one who partakes in it.” In applied logic the distinction here indicated may be of importance; a somewhat similar distinction is indicated by Mill in his treatment of “inductions improperly so-called.” But it cannot be regarded as a *formal* distinction; it depends not so much on the propositions themselves as on the manner in which they are obtained. There is no sufficient justification for Lotze’s implication that propositions of the form *all S is P* are always in his sense universal, while those of the form *S is P* are always in his sense general.

yield triple or any higher order of quantification. Thus, we have triple quantification in the proposition, *In all countries all foreigners are sometimes unpopular.*

In this way a proposition with a singular term for subject may, with reference to some secondary quantification, be classified as universal or particular as the case may be; for example, *Gladstone is always eloquent, Browning is sometimes obscure*¹.

44. *Signs of Quantity.*—A brief discussion is necessary in regard to the precise signification to be attached to the various signs of quantity which may be recognised by the logician.

Some is always understood to be exclusive of *none*, but in its relation to *all* there is ambiguity, for it is sometimes interpreted as excluding *all* as well as *none*, but sometimes it is not regarded as carrying this further implication. The word may, therefore, be defined in two conflicting senses: first, as equivalent simply to one at least, that is, as the pure contradictory of *none*, and therefore as covering every case (including *all*) which is inconsistent with *none*; secondly, as any quantity intermediate between none and all, carrying with it, therefore, the implication *not all* as well as *not none*. In ordinary speech the latter of these two meanings is the more usual². This may, however, be

¹ For a further development of the notion of multiple quantification see Mr Johnson's articles on *the Logical Calculus in Mind*, 1892.

² We might indeed go further and say that in ordinary speech *some* usually means *considerably less than all*, so that it becomes still more limited in its signification. In common language, as is remarked by De Morgan, "*some* usually means a rather small fraction of the whole; a larger fraction would be expressed by *a good many*; and somewhat more than half by *most*; while a still larger proportion would be *a great majority* or *nearly all*" (*Formal Logic*, p. 58). It may be added that in regarding *some* as implying no more than *at least one*, we are in another way departing from the ordinary usage of language, which would generally regard it as implying *more than one*, that is, *at least two*. On this point, compare Mansel's *Aldrich*, p. 59, and Venn's *Empirical Logic*, pp. 222, 3. It should perhaps be added that on rare occasions *some* may in ordinary speech carry with it a still more definite implication. For example, the proposition "Some truth is better kept to oneself" may be so emphasized as to make it perfectly clear to what particular kind of truth reference is made. This is, however, extra-logical. Logically the proposition must be treated as particular, or it must be written in another form, "All truth of a certain specified kind is better kept to oneself." Thus, Spalding remarks, "The logical 'some' is totally indeterminate in its reference to the constitutive

regarded as one of the implications or suggestions of ordinary speech that logic cannot recognise; and accordingly with most modern logicians the logical interpretation of *some* is limited to *one at least*. Using the word in this sense, if we want to express *Some, but not all, S is P*, we must make use of two propositions—*Some S is P, Some S is not P*. The particular proposition as thus interpreted is, as already suggested, *indefinite*, though with a certain limit; that is, it is indefinite in so far that it may apply to any number from a single one up to all, but on the other hand it is definite in so far as it excludes *none*¹. We shall henceforth interpret *some* in this indefinite sense unless an explicit indication is given to the contrary.

All is ambiguous, inasmuch as it may be used either distributively or collectively. In the proposition *All the angles of a triangle are less than two right angles* it is used distributively, the predicate applying to each and every angle of a triangle taken separately. In the proposition *All the angles of a triangle are equal to two right angles* it is used collectively, the predicate applying to all the angles taken together, and not to each separately. This ambiguity attaches to the symbolic form *All S is P*, but not to the form *All S's are P's*. Ambiguity may also be avoided by using *every*, instead of *all*, as our sign of quantity. In general, *all* is to be interpreted distributively, unless by the context or in some other way an indication is given to the contrary.

Any, as the sign of quantity of a categorical proposition, *e.g.*, *Any S is P*, introduces a universal statement; for *P* is affirmed indiscriminately of *S*, whatever particular *S* we may happen to have selected, and it is therefore practically affirmed of the whole of the subject. Hence in such a proposition *any* is, as a rule², equivalent to *all* in its distributive sense. When not the

objects. It is always *aliqui*, never *quidam*; it designates some objects or other of the class, not some certain objects definitely pointed out" (*Logic*, p. 63).

¹ It will hardly do to define *some* as an *indefinite quantity or number* without further explanation, since this would include *none* as a limiting case.

² This qualification is introduced because, as Miss Jones points out, a proposition commencing with *any* may sometimes be in effect a singular proposition, and, when this is so, *any* and *all* are no longer interchangeable. "*Any* may occur as subject indicator in a proposition in which, by the signi-

subject of a categorical proposition, however, *any* may have a different signification. For example, in the hypothetical proposition *If any A is B, C is D*, it has the same indefinite character which we logically ascribe to *some*¹; since the antecedent condition is satisfied if a single *A* is *B*. The proposition might indeed be written—*If one or more A is B, C is D*.

Propositions of the forms *Most S's are P's*, *Few S's are P's*, are called plurative propositions. *Most* may be logically interpreted as equivalent to *at least one more than half*². *Few* has a negative force; and *Few S's are P's* may be regarded as equivalent to *Most S's are not P's*³. Formal logicians (excepting

fication of *S* or *P*, the application of the subject is restricted to one individual; e.g., 'Any one who wins this race will have a silver cup' (General Logic, p. 70). An affirmation is still made of the whole of the subject, but the subject consists of a single individual only. Miss Jones adds as another illustration the proposition "Any one may have my ticket"; this proposition, however, seems really general, so far as the potentiality of having the ticket is concerned.

¹ It appears to have this meaning (a) in the principal clause of an interrogative sentence, e.g., Are any subscribers dissatisfied because some non-subscribers were admitted? (b) in the subordinate clause of a negative sentence, e.g., Some people do not think that any men are perfect; (c) in the antecedent clause of a pure hypothetical, e.g., If any men are perfect, some men are mistaken. This does not, however, apply to the antecedent of a conditional, in the sense in which conditionals are distinguished from hypotheticals in chapter 8; e.g., the proposition "If any flower is scarlet, it is scentless" is equivalent to the proposition "All scarlet flowers are scentless."

² We are here somewhat departing from popular usage just as when we interpret *some* as equivalent to *at least one more than none*. No doubt a larger excess over *half* and *none*, as the case may be, is usually contemplated when the words *most* and *some* are used in ordinary discourse.

³ With perhaps the further implication "although *some S's* are *P's*"; thus, *Few S's are P's* is given by Kant as an example of the *exponible* proposition, on the ground that it contains both an affirmation and a negation, though one of them in a concealed way. The proposition *Few S's are P's* may also be interpreted in a slightly different way, as meaning that an absolutely small number of *S's* are *P's*. It can then be written in the form, *The number of S's which are P's is small*. If, however, we recognise *few* as a logical sign of quantity, it is necessary to give it a fixed interpretation, and on the whole that indicated in the text seems the most satisfactory that can be adopted. It should be added that *a few* has not the same signification as *few*, but must be regarded as affirmative, and, generally, as simply equivalent to *some*; e.g., *A few S's are P's* = *Some S's are P's*. Sometimes, however, it means a small number, and in this case the proposition is perhaps best regarded as singular, the subject being collective. Thus, "a few peasants successfully

De Morgan and Hamilton) have not as a rule recognised these additional signs of quantity; and it is true that in many logical combinations they cannot be regarded as yielding more than particular propositions, *Most S's are P's* being reduced to *Some S's are P's*, and *Few S's are P's* to *Some S's are not P's*. Sometimes, however, we are able to make use of the extra knowledge given us; e.g., from *Most M's are P's*, *Most M's are S's*, we can infer *Some S's are P's*, although from *Some M's are P's*, *Some M's are S's*, we can infer nothing.

Numerically definite propositions are those in which a predication is made of some definite proportion of a class; e.g., *Two-thirds of S are P*. A certain ambiguity may lurk in numerically definite propositions; e.g., in the above proposition is it meant that *exactly two-thirds of S neither more nor less are P*, so that we are also given implicitly *one-third of S are not P*, or is it merely meant that *at least two-thirds of S but perhaps more are P*? In ordinary discourse we should no doubt mean sometimes the one and sometimes the other. If we are to fix our interpretation, it will probably be best to adopt the first alternative, on the ground that if figures are introduced at all we should aim at being quite determinate¹. But some such words as *at least* can of course be used when it is not professed to state more than the minimum proportion of *S's* that are *P's*.

45. *The Distribution of Terms in a Proposition.*—A term is said to be distributed when reference is made to all the individuals denoted by it; it is said to be undistributed when they are only referred to *partially*, i.e., when information is given

defended the citadel" may be rendered "a small band of peasants successfully defended the citadel," rather than "some peasants successfully defended the citadel," since the stress is intended to be laid at least as much on the paucity of their numbers as on the fact that they were peasants. Whilst the proposition interpreted in this way is singular, not general, it is *singular indefinite*, not singular definite; for what small band is alluded to is left indeterminate.

¹ De Morgan remarks that "a perfectly definite particular, as to quantity, would express how many X's are in existence, how many Y's, and how many of the X's are or are not Y's: as in *70 of the 100 X's are among the 200 Y's*" (*Formal Logic*, p. 58). He contrasts the definite particular with the indefinite particular which is of the form *Some X's are Y's*. It will be noticed that De Morgan's definite particular, as here defined, is still more explicit than the numerically definite proposition, as defined in the text.

with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class. It follows immediately from this definition that the subject is distributed in a universal, and undistributed in a particular¹, proposition. It can further be shewn that the predicate is distributed in a negative, and undistributed in an affirmative proposition. Thus, if I say *All S is P*, I identify every member of the class *S* with some member of the class *P*, and I therefore imply that at any rate *some P is S*, but I make no implication with regard to the whole of *P*. It is left an open question whether there is or is not any *P* outside the class *S*. Similarly if I say *Some S is P*. But if I say *No S is P*, in excluding the whole of *S* from *P*, I am also excluding the whole of *P* from *S*, and therefore *P* as well as *S* is distributed. Again, if I say *Some S is not P*, although I make an assertion with regard to a part only of *S*, I exclude this part from the whole of *P*, and therefore the whole of *P* from it. In this case, then, the predicate is distributed, although the subject is not².

Summing up our results we find that

A distributes its subject only,

I distributes neither its subject nor its predicate,

E distributes both its subject and its predicate,

O distributes its predicate only.

46. *The Distinction between the Subject and the Predicate of a Proposition.*—The nature of the distinction ordinarily drawn between the subject and the predicate of a proposition may be expressed by saying that the subject is that of which something is affirmed or denied, the predicate that which is affirmed or

¹ *Some* being used in the sense of *some*, it may be *all*. If by *some* we understand *some, but not all*, then we are not really left in ignorance with regard to the remainder of the class which forms the subject of our proposition.

² Hence we may say that the quantity of a proposition, so far as its predicate is concerned, is determined by its quality. The above results, however, no longer hold good if we explicitly quantify the predicate as in Hamilton's doctrine of the Quantification of the Predicate. According to this doctrine, the predicate of an affirmative proposition is sometimes expressly distributed, while the predicate of a negative proposition is sometimes given undistributed. For example, such forms are introduced as *Some S is all P*, *No S is some P*. This doctrine will be discussed in chapter 6.

denied of the subject; or we may say that the subject is that which we regard as the determined or qualified notion, while the predicate is that which we regard as the determining or qualifying notion¹.

It follows that the subject must be given first in idea, since we cannot assert anything, until we have something about which to assert it. Can it, however, be said that because the subject logically comes first in order of thought, it must necessarily do so in order of statement, the subject always preceding the copula, and the predicate always following it? In other words, can we consider the order of the terms in a proposition to suffice as a criterion? If the subject and predicate are pure synonyms² or if the proposition is practically reduced to an equation, as in the doctrine of the quantification of the predicate, it is difficult to see what other criterion can be taken; or it may rather be said that in these cases the distinction between subject and predicate loses all importance. The two are placed on an equality, and nothing is left by which to distinguish them except the order in which they are stated. This view is indicated by Professor Baynes in his *Essay on the New Analytic of Logical Forms*. In such a proposition, for example, as "Great is Diana of the Ephesians," he would call "great" the subject, reading the proposition, however, "(Some) great is (all) Diana of the Ephesians."

But leaving this particular doctrine on one side, it cannot be said that the order of terms is always a sufficient criterion. In the proposition just quoted, "Diana of the Ephesians" would generally be accepted as the subject. What further criterion then can be given? In the case of E and I propositions (propositions, as will be shewn, which can be simply converted) we must appeal to the context or to the question to which the proposition is an answer. If one term clearly conveys informa-

¹ Hence the subject is generally speaking that term which is comparatively unemphatic, whilst the predicate is comparatively emphatic, the point at issue being its applicability to the subject. There may, however, be exceptions to this rule, and sometimes the only emphatic word in a proposition is the sign of quantity.

² For illustrations of this point, and on the general question raised in this section, compare Venn, *Empirical Logic*, pp. 208 to 214.

tion regarding the other term, it is the predicate. It will be shewn also that it is more usual for the subject to be read in extension and the predicate in intension¹. If none of these considerations are decisive, then the order of the terms must suffice. In the case of **A** and **O** propositions (propositions, as will be shewn, which cannot be simply converted) a further criterion may be added. From the rules relating to the distribution of terms in a proposition it follows that in affirmative propositions the distributed term (if either term is distributed) is the subject; whilst in negative propositions, if only one term is distributed, it is the predicate. It is doubtful if the inversion of terms ever occurs in the case of an **O** proposition; but in **A** propositions it is not infrequent. Applying the above to such a proposition as "Workers of miracles were the Apostles," it is clear that the latter term is distributed while the former is not; the latter term is, therefore, the subject. Since a singular term is equivalent to a distributed term, it follows further as a corollary that in an affirmative proposition if one and only one term is singular it is the subject. This decides such a case as "Great is Diana of the Ephesians."

47. *Infinite or Limitative Propositions.*—In place of the ordinary twofold division of propositions in respect of quality, Kant gave a *threefold* division, recognising a class of *infinite* (or *limitative*) judgments, which are neither affirmative nor negative. Thus, *S is P* being affirmative, and *S is not P* negative, *S is not-P* is spoken of as infinite or limitative². Logically, however, the last judgment (which is equivalent to the second in meaning) must be regarded as simply affirmative. As shewn in section 30, it is impossible to say which of the terms *P* or *not-P* is really infinite; and it is, therefore, also impossible to

¹ The subject is often a substantive and the predicate an adjective. Compare section 97.

² An infinite judgment, in the sense in which the term is here used, may be described as the affirmative predication of a negative. Some writers, however, include under *propositiones infinitæ* those whose subject, as well as those whose predicate, is negative. Thus Father Clarke defines *propositiones infinitæ* as propositions in which "the subject or predicate is indefinite in extent, being limited only in its exclusion from some definite class or idea: as, *Not to advance is to recede*" (*Logic*, p. 268).

say which of the propositions *S is P* or *S is not-P* is really infinite or limitative. Hence they must be regarded as belonging to the same type of proposition, and we have to fall back upon the twofold division into affirmative and negative.

48. *Complex Propositions and Compound Propositions.*—A complex proposition may be defined as a proposition which has a complex term either as subject or predicate: for example, *AB is C, S is P or Q, XY is Z or W*. But just as terms may be combined conjunctively or alternatively, so may propositions themselves; and a compound proposition may be defined as one which consists of the conjunctive or alternative combination of other propositions: for example, *P and Q are both true, P is true or Q is true*.

The distinction between complex propositions and compound propositions is commonly overlooked. Thus, it is usual to class simply as disjunctives, (a) propositions which may be regarded as categorical with a disjunctive predicate, e.g., *Every S is P or Q, Every blood vessel is either a vein or an artery*; and (b) propositions which consist in the disjunctive (alternative) combination of two distinct propositions, e.g., *All S is P or some X is not Y, Either free will is a fact or the sense of obligation an illusion*. It will presently be shewn that the distinction is really of fundamental importance in the case both of disjunctives and hypotheticals.

Leaving this point, however, for the present, we may here notice certain classes of complex and compound propositions which have in a somewhat unsystematic way been commonly recognised in logical text-books. The distinctions indicated are of some historical interest, but for the most part they are of little scientific importance.

Exponible Propositions. Some propositions, which are not compound in form, can nevertheless be resolved into a conjunction of two or more simpler propositions which are independent of one another. Propositions which are in this way susceptible of analysis are termed exponible. One of the chief difficulties arising in the logical interpretation of a good many propositions relates to the question whether they are or are not to be regarded as exponible. For example, is the plurative proposition

Most S's are P's to be interpreted as implying not only that the majority of *S's* are *P's* but also that *some S's are not P's*? If the latter view is taken, then the proposition is *exponible*, but not otherwise. Other examples will be given below.

Copulative Propositions. Complex propositions which can be analysed into a conjunction of two or more *affirmative* propositions having the same subject are termed *copulative*. For example, the proposition *All P is QR* is in form complex, not compound, but it is obviously resolvable into the conjunction *All P is Q and all P is R*¹. Copulative propositions fall, therefore, within the class of *exponibles*.

Remotive Propositions. Complex propositions which can be analysed into a conjunction of two or more *negative* propositions having the same subject are termed *remotive*; e.g., *No P is Q or R*, which is resolvable into the conjunction *No P is Q and no P is R*. Remotives, as well as copulatives, fall within the class of *exponibles*.

Exceptive Propositions. Propositions in which the subject is limited by some such word as *unless* or *except* are termed *exceptive*; for example, *All P is Q, unless it happens to be R*. If we interpret a proposition of this type as implying not merely that *every P which is not R is Q*, but also that *every P which is R is not Q*, then exceptives must be regarded as forming another class of *exponibles*. The above proposition may on the same interpretation also be resolved into *All P is either Q or R, but no P is both of these*.

Exclusive Propositions. Propositions which contain some such word as *only* or *alone* whereby the predicate is limited to the subject, for example, *Only S is P*, *S alone is P*, are termed *exclusive*. Propositions of this kind may be written in the form *Some S is all P*; but this is not one of the forms recognised in the traditional scheme as given in section 40². In order to deal with exclusives under the traditional scheme it is necessary to replace them by one of the equivalent forms

¹ The resolution of complex propositions into compound propositions will be considered more systematically in Part iv.

² For a further discussion of propositions of the form *Some S is all P*, see chapter 6.

—*All P is S, No not-S is P.* In making this transformation, however, we have not kept the original subject and predicate; we have in fact performed upon the given proposition a process of immediate inference.

It is convenient to discuss exclusive propositions in the present section, but the view that all exclusives are exponible must be regarded as erroneous¹. When this view is taken, it is not quite clear whether it is intended to resolve such a proposition as *S alone is P* into (a) *All S is P and no not-S is P* or into (b) *Some S is P and no not-S is P*. The first of these alternatives, however, must be rejected on the ground that *S alone is P* does not necessarily² imply that *all S is P*; for example, *Graduates alone are eligible* does not imply that all graduates are so, since other qualifications may also be necessary for eligibility. The second alternative appears to overlook the fact that *Some S is P* is itself an immediate inference from *No not-S is P*, and that hence there is no resolution into two distinct propositions at all. If a proposition were considered exponible simply because it could be resolved into two propositions, whether or not these propositions were independent of one another, then every proposition would be exponible.

Discretive Propositions. Compound propositions in which we express different judgments, denoting that difference by the particles *but, nevertheless*, or the like, expressed or understood, are called discretive; e.g., "Fortune may take away wealth, but it cannot take away virtue" (*Port Royal Logic*, p. 136).

Inceptive Propositions and Desitive Propositions. "When we say that a thing has commenced or ceased to be such, we form two judgments—one what the thing was before the time of which we speak, the other what it is after; and thus these propositions, of which the one class is called inceptives, the other desitives, are compound in sense: e.g., The Jews commenced, after the return from the captivity of Babylon, to disuse their ancient characters, which are those which are now called the

¹ Exclusives and exceptives are sometimes spoken of as interchangeable forms. But in order that this may be the case we must of course interpret both as exponible, or *neither*.

² No doubt it may in ordinary discourse be often so understood.

Samaritan; The Latin language has for five hundred years ceased to be common in Italy" (*Port Royal Logic*, p. 143). It is clear that, as here interpreted, inceptives and desitives belong to the class of exponible.

49. *The Modality of Propositions.*—Different accounts of the modality of propositions are given by different writers. Whately (*Logic*, Book ii. chapter 2, § 1) divides categorical propositions into (a) *pure* and (b) *modal*, according as they assert *simply* that the subject does or does not agree with the predicate, or express in what *mode* or manner it agrees: *Brutus killed Cæsar*, *An intemperate man will be sickly*, are given as examples of pure propositions; *Brutus killed Cæsar justly*, *An intemperate man will probably be sickly*, as examples of modals. A clear distinction ought, however, to be drawn between these examples. In passing from the statement that Brutus killed Cæsar to the statement that Brutus killed Cæsar justly, the addition is obviously a qualification or modification of the predicate, and there is no reason why it should not logically be included in the predicate. Compare the propositions, *Browning is a poet*, *Browning is a great poet*. But the distinction between affirming something as a fact and affirming its probability is a distinction of quite a different kind, and one that does not admit of so summary a treatment. Whately's account of modal propositions may, therefore, be at once rejected, on the ground that it classes together propositions which fundamentally differ from one another in their character.

The Aristotelian doctrine of modals, which was also the scholastic doctrine, gave a fourfold division into (a) *necessary*, (b) *contingent*, (c) *possible*, and (d) *impossible*, according as a proposition expresses (a) that which is necessary and unchangeable, and which cannot therefore be otherwise; or (b) that which happens to be at any given time, but might have been otherwise; or (c) that which is not at any given time, but may be at some other time; or (d) that which cannot be. The point of view here taken is objective, not subjective; that is to say, the distinctions indicated depend upon material considerations, and do not relate to the varying degrees of belief with which different propositions are accepted. The resulting

classification, so far as it can be clearly interpreted in accordance with modern ideas, corresponds broadly with the ordinary fourfold classification of propositions into universal affirmative, particular affirmative, particular negative, and universal negative; and independently of this it seems to have little or no value. On this ground, taken in connexion with its inherent vagueness and obscurity, the scholastic doctrine of modals may now be regarded as obsolete and as having only an historical interest¹.

Kant's doctrine of modality is distinguished from the scholastic doctrine in that the point of view taken is subjective, not objective, according to one of the senses in which Kant uses these terms. Kant divides judgments according to modality into (a) *apodeictic* judgments—*S must be P*, (b) *assertoric* judgments—*S is P*, and (c) *problematic* judgments—*S may be P*; and the distinctions between these three classes have come to be interpreted as depending upon the character of the belief with which the judgments are accepted.

In criticizing this division from the point of view of the logician, Dr Venn urges that whereas the distinction between the problematic judgment and the assertoric concerns the *quantity* of belief with which the judgments are entertained, that between the apodeictic judgment and the assertoric concerns *quality* rather than *quantity* of belief, and this is a distinction of which the logician cannot properly take account. "The belief with which an assertory judgment is entertained is full belief, else it would not differ from the problematic; and

¹ The consideration of modality as above conceived has sometimes been regarded as extra-logical on the ground that necessity, contingency, possibility, and impossibility depend upon matters of fact with which the logician as such has no concern. But it also depends upon matters of fact whether any given predicate can rightly be predicated affirmatively or negatively, universally or particularly, of any given subject. Distinctions of quality and quantity can nevertheless be formally expressed, and if distinctions of modality can also be formally expressed, there is no initial reason why they should not be recognised by the logician, even though he is not competent to determine the validity of any given modal. In so far, however, as the modality of a proposition is something that cannot be formally expressed, so that propositions of the same form may have a different modality, then the argument that the doctrine of modals is extra-logical is perfectly sound.

therefore in regard to the quantity of belief, as distinguished from the quality or character of it, there is no difference between it and the apodeictic" (*Logic of Chance*, p. 313). It is further urged that problematic judgments form a class admitting of all degrees and that they cannot be satisfactorily treated except in connexion with the general theory of probability¹.

On the whole, if the doctrine of modals is to be retained at all in logic it must be by regarding it as having relation to the *grounds* upon which judgments are formed. This view and its consequences are excellently stated by Mr Johnson. "Modality refers to the grounds on which the thinker forms his judgment. It, therefore, expresses a relation between the thinker on the one hand and a certain proposition on the other hand. The real terms, then, of the modal proposition are the thinker and his relation to some judgment which is propounded to him. Thus the proposition *S must be P* asserts (say) that *Any rational being is bound by his rationality* (or, it may be, by his spatial or moral intuitions) *to judge that S is P.* Now the contradictory of a modal proposition such as *S must be P* is always another modal proposition such as *S may be not-P*, which would mean on the above shewing *A rational being is not bound by his rationality* (or by his spatial or moral intuitions, as the case may be) *to judge that S is P.* The modal proposition is, therefore, simply an assertoric on a different plane—concerned with the relations between different sorts of terms. It follows, then, that whereas a modal must always be contradicted by a modal, an assertoric must always be contradicted by an assertoric" (*Mind*, 1892, pp. 18, 19)².

¹ On the whole subject of modality compare Venn's *Logic of Chance*, chapter 13.

² An account of modality, which somewhat resembles the above, but which can be only partially accepted, is given by Mr Bradley, who identifies the assertoric proposition with the categorical, and regards the apodeictic and the problematic as different phases of the hypothetical or conditional, what must be and what may be not being declared as actual facts, but both being inferred on the strength of a condition and subject to a condition. "It is easy to give the general sense in which we use the term *necessity*. A thing is necessary if it is taken not simply in and by itself, but by virtue of something else and because of something else. Necessity carries with it the idea of mediation, of dependency, of inadequacy to maintain an isolated position and to stand and

EXERCISES.

50. Determine the quality of each of the following propositions, and the distribution of its terms: (a) A few distinguished men have had undistinguished sons; (b) Few very distinguished men have had very distinguished sons; (c) Not a few distinguished men have had distinguished sons. [J.]

51. *Everything is either X or Y; X and Y are coextensive; Only X is Y; The class X comprises the class Y and something more.* Express each of these statements by means of ordinary A, I, E, O categorical propositions. [C.]

52. Express each of the following statements in one or more of the strict categorical forms admitted in logic: (i) No one can be rich and happy unless he is also temperate and prudent, and not always then; (ii) No child ever fails to be troublesome if ill taught and spoilt; (iii) It would be equally false to assert that the rich alone are happy, or that they alone are not. [V.]

act alone and self-supported. A thing is not necessary when it simply *is*; it is necessary when it is, or is said to be, *because of* something else" (*Principles of Logic*, p. 183). In a conditional or hypothetical proposition, however, the statement that the consequent follows from the antecedent may be itself merely assertoric. If, for example, we compare the propositions *All spoilt children are troublesome* and *If a child is spoilt then he will be troublesome*, it is clear that the latter is no more apodeictic than the former. Each affirms a matter of fact and the same matter of fact. This point will arise again when we are considering the import of the conditional proposition as contrasted with the categorical. But it seems desirable to make it clear at this stage that there is no justification for considering that there is always a modal distinction between these two kinds of propositions. Possibility is treated by Mr Bradley as a form of hypothetical necessity. When we say that anything is possible we mean that it would exist as fact, if something else were fact. What separates the species *possible* from the genus *necessary* is the implication that part of the antecedent exists but that we are in ignorance as regards the remaining part. "Take a judgment such as this, Given *abcd* then *E* must follow. Add to it the judgment, or the supposition, that *ab* exists, while *cd* is not known to exist, and we get the possible. *E* is now a possibility" (p. 187). But are we not here assuming that *cd* is a possibility? The argument does not seem to be pursued far enough back. A far more consistent and satisfactory account of the problematic judgment is given when it is regarded as the form which the contradictory of an apodeictic judgment will take, as explained in the text.

CHAPTER II.

THE OPPOSITION OF CATEGORICAL PROPOSITIONS¹.

53. *The Square of Opposition.*—Two propositions are technically said to be *opposed* to each other when they have the same subject and predicate respectively, but differ in quantity or quality or both².

Taking the propositions *SaP*, *SiP*, *SeP*, *SoP*, in pairs we find that there are four possible kinds of relation between them.

(1) The pair of propositions may be such that they can neither both be true nor both false. This is called *contradictory opposition*, and subsists between *SaP* and *SoP*, and between *SeP* and *SiP*.

(2) They may be such that whilst both cannot be true, both may be false. This is called *contrary opposition*. *SaP* and *SeP*.

(3) They may be such that they cannot both be false, but may both be true. *Subcontrary opposition*. *SiP* and *SoP*³.

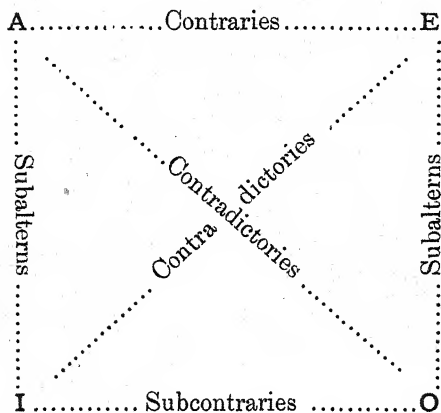
¹ Complications arising in connexion with the implication or non-implication of existence in propositions will for the present be postponed. For a further discussion of the doctrine of opposition, see section 119.

² This definition, according to which opposed propositions are not necessarily incompatible with one another, is given by Aldrich (p. 53 in Mansel's edition). Ueberweg (*Logic*, § 97) defines opposition in such a way as to include only contradiction and contrariety; and Mansel remarks that "subalterns are improperly classed as *opposed* propositions" (*Aldrich*, p. 59). Modern logicians, however, usually adopt Aldrich's definition, and this seems on the whole the best course. Some term is wanted to signify the above general relation between propositions; and though it might be possible to find a more convenient term, no confusion is likely to result from the use of the term *opposition* if the student is careful to notice that it is here employed in a technical sense.

³ Mr Stock writes, "When we say that of two subcontrary propositions, if one be false, the other is true, we are not taking the propositions **I** and **O** in

(4) From a given universal proposition, the truth of the particular having the same quality follows, but not *vice versa*¹. This is *subaltern opposition*, the universal being called the *subalternant*, and the particular the *subalternate* or *subaltern*. *SaP* and *SiP*. *SeP* and *SoP*.

All the above relations are indicated in the ancient square of opposition.



their now accepted logical meaning as indefinite, but rather in their popular sense as 'strictly particular' propositions. For if I and O were taken as indefinite propositions, meaning 'some, if not all,' the truth of I would not exclude the possibility of the truth of A, and similarly the truth of O would not exclude the possibility of the truth of E. Now A and E may both be false. Therefore, I and O, being possibly equivalent to them, may both be false also. In this case the doctrine of contradiction breaks down as well. For I and O may, on this shewing, be false, without their contradictories E and A being thereby rendered true" (*Deductive Logic*, pp. 139, 140). This criticism of the received doctrine of opposition is based on a fallacy. If *some* is interpreted in its strict logical sense, then I may be true along with either A or O, but both these coincidences cannot occur together. In other words, whilst I and O are always formally consistent, we can never infer the one from the other, inasmuch as additional information may at any time shew that they are not as a matter of fact both true. We have this case whenever I is an understatement in the sense that A might have been affirmed in its place. Mr Stock does not appear to recognise that to say that either of two things may occur is not the same thing as to say that they may both occur together. We may add that it is in any case infelicitous to speak of I and O as being possibly equivalent to A and E.

¹ It is perhaps desirable to warn the student at once that this result and some of our other results may need to be modified when we take into account

Propositions must of course be brought to such a form that they have the same subject and the same predicate before the terms of opposition can be directly applied to them; for example, *All S is P* and *Some P is not S* are not contradictories.

We may distinguish between two different points of view from which the doctrine of opposition may be regarded, namely, *first*, as a relation between two *given* propositions; and, *secondly*, as a process of inference by which one proposition being given either as true or as false, the truth or falsity of certain other propositions may be determined. Taking the second of these points of view, we have the following table:—

A being given *true*, E is *false*, I *true*, O *false*;
 E being given *true*, A is *false*, I *false*, O *true*;
 I being given *true*, A is unknown, E *false*, O unknown;
 O being given *true*, A is *false*, E unknown, I unknown;
 A being given *false*, E is unknown, I unknown, O *true*;
 E being given *false*, A is unknown, I *true*, O unknown;
 I being given *false*, A is *false*, E *true*, O *true*;
 O being given *false*, A is *true*, E *false*, I *true*.

The legitimacy of the above inferences may be considered to depend exclusively on the three fundamental laws of thought: namely, the Law of Identity—*A is A*; the Law of Contradiction—*A is not not-A*; the Law of Excluded Middle—*A is either B or not-B*. Thus, from the truth of *All S is P* we may infer the truth of *Some S is P* by the Law of Identity¹, and the falsity of *Some S is not P* by the Law of Contradiction; from the falsity of *All S is P* we may infer the truth of *Some S is not P* by the Law of Excluded Middle; and similarly in other cases.

54. Contradictory Opposition.—The doctrine of opposition given in the preceding section is primarily applicable only to the fourfold schedule of propositions ordinarily recognised; but it is of essential importance to understand clearly the nature of

later on the existential import of propositions. But, as stated in the note at the beginning of the chapter, all complications resulting from considerations of this kind are for the present put on one side.

¹ This of course is on the assumption that the existential import of universals and particulars is the same.

contradictory opposition whatever may be the schedule of propositions with which we are dealing.

To deny the truth of a proposition is equivalent to affirming the truth of its *contradictory*; and *vice versâ*. The criterion of contradictory opposition is that of the two propositions, one must be true and the other must be false; they cannot be true together, but on the other hand no mean is possible between them. The relation between two contradictories is mutual; it does not matter which is given true or false, we know that the other is false or true accordingly. Every proposition has its contradictory, which may however be more or less complicated in form.

The nature of contradictory opposition may be illustrated by asking what are the contradictories of the following propositions—*Few S are P*; *Two-thirds of the army are abroad*; *None but the brave deserve the fair*.

Few S are P is equivalent to *Most S are not P*, and we might hastily be inclined to give as the contradictory *Most S are P*. Both these propositions would, however, be false in the case in which exactly one half *S* was *P*. The true contradictory, therefore, is *At least one half S is P*. This example shews, that if we once travel outside the limits set by the old logic, and recognise the signs of quantity *most* and *few* as well as *all* and *some*, we soon become involved in numerical statements¹. Propositions of the above kind are, therefore, usually relegated to what has been called numerical logic, a topic discussed at length by De Morgan and to some extent by Jevons.

Two-thirds of the army are abroad might mean that at least two-thirds of the army are abroad or that exactly two-thirds of the army are abroad². On the first interpretation, the contradictory is—Less than two-thirds of the army are abroad.

¹ It is obvious that in seeking the contradictory of *Few S are P*, we cannot treat the proposition as simply equivalent to *Some S is not P*; for this would give for the required contradictory *All S is P*, and this and the original proposition might both be false.

² We have already indicated a preference for the latter of these interpretations if numerically definite propositions are to receive a systematic logical treatment; but there can be no doubt that in ordinary discourse the former might happen to be the correct interpretation.

On the second interpretation, it becomes—Not exactly two-thirds of the army are abroad, *i.e.*, Either more or less than two-thirds are abroad.

None but the brave deserve the fair is, again, a proposition that raises a question of interpretation. If this proposition leaves it an open question whether or not *all* who *are* brave are deserving of the fair, then the contradictory is simply *Some who are not brave deserve the fair*. But if the proposition is *exponible* and capable of being analysed into the two distinct statements that all who are brave deserve the fair but none who are not brave deserve them, then a disjunctive proposition is required to express its contradictory, namely, *Either some who are brave do not deserve the fair or some who are not brave do deserve the fair*.

The above examples serve to illustrate the way in which attention is almost inevitably called to any ambiguity in a proposition as soon as we seek to determine its contradictory. It has been truly said that we can never fully understand the meaning of a proposition until we know precisely what it denies; and we shall find later on that the problem of the import of propositions sometimes resolves itself at least partly into the question how propositions of a given form are to be contradicted.

Further light may be thrown upon the nature of contradictory opposition by reference to a discussion entered into by Jevons (*Studies in Deductive Logic*, p. 116) as to the precise meaning of the assertion that a proposition—say, *All grasses are edible*—is false. After raising this question, Jevons begins by giving an answer, which may be called the orthodox one, and which, in spite of what he goes on to say, must also be considered the correct one. When I assert that a proposition is false, I mean that its contradictory is true. The given proposition is of the form **A**, and its contradictory is the corresponding **O** proposition—*Some grasses are not edible*. When, therefore, I say that it is false that all grasses are edible, I mean that some grasses are not edible. Jevons, however, continues, “But it does not seem to have occurred to logicians in general to inquire how far similar relations could be detected

in the case of disjunctive and other more complicated kinds of propositions. Take, for instance, the assertion that 'all endogens are *all* parallel-leaved plants.' If this be false, what is true? Apparently that one or more endogens are not parallel-leaved plants, or else that one or more parallel-leaved plants are not endogens. But it may also happen that no endogen is a parallel-leaved plant at all. There are three alternatives, and the simple falsity of the original does not shew which of the possible contradictories is true."

This statement is open to criticism in two respects. In the first place, in saying that one or more endogens are not parallel-leaved plants, we do not mean to exclude the possibility that no endogen is a parallel-leaved plant at all. Symbolically, *Some S is not P* does not exclude *No S is P*. The three alternatives are, therefore, at any rate reduced to the two first given. But in the second place, Jevons is incorrect in speaking of each of these alternatives as being by itself a contradictory of the original proposition. The true logical contradictory is the affirmation of the truth of *one or other* of these alternatives. If the original proposition is false, we certainly know that the new proposition limiting us to such alternatives is true, and *vice versa*.

The point at issue may be made clearer by taking the proposition in question in a symbolic form. *All S is all P* is a condensed expression, resolvable into the form, *All S is P and all P is S*. It has but one contradictory, namely, *Either some S is not P or some P is not S*¹. If either of these alternatives holds good, the original statement must in its entirety be false; and on the other hand, if the latter is false, one at least of these alternatives must be true. *Some S is not P* is not by itself a contradictory of *All S is all P*. These two propositions are indeed inconsistent with one another; but they may both be false.

It follows that we must reject Jevons's further statement

¹ The contradictory of *All S is all P* may indeed be expressed in a different form, namely, *S and P are not coextensive*, but this has precisely the same force as the contradictory given in the text. It must always be the case that two different forms of the contradictory of the same proposition are equivalent to one another.

that "a proposition of moderate complexity has an almost unlimited number of contradictory propositions, which are more or less in conflict with the original. The truth of any one or more of these contradictories establishes the falsity of the original, but the falsity of the original does not establish the truth of any one or more of its contradictories." No doubt a proposition which is complicated in form may yield an indefinite number of other propositions the truth of any one of which is *inconsistent with* its own. But it has only one logical *contradictory*, which contradictory may be still more complicated in form, affirming a number of alternatives one or other of which must hold good if the original proposition is false¹.

In connexion with the same point, Jevons raises another question, in regard to which his view is also very misleading. He says, "But the question arises whether there is not confusion of ideas in the usual treatment of this ancient doctrine of opposition, and whether a contradictory of a proposition is not any proposition which involves the falsity of the original, but is not the sole condition of it. I apprehend that any assertion is false which is made without sufficient grounds. It is false to assert that the hidden side of the moon is covered with mountains, not because we can prove the contradictory, but because we know that the assertor must have made the assertion without evidence. If a person ignorant of mathematics were to assert that 'all involutes are transcendental curves,' he would be making a false assertion, because, whether they are so or not, he cannot know it." We should, however, involve ourselves in hopeless confusion were we to consider the truth or

¹ No doubt logicians have often used the word *contradict* somewhat loosely. For example, in the *Port Royal Logic*, we find the following: "*Except the wise man (said the Stoics) all men are truly fools.* This may be contradicted (1) by maintaining that the wise man of the Stoics was a fool as well as other men; (2) by maintaining that there were others, besides their wise man, who were not fools; (3) by affirming that the wise man of the Stoics was a fool, and that other men were not" (p. 140). The affirmation of any one of these three propositions certainly renders it necessary to deny the truth of the given proposition, but no one of them is by itself the strict logical contradictory of the given proposition. The true contradictory is the alternative proposition: Either the wise man of the Stoics is a fool or some other men are not fools.

falsity of a proposition to depend upon the knowledge of the person affirming it, so that the same proposition would be now true, now false. Logical problems would be simply insoluble were we not allowed to proceed, for example, from the falsity of *All S is P* to the truth of *Some S is not P*.

55. *Contrary Opposition*.—Generalising the relation between **A** and **E**, we should naturally characterize the contrary of a given proposition by saying that it goes beyond mere denial, and sets up a further assertion as far as possible removed from the original assertion, declaring not merely the falsity of the given proposition taken as a whole, but the falsity of every part of it; so that, whilst the contradictory of a proposition denies its entire truth, its contrary may be said to assert its entire falsehood.

The notion of contrariety, however, as thus defined cannot very satisfactorily be extended beyond the particular case contemplated in the ordinary square of opposition. Taking, for example, such a proposition as "Two-thirds of the army are abroad," and interpreting it to mean that exactly two-thirds of the army are abroad, it would seem that seeking for an assertion as far as possible removed from the original assertion we might proceed in either of two directions and take our choice between saying that all the army are abroad and that none of the army are abroad. It is certainly difficult to see on what principle we ought to select either of these alternatives in preference to the other as being the *diametrical opposite* of the given proposition.

Hence if it is desired to define contrariety so that the conception may be generally applicable, the idea of two propositions standing, as it were, furthest apart from each other must be given up, and any two propositions must be described as contraries if they are inconsistent with one another without at the same time exhausting all possibilities. Contraries must on this definition always admit of a mean, but they may not always be what we should speak of as diametrical opposites, and any given proposition may have not one only, but an indefinite number of contraries. It will be observed, however, that this definition still suffices to identify **A** and **E** as a pair of contraries, and as the only pair in the traditional scheme of opposition.

With either definition of contrariety it follows that if the contrary of a given proposition is true, its contradictory must also be true, but not *vice versa*. Hence in controversy it is under ordinary circumstances better to refute a statement by its contradictory rather than by its contrary. The contradictory is sufficient for disproof, and therefore as effective as the contrary for purposes of refutation; at the same time it commits the controversialist to less, and is not liable—as the contrary may be—to be upset by evidence which is quite inadequate so far as establishing the original proposition is concerned¹.

56. *The Opposition of Singular Propositions*.—Taking the proposition *Socrates is wise*, its contradictory is *Socrates is not wise*²; and so long as we keep to the same terms, we cannot go beyond this simple denial. The proposition has, therefore, no formal contrary³. This opposition of singulars has been called secondary opposition (Mansel's *Aldrich*, p. 56).

If, however, there is secondary quantification in a proposition having a singular subject, then we may obtain the ordinary square of opposition. Thus, if our original proposition is *Socrates is always (or in all respects) wise*, it is contradicted by the statement that Socrates is sometimes (or in some respects) not wise, while it has for its contrary, Socrates is never (or in no respects) wise, and for its subaltern, Socrates is

¹ For this reason, Bain characteristically observes that the contradictory “possesses the imposing circumstance of securing great ends by small means” (*Deductive Logic*, p. 94).

² This must be regarded as the correct contradictory from the point of view reached in the present chapter. But it will be shewn in section 123, that if the original proposition is understood (as it probably would be understood) to imply the existence of Socrates, then a strict application of the criterion of contradiction requires that our contradictory be written—If there is such a man as Socrates, he is unwise.

³ We can obtain what may be called a material contrary of the given proposition by making use of the contrary of the predicate instead of its mere contradictory; thus, Socrates has not a grain of sense. This is spoken of as *material contrariety* because it necessitates the introduction of a fresh term that could not be formally obtained out of the given proposition. It should be added that the distinction between formal and material contrariety might also be applied in the case of general propositions.

sometimes (or in some respects) wise. It may be said that when we thus regard Socrates as having different characteristics at different times or under different conditions, our subject is not strictly singular, since it is no longer a whole indivisible. This is in a sense true, and we might no doubt replace our proposition by one having for its subject "the judgments or the acts of Socrates." But it does not appear that this resolution of the proposition is in any way necessary for its logical treatment.

The possibility of implicit secondary quantification, although no such quantification is explicitly indicated, is a not unfruitful source of fallacy in the employment of propositions having singular subjects. If we take such propositions as *Browning is obscure*, *Epimenides is a liar*, *That flower is blue*, and give as their contradictories *Browning is not obscure*, *Epimenides is not a liar*, *That flower is not blue*, shall we say that the original proposition or its contradictory is true in case Browning is sometimes (but not always) obscure, or in case Epimenides sometimes (but not often) speaks the truth, or in case the flower is partly (but not wholly) blue? There is certainly a considerable risk in such instances as these of confusing contradictory and contrary opposition, and this will be avoided if we make the secondary quantification of the propositions explicit at the outset by writing them in the form *Browning is always* (or *sometimes*) *obscure*, &c.¹ The contradictory will then be particular or universal accordingly.

57. *Possible Relations of Propositions into which the same Terms or their Contradictories enter*².—If we no longer confine ourselves to propositions having the same subject (*S*) and predicate (*P*) respectively, but consider propositions into which two terms or their contradictories (*S*, *not-S*, *P*, *not-P*) enter, either as subjects or predicates, it becomes necessary to amplify

¹ Or we might reduce them to the forms—All (or some) of the poems of Browning are obscure, All (or some) of the statements of Epimenides are false, All (or some) of the surface of the flower is blue.

² The illustrations given in this section presuppose a knowledge of immediate inferences. The section may accordingly on a first reading be postponed until part of the following chapter has been read.

the list of formal relations recognised in the square of opposition, and also to extend the meaning of certain terms. We may give the following classification :—

(1) Two propositions into which only the same terms or their contradictories enter may be *equivalent* or *equipollent*, each proposition being formally inferable from the other. Hence if either one of the propositions is true, the other is also true. For example, as will presently be shewn, *All S is P* and *All not-P is not-S* stand to each other in this relation.

(2) One of the two propositions may be formally *inferable* from the other, but not *vice versâ*. Thus the truth of one of the propositions carries the truth of the other with it, but not conversely. Ordinary subaltern propositions with their subalternants fall into this class; and it will be found in our treatment of immediate inferences that other pairs of propositions fulfilling the above condition are in all cases equivalent to some pair which have the same subject and predicate respectively. The relation between them may accordingly be said to be *implicitly subaltern*, the term *subaltern*, however, when used without qualification, being still applied only as in the ordinary square of opposition. Thus *Some P is S*—which is equivalent to *Some S is P*—is inferable from *All S is P*, but not *vice versâ*. It is, therefore, implicitly subaltern to *All S is P*. Again, *All S is P* and *Some not-S is not P* are implicitly subalternant and subalternate. Here it is not so immediately obvious in what direction we are to look for a pair of equivalent propositions the relation between which is explicitly subaltern. *No not-P is S* and *Some not-P is not S* will, however, be found to satisfy the required conditions.

(3) The propositions may be such that *one or other* of them *must be* true while *both may be* true. A pair of propositions outside the ordinary square of opposition which are thus related—for example, *Some S is P* and *Some not-S is P*—may be said to be *implicitly subcontrary*. It can be shewn that any pair of implicit subcontraries are equivalent to some pair of explicit subcontraries; thus, the above pair are equivalent to *Some P is S* and *Some P is not S*.

(4) The propositions may be such that they can both

be true together, or both false, or either one true and the other false. For example, *All S is P* and *All P is S*. Such propositions may be called *independent* in their relation to one another.

(5) The two propositions may be *contrary* to one another, in the sense that they cannot both be true, but can both be false. We may again distinguish between explicit contraries and implicit contraries; and as before it can be shewn that any pair of implicit contraries are equivalent to some pair of explicit contraries. For example, the implicit contraries *All S is P* and *All not-S is P* are equivalent to the explicit contraries *No not-P is S* and *All not-P is S*.

(6) The two propositions may be *contradictory* to one another according to the definition given in section 54, that is, they can neither both be true nor both false. *All S is P* and *Some not-P is S* afford an example outside the ordinary square of opposition, and may on the same principle as before be called implicit contradictories, an equivalent pair of explicit contradictories being *All S is P* and *Some S is not P*.

Two propositions, then, into which only the same terms or their contradictories enter may, in respect of their mutual relation, be (1) equivalent, (2) explicitly or implicitly subaltern, (3) explicitly or implicitly subcontrary, (4) independent, (5) explicitly or implicitly contrary, (6) explicitly or implicitly contradictory. What pairs of propositions actually fall into these categories respectively will be shewn in sections 72 and 73.

EXERCISES.

58. Explain the nature of the opposition between each pair of the following propositions:—None but Liberals voted against the motion; Amongst those who voted against the motion were some Liberals; It is untrue that those who voted against the motion were all Liberals. [K.]

59. If *some* were used in its ordinary colloquial sense, how would the scheme of opposition between propositions have to be modified? [J.]

60. Give the contradictory of each of the following propositions:—*Some but not all S is P; All S is P and some P is not R; Either all S is P or some P is not R; Wherever the property A is found, either the property B or the property C will be found with it, but not both of them together*¹. [K.]

61. Shew that any term distributed in a general proposition² is undistributed in its contradictory, and *vice versa*. [K.]

¹ For a general discussion of the opposition of complex terms and complex propositions see Part iv.

² Including under generals both universals and particulars, but excluding singulars.

CHAPTER III.

IMMEDIATE INFERENCES¹.

62. *The Conversion of Categorical Propositions.*—By *conversion*, in a broad sense, is meant a change in the position of the terms of a proposition². Logic, however, is concerned with conversion only in so far as the truth of the new proposition obtained by the process is a legitimate inference from the truth of the original proposition. For example, the change from *All S is P* to *All P is S* is not a legitimate logical conversion, since the truth of the latter proposition does not follow from the truth of the former. In other words, logical conversion is a case of immediate inference, which may be defined as the inference of a proposition from a single other proposition³.

¹ In this chapter, except where an explicit statement is made to the contrary, we proceed on the assumption that each class represented by a simple term exists in the universe of discourse, while at the same time it does not exhaust that universe. This assumption appears always to have been made implicitly in the traditional treatment of logic.

² Ueberweg (*Logic*, § 84) defines conversion thus. Compare also De Morgan, *Formal Logic*, p. 58. In geometry, *all equiangular triangles are equilateral* would be regarded as the converse of *all equilateral triangles are equiangular*. In this sense of the term conversion, which is its ordinary non-technical sense, we may say—as we frequently do say—“Yes, such and such a proposition is true; but its converse is not true.”

³ In discussing immediate inferences we “pursue the content of an enunciated judgment into its relations to judgments not yet uttered” (Lotze). Instead of “immediate inferences” Professor Bain prefers to speak of “equivalent propositional forms.” It will be found, however, that the new propositions obtained by immediate inference are not always equivalent to the original propositions, *e.g.*, in conversion *per accidens*. Miss Jones suggests the term *eduction* as a synonym for *immediate inference* (*General Logic*, p. 79); and she then distinguishes between *eversions* and *transversions*, an *eversion* being an

The simplest form of logical conversion, and that which is understood in logic when we speak of conversion without further qualification, may be defined as a *process of immediate inference in which from a given proposition we infer another, having the predicate of the original proposition for subject, and its subject for predicate*. Thus, given a proposition having *S* for its subject and *P* for its predicate, our object in the process of conversion is to obtain by immediate inference a new proposition having *P* for its subject and *S* for its predicate. The original proposition may be called the *convertend*, and the inferred proposition the *converse*¹.

The process will be valid if the two following rules are observed:

(1) The converse must be the same in quality as the convertend (*Rule of Quality*);

(2) No term must be distributed in the converse unless it was distributed in the convertend (*Rule of Distribution*).

Applying these rules to the four fundamental forms of proposition, we have the following table:—

<i>Convertend.</i>	<i>Converse.</i>
All <i>S</i> is <i>P</i> . A.	Some <i>P</i> is <i>S</i> . I.
Some <i>S</i> is <i>P</i> . I.	Some <i>P</i> is <i>S</i> . I.
No <i>S</i> is <i>P</i> . E.	No <i>P</i> is <i>S</i> . E.
Some <i>S</i> is not <i>P</i> . O.	(None.)

It is desirable at this stage briefly to call attention to a point which will receive fuller consideration later on in connexion with the reading of propositions in extension and inten-

duction from categorical form to categorical, or from hypothetical to hypothetical, &c., and a *transversion* an eduction from categorical form to conditional, or from conditional to categorical, &c. For the present we shall be concerned with eversions only.

¹ The process of conversion will be considered in a more generalised form in Part iv.

sion¹; namely, that, generally speaking, in any judgment we have naturally before the mind the objects denoted by the subject, but the qualities connoted by the predicate. In converting a proposition, however, the extensive force of the predicate is made prominent, and an import is given to the predicate similar to that of the subject. In other words, the proposition is taken wholly in extension. It is in passing from the predicative to the class reading (e.g., from *all men are mortal* to *all men are mortals*), that the difficulty sometimes found in correctly converting propositions probably consists. We shall at any rate do well to recognise clearly that conversion and other immediate inferences usually involve a distinct mental act of the above nature.

63. Simple Conversion and Conversion per accidens.—It will be observed that in the case of **I** and **E**, the converse is of exactly the same form as the original proposition; we do not lose any part of the information given us by the convertend, and we can pass back to it by re-conversion of the converse. The convertend and its converse are accordingly equivalent propositions. The conversion in both these cases is said to be simple.

In the case of **A**, it is different; we cannot pass by immediate inference from *All S is P* to *All P is S*, inasmuch as *P* is distributed in the latter of these propositions but undistributed in the former². Hence, although we start with a universal proposition, we obtain by conversion a particular one only³, and by no means of operating upon the converse can we regain the original proposition. The convertend and its converse are accordingly non-equivalent propositions. The con-

¹ See chapter 5.

² *All S is P* and *All P is S* may of course happen to be true together, as in the case of the two propositions *All equilateral triangles are equiangular triangles* and *All equiangular triangles are equilateral triangles*. "But it is only knowledge of the matter of fact contained in the judgment in question which can assure us that the relation, upon which this possibility depends, holds good between *S* and *P* in any particular instance" (Lotze, *Logic*, § 80). When it also happens that *All P is S*, the judgment *All S is P* is sometimes said to be *reciprocal*. If this is to be *formally* expressed in a single judgment, we must make use of the form *All S is all P*.

³ The failure to recognise or to remember that universal affirmative propositions are not simply convertible is one of the most fertile sources of fallacy.

version in this case is called conversion per accidens¹, or conversion by limitation².

For concrete illustrations of the process of conversion, we may take the propositions—A stitch in time saves nine; None but the brave deserve the fair. The first of these may be written in logical form—All stitches in time are things that save nine stitches. This, being an **A** proposition, is only convertible *per accidens*, and we have for our converse—Some things that save nine stitches are stitches in time. The second of the given propositions may be written—No one who is not brave is deserving of the fair. This, being an **E** proposition, may be converted simply, giving, No one deserving of the fair is not brave. Our results may be expressed in a more natural form as follows: One way of saving nine stitches is by a stitch in time; No one deserving of the fair can fail to be brave.

No difficulty ought ever to be found in converting or performing other immediate inferences upon any given proposition when once it has been brought into ordinary logical form, its quantity and quality being determined, its subject, copula, and predicate being definitely distinguished from one another, and its predicate as well as its subject being read in extension. If, however, this rule is neglected, mistakes are pretty sure to follow.

64. Inconvertibility of Particular Negative Propositions.—It follows immediately from the rules of conversion given in section 62 that *Some S is not P* does not admit of ordinary conversion; for *S* which is undistributed in the convertend would become the predicate of a negative proposition in the converse, and would therefore be distributed³. It will be shewn presently,

¹ The conversion of **A** is said by Mansel to be called conversion *per accidens* "because it is not a conversion of the universal *per se*, but by reason of its containing the particular. For the proposition 'Some B is A' is *primarily* the converse of 'Some A is B,' *secondarily* of 'All A is B'" (Mansel's *Aldrich*, p. 61). Professor Baynes seems to deny that this is the correct explanation of the use of the term (*New Analytic of Logical Forms*, p. 29); but however this may be, we certainly need not regard the converse of **A** as necessarily obtained through its subaltern. It is possible to proceed directly from *All A is B* to *Some B is A* without the intervention of *Some A is B*.

² Simple conversion and conversion *per accidens* are also called respectively *conversio pura* and *conversio impura*. Compare Lotze, *Logic*, § 79.

³ As regards the inconvertibility of **O** see also sections 65 and 89.

however, that although we are unable to infer anything about *P* in this case, we are able to draw an inference concerning *not-P*.

Jevons considers that the fact that the particular negative proposition is incapable of ordinary conversion "constitutes a blot in the ancient logic" (*Studies in Deductive Logic*, p. 37). There is, however, no sufficient justification for this criticism. We shall find subsequently that just as much can be inferred from the particular negative as from the particular affirmative (since the latter unlike the former does not admit of contraposition). No logic, symbolic or other, can actually obtain more from the given information than the ancient logic does. It has been suggested that what Jevons means is that the inconvertibility of *O* results in a want of symmetry and that logicians ought specially to aim at symmetry. With this last contention we may heartily agree. The want of symmetry, however, in the case before us is apparent only and results from taking an incomplete view. It will be found that symmetry reappears later on¹.

65. Legitimacy of Conversion.—Aristotle proves the conversion of *E* indirectly, as follows²: *No S is P*, therefore, *No P is S*; for if not, *Some individual P, say Q, is S*; and hence *Q is both S and P*; but this is inconsistent with the original proposition.

Having shewn that the simple conversion of *E* is legitimate, we can prove that the conversion *per accidens* of *A* is also legitimate. *All S is P*, therefore, *Some P is S*; for, if not, *No P is S*, and therefore (by conversion) *No S is P*; but this is inconsistent with the original supposition. The legitimacy of the simple conversion of *I* follows similarly.

It might appear that nothing is required in the above proof beyond the principles of contradiction and excluded middle; the proof is not however satisfactory, for it may be plausibly maintained that in passing from *Some individual P, say Q, is S*, to *Q is both S and P*, we have already practically

¹ See section 71.

² "By the method called *ἐκθεσις*, i.e., by the exhibition of an individual instance." See Mansel's *Aldrich*, pp. 61, 2.

assumed the process of conversion, that is to say, we have assumed that because P is Q , therefore, Q is P ¹.

But, however this may be, it is clear that conversion is capable of being justified without any explicit reference to the above-mentioned principles. For it seems sufficient to say that in the case of each of the four fundamental forms of proposition, its conversion (or in the case of an **O** proposition, the impossibility of converting it) is self-evident. Thus, taking an **E** proposition, it is self-evident that if one class is entirely excluded from another class, this second class is entirely excluded from the first². In the case of an **A** proposition it is clear on reflection that the statement *All S is P* is consistent with either of two relations of the classes S and P , namely, S and P coincident, or P containing S and more besides, and

¹ This I imagine would be the line taken by De Morgan who denies that conversion can be based exclusively on the three fundamental laws of thought. He remarks, "When any writer attempts to shew *how* the perception of convertibility ' A is B gives B is A ' follows from the principles of identity, difference, and excluded middle, I shall be able to judge of the process; as it is, I find that others do not go beyond the simple assertion, and that I myself can detect the *petitio principii* in every one of my own attempts" (*Syllabus*, p. 47). At any rate in basing conversion on the laws of contradiction and excluded middle, recourse must be had to an indirect method of proof; and I do not see how any direct application of the three laws of thought will establish the inconvertibility of an **O** proposition.

² "Aldrich assumes the distribution of the predicate in a negative to prove the simple conversion of **E**. Those who adopt Aristotle's proof of the latter might deduce the former from it. Both however may fairly be allowed to stand on their own evidence" (Mansel's *Aldrich*, p. 52). It is impossible to agree with Professor Bain who would establish the rules of conversion by a kind of inductive proof. He writes as follows :—"When we examine carefully the various processes in Logic, we find them to be material to the very core. Take *Conversion*. How do we know that, if No X is Y , No Y is X ? By examining cases in detail, and finding the equivalence to be true. Obvious as the inference seems on the mere formal ground, we do not content ourselves with the formal aspect. If we did, we should be as likely to say, All X is Y gives All Y is X ; we are prevented from this leap merely by the examination of cases" (*Logic, Deduction*, p. 251). But no one would on reflection maintain it to be self-evident that the simple conversion of **A** is legitimate; for when the case is put to us we recognise immediately that the *contradictory* of *All P is S* is compatible with *All S is P*. On the other hand, no one can deny that in the case of **E** the legitimacy of the process of conversion is self-evident.

further that these are the only two possible relations with which it is consistent. It is self-evident that in each of these cases *Some P is S*; and hence the inference by conversion from an **A** proposition is shewn to be justified¹. In the case of an **O** proposition, if we consider all the relationships of classes in which it holds good, we find that nothing is true of *P* in terms of *S* in *all* of them. Hence **O** is inconvertible². The inconvertibility of **O** can also be established by shewing that *Some S is not P* is compatible with every one of the following propositions—*All P is S*, *Some P is S*, *No P is S*, *Some P is not S*.

66. *Table of Propositions connecting any two terms.*—There are—connecting any two terms *S* and *P*—eight propositions of the forms **A**, **E**, **I**, **O**, namely, four with *S* as subject, and four with *P* as subject. The results at which we have arrived concerning the conversion of propositions shew that of these eight, the two **E** propositions are equivalent to one another, and that the same is true of the two **I** propositions, **E** and **I** being simply convertible; also that these are the only equivalences obtainable. We have, therefore, the following table of propositions connecting any two terms *S* and *P*:—

SaP,
PaS,
SeP = *PeS*,
SiP = *PiS*,
SoP,
PoS.

The pair of propositions *SaP* and *PaS* are independent (see section 57); and the same is true of the pairs *SoP* and *PoS*, *SaP* and *PoS*, *PaS* and *SoP*. The first pair taken together indicate that the classes *S* and *P* are coextensive, and they may be called complementary propositions. The second pair taken together indicate that the classes *S* and *P* are neither coextensive nor either included within the other; they may be called sub-complementary propositions. The third pair taken together

¹ Compare section 89, where this and other similar inferences are illustrated by the aid of the Eulerian diagrams.

² Again, compare section 89.

indicate that the class *S* is included within the class *P* but that it does not exhaust that class; they may be called contra-complementary propositions. The fourth pair taken together indicate that the class *P* is included within the class *S* but that it does not exhaust that class; they are, therefore, also contra-complementary¹.

The above table will be supplemented in section 72 by a table of propositions connecting any two terms and their contradictories, *S*, *P*, *not-S*, *not-P*. It will then be found that we have a complete symmetry which is at present wanting.

67. *The Obversion of Categorical Propositions*².—Obversion is a process of immediate inference in which the inferred proposition (or obverse), whilst retaining the original subject, has for its predicate the contradictory of the predicate of the original proposition (or obvertend). This process is legitimate for a proposition of any form if at the same time the quality of the proposition is changed. The inferred proposition is, moreover, in all cases equivalent to the original proposition, so that we can always pass back from the obverse to the obvertend.

¹ The new technical terms here introduced have been suggested by Mr Johnson. For diagrammatic illustration see p. 181.

² The process of immediate inference discussed in this section has been called by a good many different names. The term obversion, which is used by Professor Bain, is the most convenient. Other names which have been used are permutation (Fowler), equipollence (Ueberweg), infitation (Bowen), immediate inference by privative conception (Jevons), contraversion (De Morgan), contraposition (Spalding). Professor Bain distinguishes between *formal obversion* and *material obversion*. By *formal obversion* is meant the kind of obversion discussed in the above section, and this is the only kind of obversion that can properly be recognised by the formal logician. *Material obversion* is described as the process of making "obverse inferences which are justified only on an examination of the matter of the proposition" (*Logic*, vol. i., p. 111); and the following are given as examples—"Warmth is agreeable; therefore, cold is disagreeable. War is productive of evil; therefore, peace is productive of good. Knowledge is good; therefore, ignorance is bad." It is very doubtful if these are legitimate inferences, formal or otherwise. The conclusions appear to require quite independent investigations to establish them. Apart from this, however, it is a mistake to regard the process as analogous to formal obversion. In the latter, the inferred proposition has the same subject as the original proposition, whilst its quality is different; but neither of these conditions is fulfilled in the above examples. The process is really more akin to the immediate inference presently to be discussed under the name of *inversion*.

We have the following table:—

<i>Obvertend.</i>	<i>Obverse.</i>
All <i>S</i> is <i>P</i> . A.	No <i>S</i> is not- <i>P</i> . E.
Some <i>S</i> is <i>P</i> . I.	Some <i>S</i> is not not- <i>P</i> . O.
No <i>S</i> is <i>P</i> . E.	All <i>S</i> is not- <i>P</i> . A.
Some <i>S</i> is not <i>P</i> . O.	Some <i>S</i> is not- <i>P</i> . I.

It will be observed that the obversion of *All S is P* depends upon the principle of contradiction, which tells us that if anything is *P* then it is not *not-P*; but that we pass back from *No S is not-P* to *All S is P* by the principle of excluded middle, which tells us that if anything is not *not-P* then it is *P*. The remaining inferences by obversion also depend upon one or other of these two principles¹.

68. *The Contraposition of Categorical Propositions*².—Contraposition may be defined as a process of immediate inference in which from a given proposition another proposition is inferred having for its subject the contradictory of the original predicate. Thus, given a proposition having *S* for its subject and *P* for its predicate, we seek to obtain by immediate inference a new proposition having *not-P* for its subject³.

It will be observed that in the above definition it is left an open question whether the contrapositive of a proposition has the original subject or the contradictory of the original subject for its predicate; and every proposition which admits of contraposition will accordingly have two contrapositives, each of

¹ Some writers express themselves as if the validity of obversion rested exclusively upon the principle of excluded middle. But the above will shew that this is not correct.

² This form of immediate inference is called by some logicians *conversion by negation*; Miss Jones suggests the name *contraversion*.

³ For a generalisation of the process of contraposition see Part iv.

which is the obverse of the other. For example, in the case of *All S is P* there will be the two forms *No not-P is S* and *All not-P is not-S*. So far as it is necessary to distinguish these forms we may call that one in which *S* is the predicate the contrapositive, and the one in which *not-S* is the predicate the obverted contrapositive¹. For many purposes, however, the distinction may be practically neglected without risk of confusion.

The following rule may be adopted for obtaining the contrapositive of a given proposition:—Obvert the original proposition and then convert the proposition thus obtained. For given a proposition with *S* for subject and *P* for predicate, obversion will yield an equivalent proposition with *S* for subject and *not-P* for predicate, and the conversion of this will make *not-P* the subject and *S* the predicate.

Applying this rule, we have the following table:—

<i>Original Proposition</i>	<i>Obverse</i>	<i>Contrapositive</i>
All <i>S</i> is <i>P</i> . A.	No <i>S</i> is not- <i>P</i> . E.	No not- <i>P</i> is <i>S</i> . E.
Some <i>S</i> is <i>P</i> . I.	Some <i>S</i> is not not- <i>P</i> . O.	(None.)
No <i>S</i> is <i>P</i> . E.	All <i>S</i> is not- <i>P</i> . A.	Some not- <i>P</i> is <i>S</i> . I.
Some <i>S</i> is not <i>P</i> . O.	Some <i>S</i> is not- <i>P</i> . I.	Some not- <i>P</i> is <i>S</i> . I.

It will be observed that in the case of **A** and **O**, the quantity of the contrapositive is the same as that of the original proposition, whereas in the case of **E** we pass from a universal to a

¹ In previous editions this distinction was explicitly included in the definition of contraposition; but, on the whole, it seems better to give the wider definition. When *not-S* is taken as the predicate of the contrapositive, the quality of the original proposition is preserved, and there is altogether greater symmetry. From certain standpoints this is an important consideration. On the other hand, if we regard contraposition as compounded out of obversion and conversion, the form with *S* as predicate is the more readily obtained. The following is from Mansel's *Aldrich*, p. 61,—“Conversion by contraposition, which is not employed by Aristotle, is given by Boethius in his first book,

particular¹. In order to emphasize this difference, and following the analogy of ordinary conversion, the contraposition of **A** and **O** has been called *simple contraposition*, and that of **E** *contraposition per accidens*².

That **I** has no contrapositive follows from the inconvertibility of **O**. For when *Some S is P* is obverted it becomes a particular negative, and the conversion of this proposition would be necessary in order to render the contraposition of the original proposition possible.

As regards the utility of the investigation as to what contrapositives are logically inferable from given propositions, De Morgan writes as follows:—"The uneducated acquire easy and accurate use of the very simplest cases of transformation of propositions and of syllogisms. The educated, by a higher kind of practice, arrive at equally easy and accurate use of some more complicated cases: but not of all those which are treated in ordinary logic. Euclid may have been ignorant of the identity of 'Every *X* is *Y*' and 'Every not-*Y* is not-*X*,' for anything that appears in his writings: he makes the one follow from the other by a new proof each time" (*Syllabus*, p. 32).

69. *The Inversion of Categorical Propositions.*—In dis-

De Syllogismo Categorico. He is followed by Petrus Hispanus. It should be observed, that the old logicians, following Boethius, maintain that in conversion by contraposition, as well as in the others, the *quality* should remain unchanged. Consequently the converse of 'All *A* is *B*' is 'All not-*B* is not-*A*,' and of 'Some *A* is not *B*,' 'Some not-*B* is not not-*A*.' It is simpler, however, to convert **A** into **E**, and **O** into **I**, ('No not-*B* is *A*,' 'Some not-*B* is *A*'), as is done by Wallis and Abp. Whately; and before Boethius by Apuleius and Capella, who notice the conversion, but do not give it a name. The principle of this conversion may be found in Aristotle, *Top.* II. 8. 1, though he does not employ it for logical purposes."

¹ In most text books, no *definition* of contraposition is given at all, and it may be pointed out that in the attempt to generalise from special examples, Jevons in his *Elementary Lessons in Logic* involves himself in difficulties. For the contrapositive of **A** he gives *All not-P is not-S*; **O** he says has no contrapositive (but only a converse by negation, *Some not-P is S*); and for the contrapositive of **E** he gives *No P is S*. It is impossible to discover any definition of contraposition that can yield these results. Assuming that in contraposition the quality of the proposition is to remain unchanged as in Jevons's contrapositive of **A**, then the contrapositive of both **E** and **O** is *Some not-P is not not-S*.

² Compare Ueberweg, *Logic*, § 90.

cussing conversion and contraposition we have enquired in what cases it is possible, having given a proposition with *S* as subject and *P* as predicate, to infer (a) a proposition with *P* as subject, (b) a proposition with *not-P* as subject. We may now enquire further in what cases it is possible to infer (c) a proposition with *not-S* as subject.

If such a proposition can be inferred at all, it will be by a certain combination of the elementary processes of ordinary conversion and obversion. We will, therefore, take each of the fundamental forms of proposition and see what can be inferred (1) by first converting it, and then performing alternately the operations of obversion and conversion; (2) by first obverting it, and then performing alternately the operations of conversion and obversion. It will be found that in each case the process can be continued until a particular negative proposition is reached whose turn it is to be converted.

(1) The results of performing alternately the processes of conversion and obversion, commencing with the *former*, are as follows:—

(i) All *S* is *P*,

therefore (by conversion), Some *P* is *S*,

therefore (by obversion), Some *P* is not *not-S*.

Here comes the turn for conversion; but as we have to deal with an **O** proposition, we can proceed no further.

(ii) Some *S* is *P*,

therefore (by conversion), Some *P* is *S*,

therefore (by obversion), Some *P* is not *not-S*;

and again we can go no further.

(iii) No *S* is *P*,

therefore (by conversion), No *P* is *S*,

therefore (by obversion), All *P* is not-*S*,

therefore (by conversion), Some *not-S* is *P*,

therefore (by obversion), Some *not-S* is not *not-P*.

In this case either of the propositions in italics is the immediate inference that was sought.

(iv) Some *S* is not *P*.

In this case we are not able even to commence our series of operations.

(2) The results of performing alternately the processes of conversion and obversion, commencing with the *latter*, are as follows:—

(i) All *S* is *P*,

therefore (by obversion), No *S* is not-*P*,

therefore (by conversion), No not-*P* is *S*,

therefore (by obversion), All not-*P* is not-*S*,

therefore (by conversion), *Some not-S is not-P*,

therefore (by obversion), *Some not-S is not P*.

Here again we have obtained the desired form.

(ii) Some *S* is *P*,

therefore (by obversion), Some *S* is not not-*P*.

(iii) No *S* is *P*,

therefore (by obversion), All *S* is not-*P*,

therefore (by conversion), Some not-*P* is *S*,

therefore (by obversion), Some not-*P* is not not-*S*.

(iv) Some *S* is not *P*,

therefore (by obversion), Some *S* is not-*P*,

therefore (by conversion), Some not-*P* is *S*,

therefore (by obversion), Some not-*P* is not not-*S*.

We can now answer the question with which we commenced this enquiry. The required proposition can be obtained only if the given proposition is universal; we then have, according as it is affirmative or negative,—

All S is P, therefore, *Some not-S is not P* (= *Some not-S is not-P*);

No S is P, therefore, *Some not-S is P* (= *Some not-S is not not-P*).

This form of immediate inference has been more or less casually recognised by various logicians; but I do not remember that it has ever received any distinctive name. Sometimes it has been vaguely classed under contraposition (compare Jevons, *Elementary Lessons in Logic*, pp. 185, 6), but it is

really as far removed from the process to which that designation has been given as the latter is from ordinary conversion. The term *inversion* may be suggested. Inversion will accordingly be defined as *a process of immediate inference in which from a given proposition another proposition is inferred having for its subject the contradictory of the original subject*. Thus, given a proposition with *S* for subject and *P* for predicate, we obtain by inversion a new proposition with *not-S* for subject. The original proposition may be called the *invertend*, and the inferred proposition the *inverse*.

In the above definition it is not specified whether the inverse is to have for its predicate *P* or *not-P*. Hence two forms (each the obverse of the other) have been obtained as in the case of contraposition. So far as it is necessary to mark the distinction we may speak of the form in which *P* is the predicate as the inverse, and of that in which *not-P* is the predicate as the obverted inverse¹.

70. *The Validity of Inversion.*—It will be remembered that we are at present working on the assumption that each class represented by a simple term exists in the universe of discourse, while at the same time it does not exhaust that universe; in other words, we assume that *S*, *not-S*, *P*, *not-P*, all represent existing classes. This assumption is perhaps specially important in the case of inversion, and it is connected with certain difficulties that may already have occurred to the reader. In passing from *All S is P* to its inverse *Some not-S is not P* there is an apparent illicit process, which it is far from easy either to account for or explain away. For the term *P*, which is undistributed in the premiss, is distributed in the conclusion, and yet if the universal validity of obversion and conversion is granted, it is impossible to detect any flaw in the argument by which the conclusion is reached. On this ground, Professor Ray rejects the validity of the above inference. "If a term is not distributed in the premiss, it cannot be distributed in the conclusion; that is, if a term is taken in the premiss to mean *at least one* thing denoted by it, it cannot in

¹ In earlier editions this distinction was made explicit in the definition of inversion.

the conclusion be taken to mean *all* things denoted by it. The above conclusion is, therefore, inadmissible. It is obtained from the original premiss by the processes of obversion and conversion; and the fallacy lies not in the process of conversion but in that of obversion, which assumes that the term *P* has a contradictory and is therefore limited in its sphere, although in the original premiss its limitation is not implied and it may cover the whole sphere of thought and existence" (*Deductive Logic*, p. 313). Instead, however, of thus denying altogether the validity of the inference under consideration, it is better to investigate the conditions of its validity. There can be no question that it is valid under the existential assumption upon which we have been proceeding. By the aid of diagrams this can be shewn directly and without the intervention of the processes of obversion and conversion (see pp. 130, 1). We must then deny that (under our present assumption) any illicit process whatever is involved in the inference. In other words, admitting contradictory terms, and assuming that the original terms and their contradictories are all represented in the universe of discourse, it is not correct to say that a term not distributed in the premiss of an immediate inference may never be distributed in the conclusion. For although a term (*P*) may be undistributed relatively to another term (*S*), it may nevertheless be distributed relatively to the contradictory of *S*. When we say *All S is P*, *P* is undistributed relatively to *S*, but implicitly it is at the same time entirely excluded from some portion of *not-S*, and is, therefore, distributed relatively to that portion of *not-S*.

71. *Summary of Results*.—It will now be useful to give a summary of results as regards the immediate inferences which we have been considering. Given two terms *S* and *P*, and admitting their contradictories *not-S* and *not-P*, we have eight possible forms of proposition as in the following scheme:—

	<i>Subject.</i>	<i>Predicate.</i>
(i)	<i>S</i>	<i>P</i>
(ii)	<i>S</i>	not- <i>P</i>
(iii)	<i>P</i>	<i>S</i>
(iv)	<i>P</i>	not- <i>S</i>
(v)	not- <i>P</i>	<i>S</i>
(vi)	not- <i>P</i>	not- <i>S</i>
(vii)	not- <i>S</i>	<i>P</i>
(viii)	not- <i>S</i>	not- <i>P</i>

Taking the first of the above as the original proposition, the others may be designated respectively (ii) the obverse, (iii) the converse, (iv) the obverted converse, (v) the contrapositive, (vi) the obverted contrapositive, (vii) the inverse, (viii) the obverted inverse.

It has been shewn in sections 62, 67, 68, and 69, that if the original proposition is universal, we can infer from it propositions of all the remaining seven forms: but that if it is particular, we can infer only three others.

Working out the different cases in detail we have:—

- A. (i) Original proposition, *All S is P.*
(ii) Obverse, *No S is not-P.*
(iii) Converse, *Some P is S.*
(iv) Obverted Converse, *Some P is not not-S.*
(v) Contrapositive, *No not-P is S.*
(vi) Obverted Contrapositive, *All not-P is not-S.*
(vii) Inverse, *Some not-S is not P.*
(viii) Obverted Inverse, *Some not-S is not-P.*
- I. (i) Original proposition, *Some S is P.*
(ii) Obverse, *Some S is not not-P.*
(iii) Converse, *Some P is S.*
(iv) Obverted Converse, *Some P is not not-S.*

No contrapositive in either of its forms is obtainable, nor any inverse.

- E.** (i) Original proposition, *No S is P.*
(ii) Obverse, *All S is not-P.*
(iii) Converse, *No P is S.*
(iv) Obverted Converse, *All P is not-S.*
(v) Contrapositive, *Some not-P is S.*
(vi) Obverted Contrapositive, *Some not-P is not not-S.*
(vii) Inverse, *Some not-S is P.*
(viii) Obverted Inverse, *Some not-S is not not-P.*
- O.** (i) Original proposition, *Some S is not P.*
(ii) Obverse, *Some S is not-P.*
(v) Contrapositive, *Some not-P is S.*
(vi) Obverted Contrapositive, *Some not-P is not not-S.*

No converse in either of its forms is obtainable, nor any inverse.

All the above is summed up in the following table :—

		A.	I.	E.	O.
i	Original proposition.....	<i>SaP</i>	<i>SiP</i>	<i>SeP</i>	<i>SoP</i>
ii	Obverse	<i>SeP'</i>	<i>SoP'</i>	<i>SaP'</i>	<i>SiP'</i>
iii	Converse	<i>PiS</i>	<i>PiS</i>	<i>PeS</i>	
iv	Obverted Converse	<i>PoS'</i>	<i>PoS'</i>	<i>PaS'</i>	
v	Contrapositive	<i>P'eS</i>		<i>P'iS</i>	<i>P'iS</i>
vi	Obverted Contrapositive ...	<i>P'aS'</i>		<i>P'oS'</i>	<i>P'oS'</i>
vii	Inverse	<i>S'oP</i>		<i>S'iP</i>	
viii	Obverted Inverse	<i>S'iP'</i>		<i>S'oP'</i>	

It may be pointed out that the following rules apply to all the above immediate inferences:—

Rule of Quality.—The total number of negatives admitted or omitted in subject, predicate, or copula must be even.

Rules of Quantity.—If the new subject is *S*, the quantity may remain unchanged; if *S'*, the quantity must be depressed¹; if *P*, the quantity must be depressed in **A** and **O**; if *P'*, the quantity must be depressed in **E** and **I**.

72. *Table of Propositions connecting any two terms and their contradictories.*—Taking any two terms and their contradictories, *S*, *P*, *not-S*, *not-P*, and combining them in pairs, we obtain thirty-two propositions of the forms **A**, **E**, **I**, **O**. The following table, however, shews that only eight of these thirty-two propositions are non-equivalent.

	(i)	(ii)	(iii)	(iv)
	Universals			
A	SaP	$= SeP'$	$= P'eS$	$= P'aS'$
A'	$S'aP'$	$= S'eP$	$= PeS'$	$= PaS$
E	SaP'	$= SeP$	$= PeS$	$= PaS'$
E'	$S'aP$	$= S'eP'$	$= P'eS'$	$= P'aS$
	Particulars			
O	SoP	$= SiP'$	$= P'iS$	$= P'oS'$
O'	$S'oP'$	$= S'iP$	$= PiS'$	$= PoS$
I	SoP'	$= SiP$	$= PiS$	$= PoS'$
I'	$S'oP$	$= S'iP'$	$= P'iS'$	$= P'oS$

In this table, columns (i) and (ii) give the propositions in which *S* or *S'* is subject, and columns (iii) and (iv) the propositions in which *P* or *P'* is subject. Columns (i) and (iv) give the forms which admit of simple contraposition (*i.e.*, **A** and **O**), and columns (ii) and (iii) those which admit of simple conversion (*i.e.*, **E** and **I**). Contradictories are shewn by identical places in the universal and particular rows. We pass from column (i) to column (ii) by obversion; from column (ii) to column (iii) by simple conversion; and from column (iii) to column (iv) by obversion.

¹ In speaking of the quantity as depressed, it is meant that a universal yields a particular, and a particular yields nothing.

The forms in black type shew that we may take for our eight non-equivalent propositions the four propositions connecting S and P , and a similar set connecting $not-S$ and $not-P$ ¹. To establish their non-equivalence we may proceed as follows: SaP and SeP are already known to be non-equivalent, and the same is true of $S'aP'$ and $S'eP'$; but no universal proposition can yield a universal inverse; therefore, no one of these four propositions is equivalent to any other. Again, SiP and SoP are already known to be non-equivalent, and the same is true of $S'iP'$ and $S'oP'$; but no particular proposition has any inverse; therefore, no one of these propositions is equivalent to any other. Finally, no universal proposition can be equivalent to a particular proposition.

73. *Mutual Relations of the non-equivalent propositions connecting any two terms and their contradictories*².—We may now investigate the mutual relations of our eight non-equivalent propositions. SaP , SeP , SiP , SoP form an ordinary square of opposition; and so do $S'aP'$, $S'eP'$, $S'iP'$, $S'oP'$. Reference to columns (iii) and (iv) in the table will shew further that SaP , $S'eP'$, $S'iP'$, SoP are equivalent to another square of opposition; and that the same is true of $S'aP'$, SeP , SiP , $S'oP'$. This leaves only the following pairs unaccounted for: SaP , $S'aP'$; SeP , $S'eP'$; SoP , $S'oP'$; SiP , $S'iP'$; SaP , $S'oP'$; $S'aP'$, SoP ; SeP , $S'iP'$; $S'eP'$, SiP ; and it will be found that in each of these cases we have an independent pair.

SaP and $S'aP'$ (which are equivalent to SaP , PaS , and also to $P'aS'$, $S'aP'$) taken together serve to *identify* the classes S and P , and also the classes S' and P' . They are therefore *complementary* propositions, in accordance with the definition given in section 66. Similarly, SeP and $S'eP'$ (which are equivalent to SaP' , $P'aS$, and also to PaS' , $S'aP$) are complementary; they serve to identify the classes S and P' , and also the classes S' and P . It will be observed that the complementary of any universal proposition may be obtained by replacing the subject and predicate respectively by their

¹ The former set being denoted by A , E , I , O , the latter set may be denoted by A' , E' , I' , O' .

² This section may be omitted on a first reading.

contradictories. No fallacy is more common than the tacit substitution of the complementary of a proposition for the proposition itself.

The complementary relation holds only between universals. Particulars between which there is an analogous relation (the subject and predicate of the one being respectively the contradictories of the subject and predicate of the other) will be found to be sub-complementary in accordance with the definition in section 66; this relation holds between SoP and $S'oP'$, and between SiP and $S'iP'$. SoP and $S'oP'$ (which are equivalent to SoP , PoS , and also to $P'oS'$, $S'oP'$) indicate that the classes S and P are neither coextensive nor either included within the other, and also that the same is true of S' and P' ; SiP and $S'iP'$ (which are equivalent to SoP , PoS , and also to PoS' , $S'oP$) indicate the same thing as regards S and P' , S' and P .

The four remaining pairs are contra-complementary, each pair serving conjointly to subordinate a certain class to a certain other class; or, rather, since each such subordination implies a supplementary subordination, we may say that each pair subordinates two classes to two other classes. Thus, SaP and $S'oP'$ (which are equivalent to SaP , PoS , and also to $P'aS'$, $S'oP'$) taken together shew that the class S is contained in but does not exhaust the class P , and also that the class P' is contained in but does not exhaust the class S' ; $S'aP'$ and SoP (which are equivalent to $S'aP'$, $P'oS'$, and also to PaS , SoP) the same as regards the classes S' and P' , and the classes P and S ; SeP and $S'iP'$ (which are equivalent to SaP' , $P'oS$, and also to PaS' , $S'oP$) as regards S and P' , and P and S' ; and $S'eP'$ and SiP (which are equivalent to $S'aP$, PoS' , and also to $P'aS$, SoP) as regards S' and P , P' and S .

Denoting the complementaries of **A** and **E** by **A'** and **E'**, and the sub-complementaries of **I** and **O** by **I'** and **O'**, the various relations between the non-equivalent propositions connecting any two terms and their contradictories may be exhibited in the following octagon of opposition :

the ordinary type we cannot eliminate a negative term (without introducing another in its place) where such a term occurs as subject either (a) in a universal affirmative or a particular negative with a positive term as predicate, or (b) in a universal negative or a particular affirmative with a negative term as predicate also.

The validity of the above results is at once shewn by reference to the table of equivalences given in section 72. At least one proposition in which there is no negative term will be found in each line of equivalences except the fourth and the eighth, which are as follows :

$$\begin{aligned} S'aP &= S'eP' = P'eS' = P'aS; \\ S'oP &= S'iP' = P'iS' = P'oS. \end{aligned}$$

In these cases we may indeed get rid of S' (as, for example, from $S'aP$), but it is only by introducing P' (thus, $S'aP = P'aS$); there is no getting rid of negative terms altogether. We may here refer back to the results obtained in sections 66 and 72; with two terms *six* non-equivalent propositions were obtained, with two terms and their contradictories *eight* non-equivalent propositions. The ground of this difference is now made clear.

If, however, we are allowed to enlarge our scheme of propositions by recognising certain additional types, and if we work on the assumption that universal propositions are existentially negative while particular propositions are existentially affirmative¹, then negative terms may always be eliminated. Thus, *No not-S is not-P* is equivalent to the statement that *Nothing is both not-S and not-P*, and this becomes by obversion *Everything is either S or P*. Again, *Some not-S is not-P* is equivalent to the statement that *Something is both not-S and not-P*, and this becomes by obversion *Something is not either S or P*, or, as this proposition may also be written, *There is something besides S and P*. The elimination of negative terms has now been accomplished in all cases. It will be observed further that we now have *eight* non-equivalent propositions containing only S and P —namely, *All S is P*, *No S is P*, *Some S is P*,

¹ It is necessary here to anticipate the results of a discussion that will come at a later stage. See chapter 7.

Some S is not P, All P is S, Some P is not S, Everything is either S or P, There is something besides S and P.

Following out this line of treatment, the table of equivalences given in section 72 may be rewritten as follows [columns (ii) and (iii) being omitted, and columns (v) and (vi) taking their places]:

(i)	(iv)	(v)	(vi)
$SaP = P'aS' =$	$\text{Nothing is } SP'$	$=$	$\text{Everything is } S' \text{ or } P.$
$S'aP' = PaS =$	$\text{Nothing is } S'P$	$=$	$\text{Everything is } S \text{ or } P'.$
$SaP' = PaS' =$	$\text{Nothing is } SP$	$=$	$\text{Everything is } S' \text{ or } P'.$
$S'aP = P'aS =$	$\text{Nothing is } S'P'$	$=$	$\text{Everything is } S \text{ or } P.$
$SoP = P'oS' =$	$\text{Something is } SP'$	$=$	$\text{There is something besides } S' \text{ and } P.$
$S'oP' = PoS =$	$\text{Something is } S'P$	$=$	$\text{There is something besides } S \text{ and } P'.$
$SoP' = PoS' =$	$\text{Something is } SP$	$=$	$\text{There is something besides } S' \text{ and } P'.$
$S'oP = P'oS =$	$\text{Something is } S'P'$	$=$	$\text{There is something besides } S \text{ and } P.$

Taking the propositions in two divisions of four sets each, the two diagonals from left to right give propositions containing *S* and *P* only¹.

The scheme of propositions given in this section may be brought into interesting relation with the fundamental laws of thought². The scheme is based upon the recognition of the following propositional forms and their contradictories;

Every S is P;
Every not-P is not-S;
Nothing is both S and not-P;
Everything is either P or not-S;

and these four propositions have been shewn to be equivalent to one another.

¹ The first four propositions in column (v) may be expressed symbolically $SP' = 0$, &c.; the second four $SP' > 0$, &c.; the first four in column (vi) $S' + P = 1$, &c.; and the second four $S' + P < 1$, &c.; where 1 = the universe of discourse, and 0 = nonentity, i.e., the contradictory of the universe of discourse.

² Compare Mrs Ladd Franklin in *Mind*, January, 1890, p. 87.

If in the above propositions we now write *S* for *P*, we have the following :

Every S is S ;
Every not-S is not-S ;
Nothing is both S and not-S ;
Everything is either S or not-S.

But the first two of these propositions express the law of identity, with positive and negative terms respectively, the third is an expression of the law of contradiction, and the fourth of the law of excluded middle. The scheme of propositions with which we have been dealing may, therefore, be said to be based upon the recognition of just those propositional forms which are required in order to express the fundamental laws of thought.

Since the propositional forms in question have been shewn to be mutually equivalent to one another, the further argument may suggest itself that if the validity of the immediate inferences involved be granted, then it follows that the fundamental laws of thought have been shewn to be mutually inferable from one another. But it may, on the other hand, be held that this argument is open to the charge of involving a *circulus in probando* on the ground that the validity of the immediate inferences themselves requires that the laws of thought be first postulated as an antecedent condition.

75. *Other Immediate Inferences.*—One or two other commonly recognised forms of immediate inference may be briefly considered.

(1) *Immediate inference by added determinants* is a process of immediate inference which consists in limiting both the subject and the predicate of the original proposition by means of the same determinant¹. For example, *All P is Q*, therefore, *All AP is AQ*; A negro is a fellow creature, therefore, A suffering negro is a suffering fellow creature. The formal validity of the reasoning may be shewn as follows: *AP* is a subdivision of the class *P*, namely, that part of it which also

¹ See section 33. The inference now under discussion is only a particular case of a wider class of inferences which will be more fully discussed in Part iv.

belongs to the class A ; and, therefore, whatever is true of the whole of P must be true of AP ; hence, given that $All\ P\ is\ Q$, we can infer that $All\ AP\ is\ Q$; moreover, by the law of identity, $All\ AP\ is\ A$; therefore, $All\ AP\ is\ AQ^1$.

The formal validity of immediate inference by added determinants has been denied on the ground of the obvious fallacy of arguing from such a premiss as *an elephant is an animal* to the conclusion *a small elephant is a small animal*, or from such a premiss as *cricketers are men* to the conclusion *poor cricketers are poor men*. In these cases, however, the fallacy really results from the ambiguity of language, the added determinant receiving a different interpretation when it qualifies the subject from that which it has when it qualifies the predicate. A term of comparison like *small* can indeed hardly be said to have an independent interpretation, its force always being relative to some other term with which it is conjoined. While then the inference in its symbolic form ($P\ is\ Q$, therefore, $AP\ is\ AQ$) is perfectly valid, it is specially necessary to guard against fallacy in its use when significant terms are employed. All that we have to insist upon is that the added determinant shall receive the same interpretation both in the subject and in the predicate. There is, for example, no fallacy in the following: An elephant is an animal, therefore, A small elephant is an animal which is small compared with elephants generally; Cricketers are men, therefore, Poor cricketers are men who in their capacity as cricketers are poor.

(2) *Immediate inference by complex conception* is a process of immediate inference which consists in employing the subject and predicate of the original proposition as parts of a more

¹ It must be observed, however, that the validity of this argument requires an assumption in regard to the existential import of propositions, which differs from that which we have for the most part adopted up to this point. It has to be assumed that universals do not imply the existence of their subjects. Otherwise this inference would not be valid in the case of no P being A . P might exist, and all P might be Q , but we could not pass to $AP\ is\ AQ$, since this would imply the existence of AP , which would be incorrect. It is necessary briefly to call attention to the above at this point, but our aim through all these earlier chapters has been to avoid as far as possible the various complications that arise in connexion with the difficult problem of existential import.

complex conception. Symbolically we can only express it somewhat as follows: *P is Q, therefore, Whatever stands in a certain relation to P stands in the same relation to Q.* The following is a concrete example: An elephant is an animal, therefore, the ear of an elephant is the ear of an animal. A systematic treatment of this kind of inference belongs to the special branch of formal logic known as the *logic of relatives*, any detailed consideration of which is beyond the scope of the present work. It must suffice here to call attention to the danger of committing a fallacy, akin to the fallacy of composition (see section 8), by passing from the distributive to the collective use of a term. For example, Protestants are Christians, therefore, A majority of Protestants are a majority of Christians.

(3) *Immediate inference by converse relation* is a process of immediate inference analogous to ordinary conversion but belonging to the logic of relatives. It consists in passing from a statement of the relation in which *P* stands to *Q* to a statement of the relation in which *Q* consequently stands to *P*. The two terms are transposed and the word by which their relation is expressed is replaced by its correlative. For example, *A is greater than B, therefore, B is less than A*; Alexander was the son of Philip, therefore, Philip was the father of Alexander; Freedom is synonymous with liberty, therefore, Liberty is synonymous with freedom.

Mansel gives the first two of the above as examples of *material consequence* as distinguished from *formal consequence*. "A Material Consequence is defined by Aldrich to be one in which the conclusion follows from the premisses solely by the force of the terms. This in fact means from some understood Proposition or Propositions, connecting the terms, by the addition of which the mind is enabled to reduce the Consequence to logical form.....The failure of a Material Consequence takes place when no such connexion exists between the terms as will warrant us in supplying the premisses required; i.e., when one or more of the premisses so supplied would be *false*. But to determine this point is obviously beyond the province of the Logician. For this reason, Material Consequence is rightly excluded from Logic.....Among these material,

and therefore extralogical, Consequences, are to be classed those which Reid adduces as cases for which Logic does not provide; *e.g.*, 'Alexander was the son of Philip, therefore, Philip was the father of Alexander'; '*A* is greater than *B*, therefore, *B* is less than *A*.' In both these it is our material knowledge of the relations 'father and son,' 'greater and less,' that enables us to make the inference" (*Aldrich*, p. 199).

The distinction between what is formal and what is material is not in reality so simple or so absolute as is here implied. It is usual to recognise as formal only those relations which can be expressed by the ordinary copula *is* or *is not*; and there is very good reason for proceeding upon this basis in the greater part of our logical discussions. No other relation is of the same fundamental importance or admits of an equally developed logical superstructure. But it is important to recognise that there are other relations which may remain the same while the things related vary; and wherever this is the case we may regard the relation as constituting the form and the things related the matter. Accordingly with each such relation we may have a different formal system. The logic of relatives deals with such systems as are outside the one ordinarily recognised. Each immediate inference by converse relation will, therefore, be formal in its own particular system. This point is admirably put by Miss Jones: "A proposition containing a relative term furnishes—besides the ordinary immediate inferences—other immediate inferences to any one acquainted with the system to which it refers. These inferences cannot be educed except by a person knowing the 'system'; on the other hand, no knowledge is needed of the objects referred to, except a knowledge of their place in the system, and this knowledge is in many cases coextensive with ordinary intelligence; consider, *e.g.*, the relations of magnitude of objects in space, of the successive parts of time, of family connexions, of number" (*General Logic*, p. 34).

(4) Immediate inference by modal consequence or, as it is also called, inference by change of modality is somewhat analogous to subaltern inference. It consists in nothing more than weakening a statement in respect of its modality; and hence it

is never possible to pass back from the inferred to the original proposition. Thus, from the *validity* of the apodeictic judgment we can pass to the validity of the assertoric, and from that to the validity of the problematic; but not *vice versâ*. On the other hand, from the *invalidity* of the problematic judgment we can pass to the invalidity of the assertoric, and from that to the invalidity of the apodeictic; but again not *vice versâ*¹.

76. *Reduction of immediate inferences to the mediate form*².

—Immediate inference has been defined as the inference of a proposition from a single other proposition; mediate inference, on the other hand, is the inference of a proposition from at least two other propositions.

We may briefly consider various ways of establishing the validity of immediate inferences by means of mediate inferences.

(1) One of the old Greek logicians, Alexander of Aphrodisias, establishes the conversion of **E** by means of a syllogism in *Ferio*.

No S is P,
therefore, *No P is S;*

for, if not, then by the law of contradiction, *Some P is S;* and we have this syllogism—

No S is P,
Some P is S,
therefore, *Some P is not P.*

a *reductio ad absurdum*³.

(2) It may be plausibly maintained that in Aristotle's proof of the conversion of **E** (given in section 65), there is an implicit syllogism: namely,—*Q is P, Q is S, therefore, Some S is P.*

(3) The contraposition of **A** may be established by means of a syllogism in *Camestres* as follows:—

¹ Compare Ueberweg, *Logic*, § 98.

² Students who have not already a technical knowledge of the syllogism may omit this section until they have read the earlier chapters of Part III.

³ Compare Mansel's *Aldrich*, p. 62. The conversion of **A** and the conversion of **I** may be established similarly.

Given *All S is P*,
 we have also *No not-P is P*, by the law of contradiction,
 therefore, *No not-P is S*¹.

(4) We might also obtain the contrapositive of *All S is P* as follows:—

By the law of excluded middle, *All not-P is S or not-S*, and,
 by hypothesis, *All S is P*,

therefore, *All not-P is P or not-S*;

but, by the law of contradiction, *No not-P is P*,

therefore, *All not-P is not-S*².

(5) The contraposition of **A** may also be established indirectly by means of a syllogism in *Darii*:—

All S is P,

therefore, *No not-P is S*;

for, if not, *Some not-P is S*; and we have the following syllogism—

All S is P,

Some not-P is S,

therefore, *Some not-P is P*,

which is absurd³.

All the above are interesting, as illustrating the processes of immediate inference; but regarded as proofs they labour under the disadvantage of deducing the less complex by means of the more complex.

¹ Similarly, granting the validity of obversion, the contraposition of **O** may be established by a syllogism in *Datissi* as follows:—

Given *Some S is not P*, then we have

All S is S, by the law of identity,

and *Some S is not-P*, by obversion of the given proposition,

therefore, *Some not-P is S*.

It will be found that, adopting the same method, the contraposition of **E** is yielded by a syllogism in *Darapti*.

² Compare Jevons, *Principles of Science*, chapter 6, § 2; and *Studies in Deductive Logic*, p. 44.

³ Compare De Morgan, *Formal Logic*, p. 25. Granting the validity of obversion, the contraposition of **E** and the contraposition of **O** may be established similarly.

The following are interesting only as curiosities :—

(6) Wolf obtains the subaltern of a universal proposition by a syllogism in *Darii*.

Given *All S is P*,
we have also *Some S is S*, by the law of identity,
therefore, *Some S is P*.

(Compare Mansel, *Prolegomena Logica*, p. 217.)

(7) "Still more absurd is the elaborate system which Krug, after a hint from Wolf, has constructed in which all immediate inferences appear as hypothetical syllogisms; a major premiss being supplied in the form, 'If all *A* is *B*, some *A* is *B*.' The author appears to have forgotten, that either this premiss is an additional empirical truth, in which case the immediate reasoning is not a logical process at all; or it is a formal inference, presupposing the very reasoning to which it is prefixed, and thus begging the whole question" (Mansel, *Prolegomena Logica*, p. 217).

EXERCISES.

77. Give all the logical opposites of the proposition :—Some rich men are virtuous; and also the converse of the contrary of its contradictory. How is the latter directly related to the given proposition?

Does it follow that a proposition admits of simple conversion because its predicate is distributed? [K.]

78. Point out any possible ambiguities in the following propositions, and give the contradictory and (where possible) the converse of each of them :—(i) Some of the candidates have been successful; (ii) All are not happy that seem so; (iii) All the fish weighed five pounds. [K.]

79. State in logical form and convert the following propositions :—(a) He jests at scars who never felt a wound; (b) Axioms are self-evident; (c) Natives alone can stand the climate of Africa; (d) Not one of the Greeks at Thermopylæ escaped; (e) All that glitters is not gold. [O.]

80. "The angles at the base of an isosceles triangle are equal." What can be inferred from this proposition by obversion, conversion, and contraposition respectively? [L.]

81. Give the obverse, the contrapositive, and the inverse of each of the following propositions:—The virtuous alone are truly noble; No Athenians are Helots. [M.]

82. Give the contrapositive and (where possible) the inverse of the following propositions:—(i) A stitch in time saves nine; (ii) None but the brave deserve the fair; (iii) Blessed are the peacemakers; (iv) Things equal to the same thing are equal to one another; (v) Not every tale we hear is to be believed. [K.]

83. Take the following propositions in pairs, and in regard to each pair state whether the two propositions are consistent or inconsistent with each other; in the former case, state further whether either proposition can be inferred from the other, and, if it can be, point out the nature of the inference; in the latter case, state whether it is possible for both the propositions to be false:—(a) *All S is P*; (b) *All not-S is P*; (c) *No P is S*; (d) *Some not-P is S*. [K.]

84. Transform the following propositions in such a way that, without losing any of their force, they may all have the same subject and the same predicate:—*No not-P is S*; *All P is not-S*; *Some P is S*; *Some not-P is not not-S*. [K.]

85. Describe the logical relations, if any, between each of the following propositions, and each of the others:—

(i) There are no inorganic substances which do not contain carbon;

(ii) All organic substances contain carbon;

(iii) Some substances not containing carbon are organic;

(iv) Some inorganic substances do not contain carbon. [C.]

86. "All that love virtue love angling."

Arrange the following propositions in the three following groups:—(a) those which can be inferred from the above proposition; (b) those which are consistent with it, but which cannot be inferred from it; (c) those which are inconsistent with it.

(i) None that love not virtue love angling.

(ii) All that love angling love virtue.

- (iii) All that love not angling love virtue.
- (iv) None that love not angling love virtue.
- (v) Some that love not virtue love angling.
- (vi) Some that love not virtue love not angling.
- (vii) Some that love not angling love virtue.
- (viii) Some that love not angling love not virtue. [K.]

87. Determine the logical relation between each pair of the following propositions :—

- (1) All crystals are solids.
- (2) Some solids are not crystals.
- (3) Some not crystals are not solids.
- (4) No crystals are not solids.
- (5) Some solids are crystals.
- (6) Some not solids are not crystals.
- (7) All solids are crystals. [L.]

CHAPTER IV.

THE DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS.

88. *The use of Diagrams in Logic.*—In representing propositions by geometrical diagrams, our object is not that we may have a new set of symbols, but that the relation between the subject and predicate of a proposition may be exhibited by means of a sensible representation, the signification of which is clear at a glance. Hence the first requirement which ought to be satisfied by any diagrammatic scheme is that the interpretation of the diagrams should be intuitively obvious, as soon as the principle upon which they are based has been explained¹.

A second essential requirement is that the diagrams should be adequate; that is to say, they should give a complete, and not a partial, representation of the relations which they are intended to indicate. Hamilton's use of Euler's diagrams, as

¹ Hamilton's "geometric scheme" which he himself describes as "easy, simple, compendious, all-sufficient, consistent, manifest, precise, complete" (*Logic*, vol. 2, p. 475) fails to satisfy this condition. He represents an affirmative copula by a horizontal tapering line (———), the broad end of which is towards the subject. Negation is marked by a perpendicular line crossing the horizontal one (⊥). A colon (:) placed at either end of the copula indicates that the corresponding term is distributed; a comma (,) that it is undistributed. Thus, for *All S is P* we have —

$$S : \text{———} , P ;$$

and similarly for the other propositions.

Dr Venn rightly observes that this scheme is purely symbolical, and does not deserve to rank as a diagrammatic scheme at all. "There is clearly nothing in the two ends of a wedge to suggest subjects and predicates, or in a colon and comma to suggest distribution and non-distribution" (*Symbolic Logic*, p. 432). Hamilton's scheme may certainly be rejected as valueless. The schemes of Euler and Lambert belong to an altogether different category.

described in the following section, will serve to illustrate the failure to satisfy this requirement.

In the third place, the diagrams should be capable of representing all the propositional forms recognised in the schedule of propositions which it is intended to illustrate, *e.g.*, particulars as well as universals. One scheme of diagrams may, however, be better suited for one purpose, and another scheme for another purpose. It will be found that Dr Venn's diagrams, presently to be described, can be adapted to the representation of particulars, but not very conveniently; whereas they are specially suited to the representation of universals. Their use may, therefore, for the most part be limited to occasions when we are dealing only with universals.

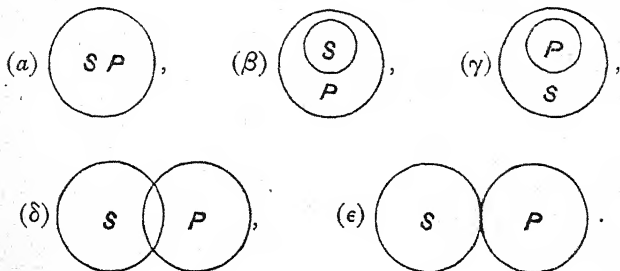
Lastly, it is advantageous that a diagrammatic scheme should be as little cumbrous as possible when it is desired to represent two or more propositions in combination with one another. This is the weak point of Euler's method. A scheme of diagrams may, however, serve a very useful function in making clear the full force of individual propositions, even when it is not well adapted for the representation of combined propositions.

A further requirement is sometimes added, namely, that each propositional form should be represented by a single diagram, not by a set of alternative diagrams. This is, however, by no means essential. For if we adopt a schedule of propositions which in some cases yields only an indeterminate relation in respect of extension between the terms involved, it is important that this should be clearly brought out, and a set of alternative diagrams may be specially helpful for this purpose. This point will be illustrated, with reference to Euler's diagrams, in the following section¹.

¹ It must be borne in mind that in all the schemes described in this chapter the terms of the propositions which are represented diagrammatically are taken in extension, not in intension. The fundamental objection raised by Mansel against the introduction of diagrammatic aids into logic appears to lose its force, even from the conceptualist standpoint, if attention is paid to this distinction. "If Logic," he says, "is exclusively concerned with Thought, and Thought is exclusively concerned with Concepts, it is impossible to approve of a practice, sanctioned by some eminent Logicians, of representing the relation

89. *Euler's Diagrams.*—We may begin with the well-known scheme of diagrams, which was first expounded by the Swiss mathematician and logician, Leonhard Euler, and which is usually called after his name¹.

Representing the individuals included in any class, or denoted by any name, by a circle, it will be obvious that the five following diagrams represent all possible relations between any two classes:—



The force of the different propositional forms is to exclude one or more of these possibilities.

All S is P limits us to one of the two α , β ;

Some S is P to one of the four α , β , γ , δ ;

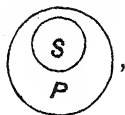
No S is P to ϵ ;

Some S is not P to one of the three γ , δ , ϵ .

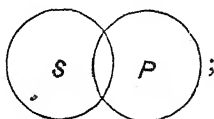
of terms in a syllogism by that of figures in a diagram. To illustrate, for example, the position of the terms in *Barbara* by a diagram of three circles, one within another, is to lose sight of the distinctive mark of a concept, that it cannot be presented to the sense, and tends to confuse the mental inclusion of one notion in the sphere of another, with the local inclusion of a smaller portion of space in a larger" (*Prolegomena Logica*, p. 55). It seems clear that in speaking of concepts as incapable of being presented to the sense, Mansel is thinking of the concepts themselves, not of their extension. Even conceptualist logicians must, however, recognise and deal with the extension of concepts. It is of course true that the local inclusion of one portion of space in another is not the same thing as the inclusion of one class in another. But the analogy is sufficiently close for purposes of illustration; and Mansel's argument does not in any degree establish the unfitness of the diagrams for the purpose for which they are intended.

¹ Euler lived from 1707 to 1783. His diagrammatic scheme is given in his *Lettres à une Princesse d'Allemagne* (Letters 102 to 105).

It is most misleading to attempt to represent *All S is P* by a single diagram, thus



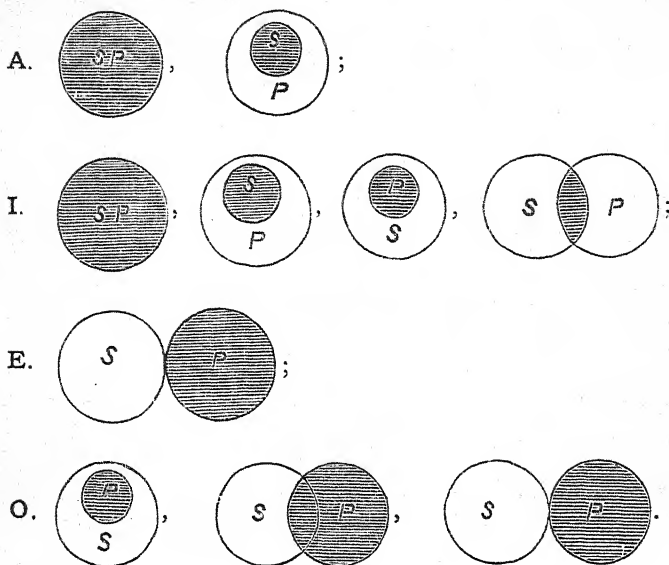
or *Some S is P* by a single diagram, thus



for in each case the proposition really leaves us with other alternatives. This method of employing the diagrams has, however, been adopted by a good many logicians who have used them, including Sir William Hamilton (*Logic*, i. p. 255), and Professor Jevons (*Elementary Lessons in Logic*, pp. 72 to 75); and the attempt at such simplification has brought their use into undeserved disrepute. Thus, Dr Venn remarks, "The common practice, adopted in so many manuals, of appealing to these diagrams—Eulerian diagrams as they are often called—seems to me very questionable. The old four propositions **A, E, I, O**, do not exactly correspond to the five diagrams, and consequently none of the moods in the syllogism can in strict propriety be represented by these diagrams" (*Symbolic Logic*, pp. 15, 16; compare also pp. 424, 425). This criticism is undoubtedly sound as regards the use of Euler's circles by Hamilton and Jevons; but it loses most of its force if the diagrams are used in the manner above described. It is true that they become somewhat cumbrous in relation to the syllogism; but the logical force of propositions and the logical relations between propositions can in many respects be well illustrated by their aid. Thus, they may be employed:—

(1) To illustrate the distribution of the predicate in a proposition. In the case of each of the four fundamental propositions we may shade the part of the predicate concerning which knowledge is given us.

We then have—

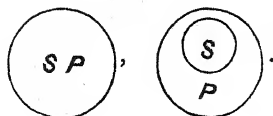


We see that with **A** and **I**, only part of *P* is in some of the cases shaded; whereas with **E** and **O**, the whole of *P* is in every case shaded; and it is thus made clear that negative propositions distribute, while affirmative propositions do not distribute, their predicates.

(2) To illustrate the opposition of propositions. Comparing two contradictory propositions, *e.g.*, **A** and **O**, we see that they have no case in common, but that between them they exhaust all possible cases. Hence the truth, that two contradictory propositions cannot be true together but that one of them must be true, is brought home to us under a new aspect. Again, comparing two subaltern propositions, *e.g.*, **A** and **I**, we notice that the former gives us all the information given by the latter and something more, since it still further limits the possibilities. The other relations involved in the doctrine of opposition may be illustrated similarly.

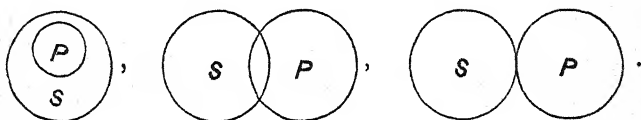
(3) To illustrate the conversion of propositions. Thus it is made clear how it is that **A** admits only of conversion

per accidens. All *S* is *P* limits us to one or other of the following—



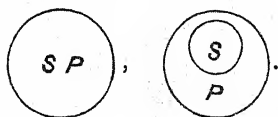
The problem of conversion is—What do we know of *P* in either case? In the first we have *All P is S*, in the second *Some P is S*; and since we do not know which of the cases holds good, we can only state what is common to them both, namely, *Some P is S*.

Again, it is made clear how it is that **O** is inconvertible. *Some S is not P* limits us to one or other of the following—



What then do we know concerning *P*? The three cases give us respectively—(i) *All P is S*; (ii) *Some P is S* and *Some P is not S*; (iii) *No P is S*. But (i) and (iii) are contraries, and (ii) is inconsistent with both of them. Hence nothing can be affirmed of *P* that is true in all three cases indifferently.

(4) To illustrate the more complicated forms of immediate inference. Taking, for example, the proposition *All S is P*, we may ask, What does this enable us to assert about *not-P* and *not-S* respectively? We have one or other of these cases—



With regard to *not-P*, these yield respectively, (i) *No not-P is S*; (ii) *No not-P is S*. And thus we obtain the contrapositive of the given proposition.

With regard to *not-S* we have, (i) *No not-S is P*, (ii) *Some*

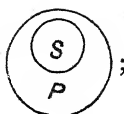
not-S is not *P*¹. Hence in either case we may infer *Some not-S* is not *P*.

E, I, O may be dealt with similarly.

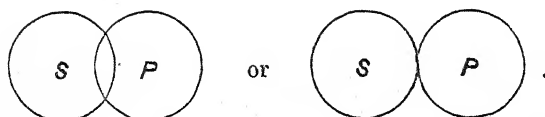
(5) To illustrate the joint force of a pair of complementary or contra-complementary or sub-complementary propositions (compare section 66). Thus, the pair of complementary propositions, *SaP* and *PaS*, taken together, limit us to



the pair of contra-complementary propositions, *SaP* and *PoS*, to



the pair of sub-complementary propositions, *SoP* and *PoS*, to



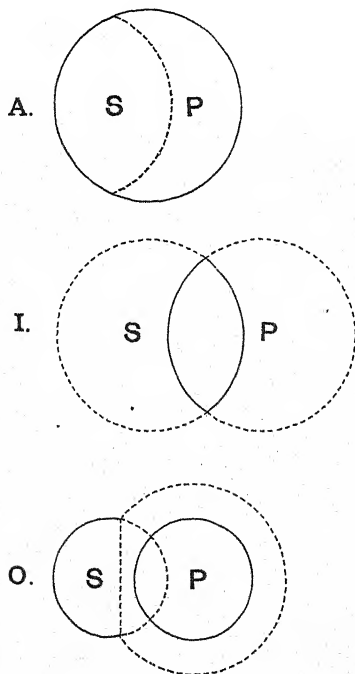
The application of the diagrams to syllogisms will be shewn in a subsequent chapter.

With regard to all the above, it may be said that the use of the circles gives us nothing that could not easily have been obtained independently. This is of course true; but no one, who has had experience of the difficulty that is sometimes found by students in really mastering the elementary principles of formal logic, and especially in dealing with immediate inferences, will despise any means of illustrating afresh the old truths, and presenting them under a new aspect.

¹ It is assumed in the use of Euler's diagrams that *S* and *P* both exist in the universe of discourse, while neither of them exhausts that universe. This assumption is the same as that upon which our treatment of immediate inferences in the preceding chapter has been based.

The fact that we have not a single pair of circles corresponding to each fundamental form of proposition is fatal if we wish to illustrate any complicated train of reasoning in this way; but in indicating the real nature of the knowledge given by the propositions themselves, it is rather an advantage than otherwise, inasmuch as it shews how limited in some cases this knowledge actually is¹.

¹ Dr Venn writes in criticism of Euler's scheme, "A fourfold scheme of propositions will not very conveniently fit in with a fivefold scheme of diagrams...What the five diagrams are competent to illustrate is the actual relation of the classes, not our possibly imperfect knowledge of that relation" (*Empirical Logic*, p. 229). The reply to this criticism is that inasmuch as the fourfold scheme of propositions gives but an imperfect knowledge of the actual relation of the classes denoted by the terms, the Eulerian diagrams are specially valuable in making this clear and unmistakeable. By the aid of dotted lines it is indeed possible to represent each proposition by a single Eulerian figure; but the diagrams then become so much more difficult to interpret that the loss is considerably greater than the gain. Ueberweg (*Logic*, § 71) gives the following:

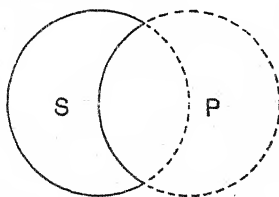


90. *Lambert's Diagrams.*—A scheme of diagrams was employed by Lambert¹ in which horizontal straight lines take the place of Euler's circles. The extension of a term is represented by a horizontal straight line, and so far as two such lines overlap it is indicated that the corresponding classes are coincident, while so far as they do not overlap these classes are shewn to be mutually exclusive. Both the absolute and the relative length of the lines is of course arbitrary and immaterial.

We may first shew how Lambert's lines may be used in such a manner as to be precisely analogous to Euler's circles. Thus, the four fundamental propositions may be represented as follows:—

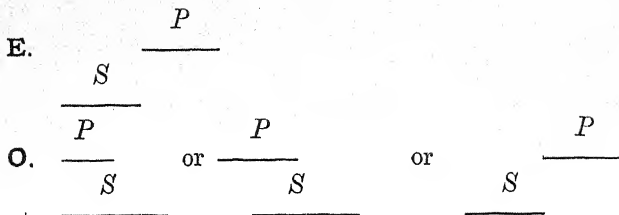
$$\begin{array}{l} \text{A. } \frac{P}{S} \text{ or } \frac{P}{S} \\ \text{I. } \frac{P}{S} \text{ or } \frac{P}{S} \text{ or } \frac{P}{S} \text{ or } \frac{P}{S} \end{array}$$

In lieu of the last of the figures on the preceding page, the following may be suggested as simpler, but equally effective:



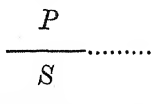
Here we get the three cases yielded by an O proposition by (1) filling in the dotted line to the left and striking out the other, (2) filling in both dotted lines, (3) filling in the dotted line to the right and striking out the other. These three cases are respectively those marked γ , δ , ϵ , on p. 127.

¹ Johann Heinrich Lambert was a German philosopher and mathematician who lived from 1728 to 1777. His *Neues Organon* was published at Leipzig in 1768. Lambert's own diagrammatic scheme differs somewhat from both of those given in the text; but it very closely resembles the one in which portions of the lines are dotted. The modifications in the text have been introduced in order to obviate certain difficulties involved in Lambert's own diagrams. See the note on p. 135.



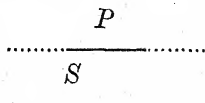
These diagrams take less room than Euler's circles. But they seem also to be less intuitively clear and less suggestive. The different cases too are less markedly distinct from one another. It is probable that one would in consequence be more liable to error in employing them.

The different cases may, however, be combined by the use of dotted lines so as to yield but a single diagram for each proposition much more satisfactorily than in Euler's scheme. Thus, *All S is P* may be represented by the diagram



where the dotted line indicates that we are uncertain as to whether there is or is not any *P* which is not *S*. We obviously get two cases according as we strike out the dots or fill them in, and these are the two cases previously shewn to be compatible with an **A** proposition.

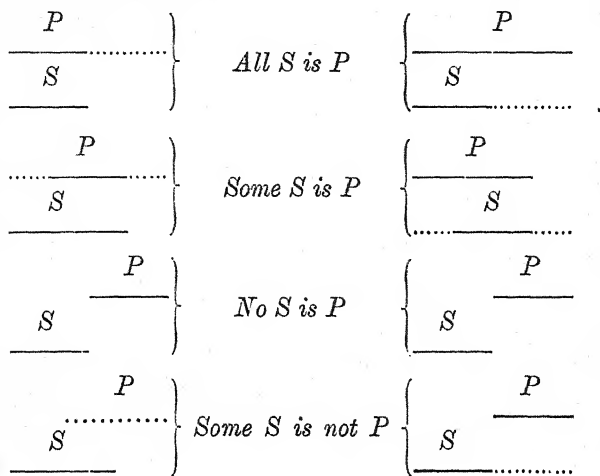
Again, *Some S is P* may be represented by the diagram



and here we get the four cases previously given for an **I** proposition by (a) filling in the dots to the left and striking out those to the right, (b) filling in all the dots, (c) striking them all out, (d) filling in those to the right and striking out those to the left.

Two complete schemes of diagrams may be constructed on this plan, in one of which no part of any *S* line, and in the

other no part of any *P* line, is dotted¹. These two schemes are given below to the left and right respectively of the propositional forms themselves.

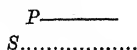


It must be understood that the two diagrams given above in the cases of **A**, **I**, and **O** are alternative in the sense that we may select which we please to represent our proposition; but either represents it completely.

We shall find later on that for the purpose of illustrating the syllogistic moods, Lambert's method is a good deal less cumbrous than Euler's². An adaptation of Lambert's diagrams in which the contradictories of *S* and *P* are introduced as well

¹ It is important to give both these schemes as it will be found that neither one of them will by itself suffice when this method is used for illustrating the syllogism. For obvious reasons the **E** diagram is the same in both schemes.

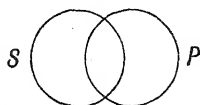
² Dr Venn (*Symbolic Logic*, p. 432) remarks, "As a whole Lambert's scheme seems to me distinctly inferior to the scheme of Euler, and has in consequence been very little employed by other logicians." The criticism offered in support of this statement is directed chiefly against Lambert's own representation of the particular affirmative proposition, namely,



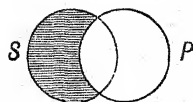
This diagram certainly seems as appropriate to **O** as it does to **I**; but the modification introduced in the text, and indeed suggested by Dr Venn himself, is not open to a similar objection.

as S and P themselves will be given in section 94. This more elaborated scheme will be found particularly valuable for illustrating the various processes of immediate inference.

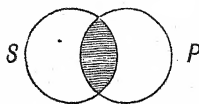
91. *Dr Venn's Diagrams.*—In the diagrammatic scheme employed by Dr Venn (*Symbolic Logic*, chapter 5) the diagram



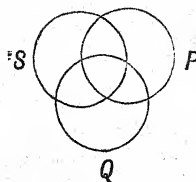
does not itself represent any proposition, but the framework into which propositions may be fitted. Denoting *not-S* by S' and what is both S and P by SP , &c., it is clear that everything must be contained in one or other of the four classes SP , SP' , $S'P$, $S'P'$; and the above diagram shews four compartments (one being that which lies outside both the circles) corresponding to these four classes. Every universal proposition denies the existence of one or more of such classes, and it may therefore be diagrammatically represented by shading out the corresponding compartment or compartments. Thus, *All S is P*, which denies the existence of SP' , is represented by



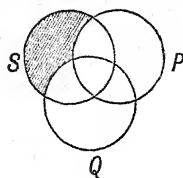
No S is P by



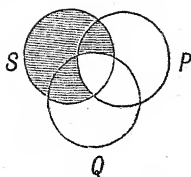
With three terms we shall have three circles and eight compartments, thus:—



All S is P or Q is represented by



All S is P and Q by



It is in cases involving three or more terms that the advantage of this scheme over the Eulerian scheme is most manifest. The diagrams are not, however, equally well adapted to the case of particular propositions. Dr Venn (in *Mind*, 1883, pp. 599, 600) suggests that we might just draw a bar across the compartment declared to be saved by a particular proposition¹; thus, *Some S is P* would be represented by drawing a bar across the SP compartment. Theoretically this plan might be worked out satisfactorily; but in representing a combination of propositions in this way special care would have to be taken in the interpretation of the diagrams. For example, if we have the diagram for three terms S , P , Q , and are given *Some S is P*, we do not know that *both* the compartments SPQ , SPQ' , are to be saved, and in a case like this a bar drawn across the SP compartment might easily be misinterpreted.

92. *Expression of the possible relations between any two classes by means of the propositional forms A, E, I, O.*—Any information given with respect to two classes limits the possible

¹ Dr Venn's scheme differs from the schemes of Euler and Lambert, in that it is not based upon the assumption that our terms and their contradictories all represent existing classes. It involves, however, the doctrine that particulars are existentially affirmative, while universals are existentially negative.

relations between them to one or more of the five which are indicated diagrammatically by Euler's circles as shewn at the beginning of section 89. It will be useful to enquire how such information may in all cases be expressed by means of the propositional forms **A**, **E**, **I**, **O**.

The five relations may, as before, be designated respectively α , β , γ , δ , ϵ ¹. Information is given when the possibility of one or more of these is denied; in other words, when we are limited to one, two, three, or four of them. Let limitation to α or β (*i.e.*, the exclusion of γ , δ , and ϵ) be denoted by α , β ; limitation to α , β , or γ (*i.e.*, the exclusion of δ and ϵ) by α , β , γ ; and so on.

In seeking to express our information by means of the four ordinary propositional forms, we find that sometimes a single proposition will suffice for our purpose; thus α , β is expressed by *All S is P*. Sometimes we require a combination of propositions; thus α is expressed by the pair of complementary propositions *All S is P and all P is S*, (since *all S is P* excludes γ , δ , ϵ , and *all P is S* further excludes β). Some other cases are still more complicated; thus the fact that we are limited to α or δ cannot be expressed more simply than by saying, *Either All S is P and all P is S, or else Some S is P, some S is not P, and some P is not S*.

Let **A** = *All S is P*, **A**₁ = *All P is S*, and similarly for the other propositions. Also let **AA**₁ = *All S is P and all P is S*, &c. Then the following is a scheme for all possible cases:—

¹ Thus, the classes being *S* and *P*, α denotes that *S* and *P* are wholly coincident; β that *P* contains *S* and more besides; γ that *S* contains *P* and more besides; δ that *S* and *P* overlap each other, but that each includes something not included by the other; ϵ that *S* and *P* have nothing whatever in common.

<i>limitation to</i>	<i>denoted by</i>	<i>limitation to</i>	<i>denoted by</i>
α	AA_1	α, β, γ	A or A_1
β	AO_1	α, β, δ	A or IO_1
γ	A_1O	α, β, ϵ	A or E
δ	IOO_1	α, γ, δ	A_1 or IO
ϵ	E	α, γ, ϵ	A_1 or E
α, β	A	α, δ, ϵ	AA_1 or OO_1
α, γ	A_1	β, γ, δ	IO or IO_1
α, δ	AA_1 or IOO_1	β, γ, ϵ	AO_1 or A_1O or E
α, ϵ	AA_1 or E	β, δ, ϵ	O_1
β, γ	AO_1 or A_1O	γ, δ, ϵ	O
β, δ	IO_1	$\alpha, \beta, \gamma, \delta$	I
β, ϵ	AO_1 or E	$\alpha, \beta, \gamma, \epsilon$	A or A_1 or E
γ, δ	IO	$\alpha, \beta, \delta, \epsilon$	A or O_1
γ, ϵ	A_1O or E	$\alpha, \gamma, \delta, \epsilon$	A_1 or O
δ, ϵ	OO_1	$\beta, \gamma, \delta, \epsilon$	O or O_1

It will be found that any other combinations of propositions than those here given either involve contradictions or redundancies, or else no information is given because all the five relations that are *à priori* possible still remain possible.

For example, AI is clearly redundant; AO is self-contradictory; A or A_1O is redundant (since the same information is given by A or A_1); A or O gives no information (since it excludes no possible case). The student is recommended to test other combinations similarly. It must be remembered that $I_1 = I$, and $E_1 = E$.

It should be noticed that if we read the first column downwards and the second column upwards we get pairs of contradictories.

93. *Euler's diagrams and the class relations between S , $not-S$, P , $not-P$.*—In Euler's diagrams, as ordinarily given, there is no explicit recognition of $not-S$ and $not-P$; but it is of course understood that whatever part of the universe lies outside S is $not-S$, and similarly for P , and it may be thought that no further account of negative terms need be taken. Further consideration, however, will shew that this is not the case; and, assuming that S , $not-S$, P , $not-P$ all represent existing classes, we shall find that *seven*, not five, determinate class relations between them are possible.

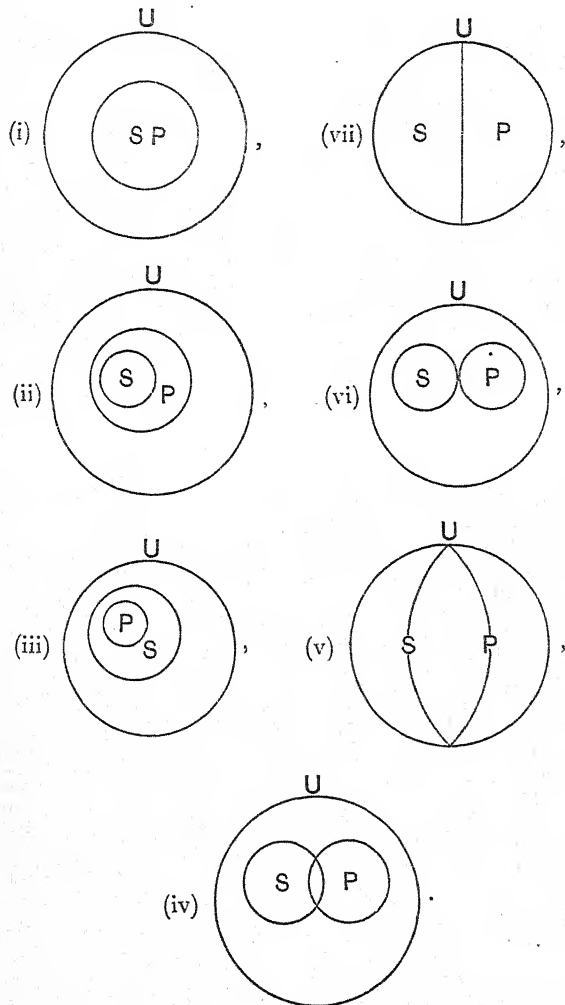
Taking the diagrams given in section 89, the above assumption clearly requires that in the cases of α , β , and γ , there should be some part of the universe lying outside both the circles, since otherwise either $not-S$ or $not-P$ or both of them would no longer be contained in the universe. But in the cases of δ and ϵ it is different. S , $not-S$, P , $not-P$ are now all of them represented within the circles; and in each of these cases, therefore, the pair of circles may or may not between them exhaust the universe.

Our results may also be expressed by saying that in the cases of α , β , and γ , there must be something which is both $not-S$ and $not-P$; whereas in the cases of δ and ϵ , there may or may not be something which is both $not-S$ and $not-P$. Euler's circles are no doubt a little apt to lead us to overlook the latter of these alternatives. If, indeed, there were always part of the universe outside the circles, every proposition, whether its form were **A**, **E**, **I**, or **O**, would have an inverse and the same inverse, namely, *Some $not-S$ is $not-P$* ; also, every proposition, including **I**, would have a contrapositive. These are erroneous results against which the student may need to be specially cautioned.

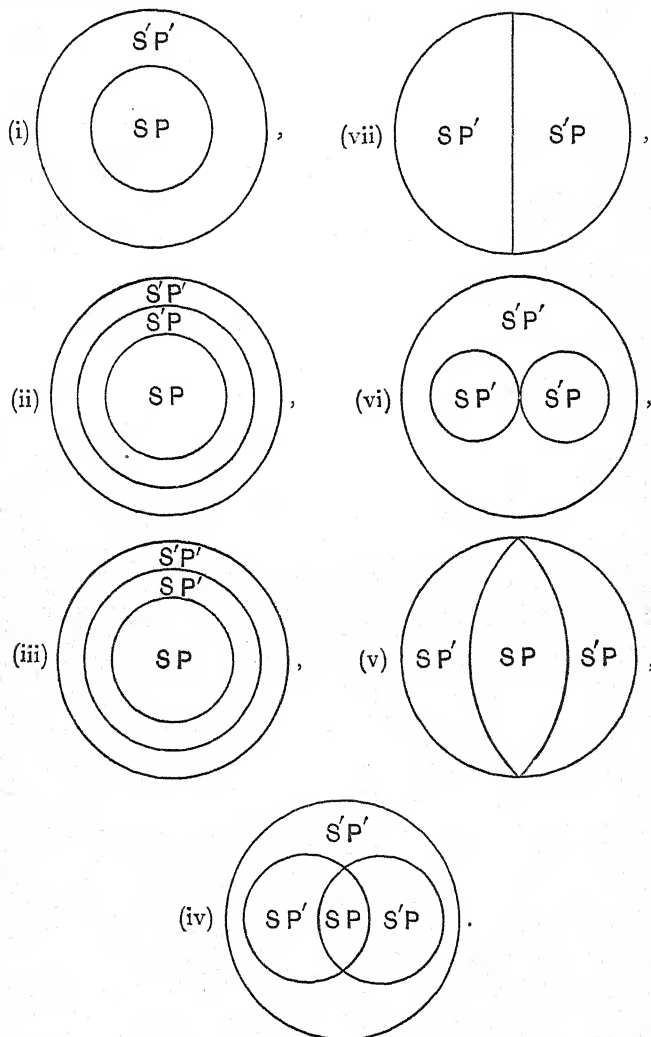
We find then that the explicit recognition of $not-S$ and $not-P$ practically leaves α , β , and γ unaffected, but causes δ and ϵ each to subdivide into two cases according as there is or is not anything that is both $not-S$ and $not-P$; and the

Eulerian fivefold division has accordingly to give place to a sevenfold division.

In the diagrammatic representation of these seven relations, the entire universe of discourse may be indicated by a larger circle in which the ordinary Eulerian diagrams (with some slight necessary modifications) are included. We shall then have the following scheme:—



It may be useful to repeat these diagrams with an explicit indication in regard to each subdivision of the universe as to whether it is *S* or *not-S*, *P* or *not-P*¹. The scheme will then appear as follows :—



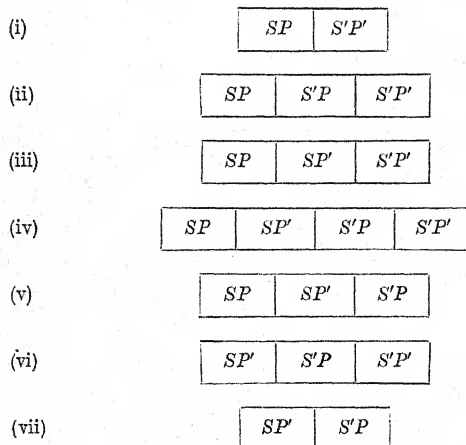
¹ We might also represent the universe of discourse by a long rectangle divided into compartments, shewing which of the four possible combinations

Comparing the above with the five ordinary Eulerian diagrams (which may be designated α , β , &c. as in section 89), it will be seen that (i) corresponds to α ; (ii) to β ; (iii) to γ ; (iv) and (v) represent the two cases now yielded by δ ; (vi) and (vii) the two yielded by ϵ .

Our seven diagrams might also be arrived at as follows:— Every part of the universe must be either S or S' , and also P or P' ; and hence the mutually exclusive combinations SP , SP' , $S'P$, $S'P'$ must between them exhaust the universe. The case in which these combinations are all to be found is represented by diagram (iv); if one but one only is absent we obviously have four cases which are represented respectively by (ii), (iii), (v), and (vi); if only two are to be found it will be seen that we are limited to the cases represented by (i) and (vii) or we should not fulfil the condition that neither S nor S' , P nor P' , is to be altogether non-existent; for the same reason the universe cannot contain less than two of the four combinations. We thus have the seven cases represented by the diagrams, and these are shewn to exhaust the possibilities.

The four fundamental propositions are related to the new scheme as follows:—

SP , SP' , $S'P$, $S'P'$ are to be found. This plan will give the following diagrams, which precisely correspond, as numbered, with those in the text:—



- A** limits us to (i) or (ii);
I to (i), (ii), (iii), (iv), or (v);
E to (vi) or (vii);
O to (iii), (iv), (v), (vi), or (vii).

Working out the further question how each diagram taken by itself is to be expressed propositionally we get the following results:

- (i) SaP and $S'aP'$;
 (ii) SaP and $S'oP'$;
 (iii) $S'aP'$ and SoP ;
 (iv) SiP , $S'iP'$, SoP , and $S'oP'$;
 (v) $S'aP$ and SoP' ;
 (vi) SaP' and $S'oP$;
 (vii) SaP' and $S'aP$.

Thus, for (i) and (vii) we have a combination of two complementaries; for (ii), (iii), (v), and (vi) a combination of two contra-complementaries; for (iv) a combination of two pairs of sub-complementaries¹.

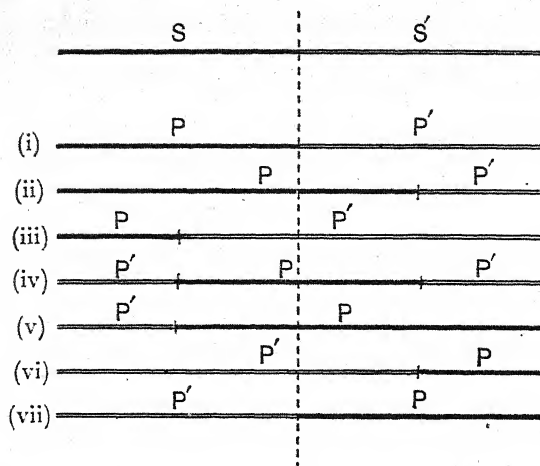
It will be observed that the new scheme is in itself more symmetrical than Euler's, and also that it succeeds better in bringing out the symmetry of the fourfold schedule of propositions². **A** and **E** give two alternatives each, **I** and **O** give five each; whereas with Euler's scheme **E** gives only one alternative, **A** two, **O** three, **I** four, and it might, therefore, seem as if **E** afforded more definite and unambiguous information than **A**, and **O** than **I**, which is not really the case. Further, the problem of expressing each diagram propositionally yields a more symmetrical result than the corresponding problem in the case of Euler's diagrams.

94. *Lambert's diagrams and the class-relations between S , not- S , P , not- P .*—The following is a compact diagrammatic representation of the seven possible class-relations between S , not- S , P , not- P , based upon Lambert's scheme³.

¹ Compare section 73.

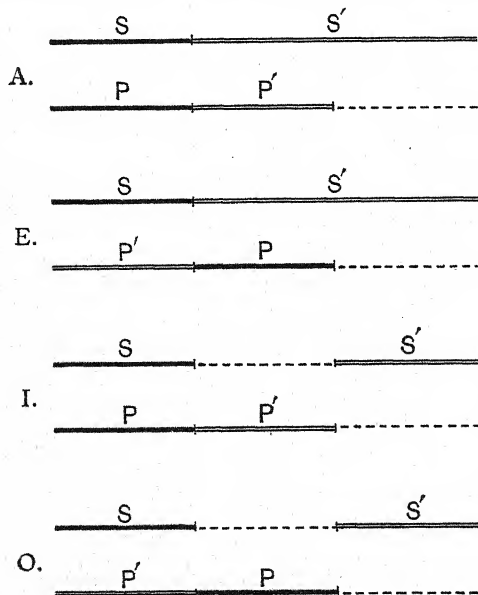
² We have seen that, similarly, in the case of immediate inferences symmetry can be gained only by the recognition of negative terms.

³ The mode of representation here adopted was first suggested to me, in a slightly different form, by my pupil, the late Mr Nicolas of Trinity College.



In this scheme each line represents the entire universe of discourse, and the first line must be taken in connexion with each of the others in turn. Further explanation will be unnecessary for the student who clearly understands the Lambertian method.

On the same principle and with the aid of dotted lines the four fundamental propositional forms may be represented as follows :



In each case the full extent of a line represents the entire universe of discourse; any portion of a line that is dotted may be either S or S' (or P or P' , as the case may be).

This last scheme of diagrams is perhaps more useful than any of the others in shewing at a glance what immediate inferences are obtainable from each proposition by conversion, contraposition, and inversion (on the assumption that S , S' , P , and P' all represent existing classes). Thus, from the first diagram we can read off at a glance SaP , PiS , $P'aS'$, $S'iP'$; from the second SeP , PeS , $P'oS'$, $S'oP'$; from the third SiP and PiS ; and from the fourth SoP and $P'oS'$. The last two diagrams are also seen at a glance to be indeterminate in respect to P' and S' , P and S' , respectively (that is to say, **I** has no contrapositive and no inverse, **O** has no converse and no inverse).

EXERCISES.

95. Illustrate by means of the Eulerian diagrams:—(1) the relation between **A** and **E**, (2) the relation between **I** and **O**, (3) the conversion of **I**, (4) the contraposition of **O**, (5) the inversion of **E**. [κ.]

96. Take all the ordinary propositions connecting any two terms, combine them as far as possible without contradiction, and represent each combination diagrammatically. [J.]

CHAPTER V.

PROPOSITIONS IN EXTENSION AND IN INTENSION.

97. *Fourfold Implication of Propositions in Connotation and Denotation.*—In dealing with the question whether propositions assert a relation between objects or between attributes or between objects and attributes, logicians have been apt to commit the fallacy of exclusiveness, selecting some one of the given alternatives, and treating the others as necessarily excluded thereby. It follows, however, from the double aspect of names—in extension and intension—that the different relations really involve one another, and hence are all of them virtually asserted in any categorical proposition whose subject and predicate are both general names.

The problem will be made more definite if we confine ourselves to a consideration of *connotation* and *denotation* in the strict sense, as distinguished from comprehension and exemplification, our terms being supposed to be defined intensively¹. Both subject and predicate will then have a denotation determined by their connotation, and hence our proposition may be considered from four different points of view, which are not indeed really independent of one another, but which serve to bring different aspects of the proposition into prominence. (1) The subject may be read in denotation

¹ With extensive definitions we might similarly work out the relations between the terms of a proposition in exemplification and comprehension; and with either intensive or extensive definitions, we might consider them in denotation and comprehension. The discussion in the text will, however, be limited to connotation and denotation, except that a separate section will be devoted to the case in which both subject and predicate are read in comprehension.

and the predicate in connotation; (2) both terms may be read in denotation; (3) both terms may be read in connotation; (4) the subject may be read in connotation and the predicate in denotation.

As an example, we may take the proposition, *All men are mortal*. According to our point of view, this proposition may be read in any of the following ways:

(1) The objects denoted by *man* possess the attributes connoted by *mortal*;

(2) The objects denoted by *man* are included within the class of objects denoted by *mortal*;

(3) The attributes connoted by *man* are accompanied by the attributes connoted by *mortal*;

(4) The attributes connoted by *man* indicate the presence of an object belonging to the class denoted by *mortal*.

It should be specially noticed that a different relation between subject and predicate is brought out in each of these four modes of analysing the proposition, the relations being respectively (i) *possession*, (ii) *inclusion*, (iii) *concomitance*, (iv) *indication*.

It may very reasonably be argued that a certain one of the above ways of regarding the proposition is (a) psychologically the most prominent in the mind in predication; or (b) the most fundamental in the sense of making explicit that relation which ultimately determines the other relations; or (c) the most convenient for a given purpose, *e.g.*, for dealing with the problems of formal logic. We need not, however, select the same mode of interpretation in each case. There would, for example, be nothing inconsistent in holding that to read the subject in denotation and the predicate in connotation is most correct from the psychological standpoint; to read both terms in connotation the most ultimate, inasmuch as connotation determines denotation and not *vice versa*; and to read both terms in denotation the most serviceable for purposes of logical manipulation. To say, however, that a certain one of the four readings alone constitutes the import of the proposition to the exclusion of the others cannot but be erroneous. They are in truth so much implicated in one another, that the

difficulty may rather be to justify a treatment which in any respect distinguishes between them¹.

(i) *Subject in denotation, predicate in connotation.*

If we read the subject of a proposition in denotation and the predicate in connotation, we have what is sometimes called the *predicative mode of interpreting the proposition*. This way of regarding propositions undoubtedly corresponds in the great majority of cases with the course of ordinary thought; that is to say, we naturally contemplate the subject as a class of objects of which a certain attribute or complex of attributes is predicated. Such propositions as *All men are mortal*, *Some violets are white*, *All diamonds are combustible* may be taken as examples. Dr Venn puts the point very clearly with reference to the last of these three propositions: "If I say that 'all diamonds are combustible,' I am joining together two connotative terms, each of which, therefore, implies an attribute and denotes a class; but is there not a broad distinction in respect of the prominence with which the notion of a class is

¹ The true doctrine is excellently stated by Mrs Ladd Franklin. "The reason that so many different views are possible in regard to the import of propositions is a very simple one. Every term is a double-edged machine—it effects the separating out of a certain group of objects and it epitomises a certain complex of marks. From this double nature of the term, it follows with mathematical rigour that a proposition, which contains two terms must have a fourfold implication (though one of the four senses may be at any time uppermost in the mind). Whoever says, for instance, that 'All politicians are statesmen' must be prepared to maintain that the objects, politicians, are the same as some of the objects, statesmen, and are in possession of all the qualities of statesmen; and also that the quality-complex, politician, entails the quality-complex, statesman, and is indicative of the presence of some of the objects statesmen...Now it is open to the logician to say that any one of these four implications is the most important or the most prominent implication of the proposition, but it is not open to him to say that less than all four of them is the complete implication...The proposition '*ab* is non-existent' does not state that the classes *a* and *b* have nothing in common, any more than it states that the qualities *a* and *b* are never found in conjunction. Mill's view of the import of the proposition is the third of these—that wherever we find certain attributes, there will be found certain other attributes, that the latter set of attributes constantly accompany the former set. The common class view is the first of these. The view that the extent of the subject and the intent of the predicate are most frequently uppermost in the mind is the view that will probably commend itself to the careful psychologist" (*Mind*, October, 1890, pp. 561, 2).

presented to the mind in the two cases? As regards the diamond, we think at once, or think very speedily, of a class of things, the distinctive attributes of the subject being mainly used to carry the mind on to the contemplation of the objects referred to by them. But as regards the combustibility, the attribute itself is the prominent thing... Combustible things, other than the diamond itself, come scarcely, if at all, under contemplation. The assertion in itself does not cause us to raise a thought whether there be other combustible things than these in existence" (*Empirical Logic*, p. 219).

cannot
share
class. 2.

Two points may be noticed as serving to confirm the view that generally speaking the predicative mode of interpreting propositions is psychologically the most prominent:

(a) The most striking difference between a substantive and an attributive (*i.e.*, an adjective or a participle) from the logical point of view is that in the former the denotation is usually more prominent than the connotation, even though it may be ultimately determined by the connotation, whilst in the latter the connotation is the more prominent, even though the name must be regarded as the name of a class of objects if it is entitled to be called a *name* in the strict logical sense at all. Corresponding to this we find that the subject of a proposition is almost always a substantive, whereas the predicate is more often an attributive.

(b) It is always the denotation of a term that we quantify, never the connotation. Whether we talk of *all men* or of *some men*, the complex of attributes connoted by *man* is taken in its totality; the distinction of quantity relates entirely to the denotation of the term. Corresponding to this, we find that we naturally regard the quantity of a proposition as pertaining to its subject, and not to its predicate. It will be shewn in the following chapter that the doctrine of the quantification of the predicate has at any rate no psychological justification.

There are, however, numerous exceptions to the statement that the subject of a proposition is naturally read in denotation and the predicate in connotation; for example, in the classificatory sciences. The following propositions may be taken as instances: *All palms are endogens, All daisies are compositæ,*

None but solid bodies are crystals, Hindoos are Aryans, Tartars are Turanians. In such cases as these most of us would naturally think of a certain class of objects as included in or excluded from another class rather than as possessing or not possessing certain definite attributes; in other words, as Dr Venn puts it, "the class-reference of the predicate is no less definite than that of the subject" (*Empirical Logic*, p. 220). In the case of such a proposition as *No plants with opposite leaves are orchids*, the position is even reversed, that is to say, it is the subject rather than the predicate that we should more naturally read in connotation. We may pass on then to other ways of regarding the categorical proposition.

(ii) *Subject in denotation, predicate in denotation.*

If we read both the subject and the predicate of a proposition in denotation, we have a relation between two classes, and hence this is called the class mode of interpreting the proposition. It must be particularly observed that the relation between the subject and the predicate is now one of *inclusion* in or *exclusion from*, not one of *possession*. It may at once be admitted that the class mode of interpreting the categorical proposition is neither the most ultimate, nor—generally speaking—that which we naturally or spontaneously adopt. It is, however, extremely convenient for manipulative purposes, and hence is the mode of interpretation usually selected, either explicitly or implicitly, by the formal logician. Attention may be specially called to the following points:

(a) When subject and predicate are both read in denotation, they are *homogeneous*.

(b) In the *diagrammatic* illustration of propositions both subject and predicate are necessarily read in denotation, since it is the denotation—not the connotation—of a term that we represent by means of a diagram.

(c) The predicate of a proposition must be read in denotation in order to give a meaning to the question whether it is or is not *distributed*.

(d) The predicate as well as the subject must be read in denotation before such a process as *conversion* is possible.

(e) In the treatment of the syllogism both subject and

predicate must be read in denotation (or else both in connotation), since either the middle term (first and fourth figures) or the major term (second and fourth figures) or the minor term (third and fourth figures) is subject in one of the propositions in which it occurs and predicate in the other.

The class mode of interpreting categorical propositions is nevertheless treated by some writers as being positively erroneous. But the arguments used in support of this view will not bear examination.

(1) It is said that to read both subject and predicate in denotation is psychologically false. It has indeed been already pointed out that the class mode of interpretation is not that which as a rule first presents itself to our mind when a proposition is given us; but we have also seen that there are exceptions to this, as, for example, in the propositions *All daisies are compositæ*, *All Hindoos are Aryans*, *All Tartars are Turanians*. It is, therefore, clearly wrong to describe the reading in question as psychologically false. On the same shewing, any other reading would equally be psychologically false, for what is immediately present to the mind in judgment varies very much in different cases. But it must be added that even when we do not spontaneously adopt the class reading, it is still a reading that is psychologically possible. Mr Bradley writes—"Judgment is not inclusion in, or exclusion from, a class. The doctrine that in saying *A is equal to B*, or *B is to the right of C*, or *To-day precedes Monday*, I have in my mind a class, either a *collection* or *description*, of *things equal to B*, or *to the right of C*, or *preceding Monday*, is quite opposed to fact" (*Principles of Logic*, pp. 22, 3). It may be readily admitted that in such cases as these we do not, when the proposition is first given to us, naturally think of the predicate as the name of a class. Still, analysis shews that in these judgments, as in others, inclusion in or exclusion from a class is really implicated along with other things, although this relation may be neither that which first impresses itself upon us nor that which is most important or characteristic.

(2) Mr Welton asks what we mean by a class, for example, by the class of birds when we say *All owls are birds*. "It is

nothing existing in space; the birds of the world are nowhere collected together so that we can go and pick out the owls from amongst them. The classification is a mental abstraction of our own, founded upon the possession of certain definite attributes. The class is not definite and fixed, and we do not find out whether any individual belongs to it by going over a list of its members, but by enquiring whether it possesses the necessary attributes" (*Logic*, p. 218). In so far as this argument applies against reading the predicate in denotation, it applies equally against reading the subject in denotation. It is in effect the argument used by Mill (*Logic*, Book i, ch. 5, § 3) in order to lead up to his position that the *ultimate* interpretation of the categorical proposition requires us to read both subject and predicate in connotation, since denotation is determined by connotation. But if we grant this, it does not prove the class reading of the proposition erroneous; it only proves that in the class reading, the analysis of the import of the proposition has not been carried as far as it admits of being carried.

(3) It is argued that when we regard a proposition as expressing the inclusion of one class within another, even then the predicate is only apparently read in denotation. "On this view," says Mr Welton, "we do not really assert *P* but 'inclusion in *P*,' and this is therefore the true predicate. For example, in the proposition, 'All owls are birds,' the real predicate is, on this view, not 'birds' but 'included in the class birds.' But this inclusion is an attribute of the subject, and the real predicate, therefore, asserts an attribute. It is meaningless to say 'Every owl *is* the class birds,' and it is false to say 'The class owls *is* the class birds'" (*Logic*, p. 218). This argument simply begs the question in favour of the predicative mode of interpretation. It overlooks the fact that the precise kind of relation brought out in the analysis of a proposition will vary according to the way in which we read the subject and the predicate. An analogous argument might also be used against the predicative reading itself. Take the proposition, "All men are mortal." It is absurd to say that "Every man *is* the attribute mortality," or that "The class men *is* the attribute mortality."

(4) It is said that a class interpretation of both *S* and *P* would lead properly to a fivefold, not a fourfold, scheme of propositions, since there are just five relations possible between any two classes, as is shewn by the Eulerian diagrams. This contention has force, however, only upon the assumption that we must have quite determinate knowledge of the class relation between *S* and *P* before being able to make any statement on the subject; and this assumption is neither justifiable in itself nor necessarily involved in the interpretation in question. It may be added that if a similar view were taken on the adoption of the predicative mode of interpretation, we should have a threefold, not a fourfold scheme. For then the quantity of our subject at any rate would have to be perfectly determinate, and with *S* and *P* for subject and predicate, the three possible statements would be—*All S is P, Some S is P and some is not, No S is P*. The point here raised will presently be considered further in connexion with the quantification of the predicate.

(iii) Subject in connotation, predicate in connotation.

If we read both the subject and the predicate of a proposition in connotation, we have what may be called the connotative mode of interpreting the proposition. In the proposition *All S is P*, the relation expressed between the attributes connoted by *S* and those connoted by *P* is one of *concomitance*—"the attributes which constitute the connotation of *S* are always found accompanied by those which constitute the connotation of *P*".¹ Similarly, in the case of *Some S is P*—"the attributes

¹ This is the only possible reading in connotation, so far as real propositions are concerned, if the term connotation is used in the strict sense as distinguished both from comprehension and from subjective intension. Unfortunately confusion is very apt to be introduced into discussions concerning the intensive rendering of propositions simply because no clear distinction is drawn between the different points of view which may be taken when terms are regarded from the intensive side. Hamilton distinguished between judgments in extension and judgments in intension, the relation between the subject and the predicate in the one case being just the reverse of the relation between them in the other. Thus, taking the proposition *All S is P*, we have in extension *S is contained under P*, and in intension *S comprehends P*. On this view the intensive reading of *All men are mortal* is "mortality is part of humanity" (the extensive reading being "the class man is part of the class mortal").

which constitute the connotation of *S* are sometimes found accompanied by those which constitute the connotation of *P*"; *No S is P*—"the attributes which constitute the connotation of *S* are never found along with those which constitute the connotation of *P*"; *Some S is not P*—"the attributes which constitute the connotation of *S* are sometimes found unaccompanied by those which constitute the connotation of *P*."

It will be noticed that in the connotative reading we have

This reading may be accepted if the term intension is used in the objective sense which we have given to *comprehension*, so that by *humanity* is meant the totality of attributes common to all men, and by *mortality* the totality of attributes common to all mortals. To this point of view we shall return in the next section. Leaving it for the present on one side, it is clear that if by *humanity* we mean only what may be called the distinctive or essential attributes of man, then in order that the above reading may be correct, the given proposition must be regarded as analytical. In other words, if *humanity* signifies only those attributes which are included in the connotation of *man*, then, if mortality is included in humanity, we shall merely have to analyse the connotation of the name *man*, in order to obtain our proposition. Hence on this view it must either be maintained that all universal affirmative propositions are analytical, or else that some universal affirmatives cannot be read in intension. But obviously the first of these alternatives must be rejected, and the second practically means that the reading in question breaks down so far as universal affirmatives are concerned. It equally fails to apply in the case of particulars and negatives. "The attributes constituting the intensions of *S* and *P* partly coincide" is clearly not equivalent to *Some S is P*; for example, the intension (in any sense) of "Englishman" has something in common with the intension of "Frenchman," but it does not follow that "Some Englishmen are Frenchmen." Again, from the fact that the intension of *S* has nothing in common with the intension of *P*, we cannot infer that *No S is P*; suppose, for example, that *S* stands for "ruminant," and *P* for "cloven-hoofed." Compare Venn, *Symbolic Logic*, pp. 391—5.

The intensive reading here criticised must be carefully distinguished from the connotative reading given in the text. The latter is not open to similar objections. Both readings must be further distinguished from the reading in comprehension discussed in the following section.

It may be added, quite independently of the above criticisms, that Hamilton is in error in speaking of the distinction between judgments in extension and judgments in intension as a *division* of judgments (*Logic*, i., p. 231). It is clear that the distinction is really between two different points of view from which the same judgment may be regarded. The view that some propositions will more naturally be interpreted extensively and others intensively may admit of justification (compare Veitch, *Institutes of Logic*, pp. 72, 3 and 224, 5); but if this is all that Hamilton means, he hardly expresses himself accurately.

always to take the attributes which constitute the connotation collectively. In other words, by the attributes constituting the connotation of a term we mean those attributes regarded as a whole. Thus, *No S is P* does not imply that none of the attributes connoted by *S* are ever accompanied by any of those connoted by *P*. This is apparent if we take such a proposition as *No oxygen is hydrogen*. It follows that when the subject is read in connotation the quantity of the proposition must appear as a separate element, being expressed by the word "always" or "sometimes," and must not be interpreted as meaning "all" or "some" of the attributes included in the connotation of the subject.

It is argued by those who deny the possibility of the connotative mode of interpreting propositions, that this is not really reading the subject in connotation at all; *always* and *sometimes* are said to reduce us to denotation at once. In reply to this, it must of course be allowed that real propositions affirm no relation between attributes independently of the objects to which they belong. The connotative reading implies the denotative, and we must not exaggerate the nature of the distinction between them. Still the connotative reading presents the import of the proposition in a new aspect, and there is at any rate a *prima facie* difference between regarding one class as included within another, and regarding one attribute as always accompanied by another, even though a little consideration may shew that the two things mutually involve one another¹.

(iv) *Subject in connotation, predicate in denotation.*

Taking the proposition *All S is P*, and reading the subject in connotation and the predicate in denotation, we have—"The

¹ Mill attaches great importance to the connotative mode of interpreting propositions as compared with the class mode or the predicative mode, on the ground that it carries the analysis a stage further; and this must be granted, at any rate so far as we consider the application of the terms involved to be determined by connotation and not by exemplification. Mill is, however, sometimes open to the charge of exaggerating the difference between the various modes of interpretation. This is apparent, for example, in his rejection of the *Dictum de omni et nullo* as the axiom of the syllogism, and his acceptance of the *Nota notæ est nota rei ipsius* in its place.

attributes connoted by *S* are an indication of the presence of an individual belonging to the class *P*." This mode of interpretation is always a possible one, but it must be granted that only rarely does the import of a proposition naturally present itself to our minds in this form. There are, however, exceptional cases in which this reading is not unnatural. The proposition *No plants with opposite leaves are orchids* has already been given as an example. Another example is afforded by the proposition *All that glitters is not gold*. Taking the subject in connotation and the predicate in denotation* we have, *The attribute of glitter does not always indicate the presence of a gold object*; and it will be found that this reading of the proverb serves to bring out its meaning really much better than any of the three other readings which we have been discussing¹.

It is worth while noticing here by way of anticipation that if the view of the existential import of propositions which will be advocated in section 122 is accepted, we shall still have a fourfold reading of categorical propositions in connotation and denotation. The universal negative and the particular affirmative may be taken as examples. The universal negative yields the following: (1) There is no individual belonging to the class *S* and possessing the attributes connoted by *P*; (2) There is no in-

¹ Mr Welton considers it impossible to take the subject of a proposition in connotation and its predicate in denotation. "It is difficult," he says, "to see how a proposition expressed and interpreted strictly in this suggested form could have any meaning whatever. 'The attributes connoted by *man* are mortal beings' (or 'are the class, or a portion of the class, mortal beings') is such an expression, and it is, surely, hopelessly devoid of any intelligible significance. The only meaning it really expresses is, of course, absurdly false; the attributes *are not* beings, or a class of beings, of any sort mortal or otherwise" (*Logic*, p. 235). It may perhaps be sufficient to say in reply to this criticism that it is based on a misapprehension of the kind of relation which is implied in a proposition between the connotation of the subject and the denotation of the predicate. A similar misapprehension might reduce to absurdity any one of the interpretations above discussed. Supposing, for example, that we read the subject in denotation and the predicate in connotation, as Mr Welton says we are bound to do, we have an equally unintelligible expression if we say that "the class denoted by *man* is the attribute mortality." The error consists in imagining that the relation expressed in a proposition must always be the same whether we regard the terms from the extensive or from the intensive standpoint.

dividual common to the two classes *S* and *P*; (3) The attributes connoted by *S* and *P* respectively are never found conjoined; (4) There is no individual possessing the attributes connoted by *S* and belonging to the class *P*. Similarly the particular affirmative yields: (1) There are individuals belonging to the class *S* and possessing the attributes connoted by *P*; (2) There are individuals common to the two classes *S* and *P*; (3) The attributes connoted by *S* and *P* respectively are sometimes found conjoined; (4) There are individuals possessing the attributes connoted by *S* and belonging to the class *P*.

The question which has been discussed in this section is, therefore, quite independent of that which will be raised in chapter 7; and no solution of the general problem raised in this chapter can afford a complete solution of the problem of the import of categorical propositions. We have here analysed the relation between the things that are signified by the terms of a proposition; but the real predicative force of the categorical proposition remains to be decided.

98. *The Reading of Propositions in Comprehension.*—If, in taking the intensional standpoint, we consider comprehension instead of connotation, our problem is to determine what relation is implied in any proposition between the comprehension of the subject and the comprehension of the predicate. Asking this question in regard to the universal affirmative proposition *All S is P*, the solution clearly is that *the comprehension of S includes the comprehension of P*. The interpretation in comprehension is thus precisely the reverse of that in denotation (the denotation of *S* is included in the denotation of *P*); and we might be led to think that, taking the different propositional forms, we should have a scheme in comprehension, analogous throughout to that in denotation. But this is not the case, for the simple reason that in our ordinary statements we do not distributively quantify comprehension in the same way in which we do denotation; in other words, comprehension is always taken in its totality. Thus, reading an *I* proposition in denotation we have—*the classes S and P partly coincide*; and corresponding to this we should have—*the comprehension of S and P partly coincide*. But this is clearly not what we express

by *Some S is P*, for it is quite compatible with *No S is P*, that is to say, the classes *S* and *P* may be mutually exclusive, and yet some attributes may be common to the whole of *S* and also to the whole of *P*; for example, *No Pembroke undergraduates are also Trinity undergraduates*. Again, given an **E** proposition we have in denotation—the classes *S* and *P* have no part in common; but for the reason just given, it does not follow that *the comprehension of S and the comprehension of P have nothing in common*.

It is indeed necessary to obvert **I** and **E** in order to obtain a correct reading in comprehension. We then have the following scheme, in which the relation of contradiction between **A** and **O** and between **E** and **I** is made clearly manifest:

All S is P, The comprehension of *S* includes the comprehension of *P*;

No S is P, The comprehension of *S* includes the comprehension of *not-P*;

Some S is P, The comprehension of *S* does not include the comprehension of *not-P*;

Some S is not P, The comprehension of *S* does not include the comprehension of *P*.

CHAPTER VI.

LOGICAL EQUATIONS AND THE QUANTIFICATION OF THE PREDICATE.

99. *The employment of the symbol of Equality in Logic.*—The symbol of equality ($=$) is frequently used in logic to express the identity of two classes. For example,

Equilateral triangles = equiangular triangles ;

Exogens = dicotyledons ;

Men = mortal men.

It is, however, important to recognise that in thus borrowing a symbol from mathematics we do not retain its meaning unaltered, and that a so-called *logical equation* is, therefore, something very different from a mathematical equation. In mathematics the symbol of equality generally means numerical or quantitative equivalence. But clearly we do not mean to express mere numerical equality when we write *equilateral triangles = equiangular triangles*. Whatever this so-called equation signifies, it is certainly something more than that there are precisely as many triangles with three equal sides as there are triangles with three equal angles. It is further clear that we do not intend to express mere similarity. Our meaning is that the denotations of the terms which are equated are absolutely identical; in other words, that the class of objects denoted by the term *equilateral triangle* is absolutely identical with the class of objects denoted by the term *equiangular triangle*¹. It is urged, however, by some writers that, if

¹ It follows that the comprehensions (but of course not the connotations) of the terms will also be identical; this cannot, however, be regarded as the primary signification of the equation.

this is what our equation comes to, then inasmuch as a statement of mere identity is absolutely empty and meaningless, it strictly speaking leaves us with nothing at all; it contains no assertion and can represent no judgment¹. The answer to this criticism is that whilst we have identity in a certain respect, it is erroneous to say that we have *mere* identity. We have *identity of denotation* combined with *diversity of connotation*, and, therefore, with *diversity of determination* (meaning thereby diversity in the ways in which the application of the two terms identified is determined)². The meaning of this will be made clearer by the aid of one or two illustrations. Taking, then, as examples the logical equations already given, we may analyse their meaning as follows. If out of all triangles we select those which possess the property of having three equal sides, and if again out of all triangles we select those which possess the property of having three equal angles, we shall find that in either case we are left with precisely the same set of triangles. Thus, each side of our equation denotes precisely the same class of objects, but the class is determined or arrived at in two different ways. Similarly, if we select all plants that are exogenous and again all plants that are dicotyledonous, our results are precisely the same although our mode of arriving at them has been different. Once more, if we simply take the class of objects which possess the attribute of humanity, and again the class which possess both this attribute and also the attribute of mortality, the objects selected will be just the same, none will be excluded by our second method of selection although an additional attribute is taken into account.

Since the identity primarily signified by a logical equation is an identity in respect of denotation, any equational mode of reading propositions must be regarded as a modification of the

¹ Compare Bradley, *Principles of Logic*, pp. 23 to 27.

² I have practically borrowed the above mode of expression from Miss Jones, who describes an affirmative categorical proposition as "a proposition which asserts identity of application in diversity of signification" (*General Logic*, p. 20). Miss Jones's meaning may, however, be slightly different from that intended in the text, and I am unable to agree with her general treatment of the import of categorical propositions, as she does not appear to allow that before we can regard a proposition as asserting identity of application we must implicitly, if not explicitly, have quantified the predicate.

["class" mode. What has been said above, however, will make it clear that here as elsewhere denotation is considered not to the exclusion of connotation but as dependent upon it; and we again see how denotative and connotative readings of propositions are really involved in one another, although one side or the other may be made the more prominent according to the point of view which is taken.

Another point to which attention may be called before we pass on to consider different types of logical equations is that in so far as a proposition is regarded as expressing an identity between its terms the distinction between subject and predicate practically disappears. We have seen that when we have the ordinary logical copula *is*, propositions cannot always be simply converted, the reason being that the relation of the subject to the predicate is not the same as the relation of the predicate to the subject. But when two terms are connected by the sign of equality, they are similarly, and not diversely, related to each other. Such an equation, for example, as $S = P$ can be read either forwards or backwards without any alteration of meaning. There can accordingly be no distinction between subject and predicate except the mere order of statement, and that must be regarded as for all practical purposes immaterial.

100. *Types of Logical Equations*¹.—Jevons (*Principles of Science*, chapter 3) recognises three types of logical equations, which he calls respectively *simple identities*, *partial identities*, and *limited identities*.

Simple identities are of the form $S = P$; for example, *Exogens = dicotyledons*. Whilst this is the simplest case equationally, the information given by the equation requires two propositions in order that it may be expressed in ordinary predicative form. Thus, *All S is P* and *All P is S*; *All exogens are dicotyledons* and *All dicotyledons are exogens*. If, however, we are allowed to quantify the predicate as well as the subject, a single proposition will suffice. Thus, *All S is all P*, *All exogens are all dicotyledons*. We shall return presently to a consideration of this type of proposition.

¹ This section may be omitted on a first reading.

Partial identities are of the form $S = SP$, and are the expression equationally of ordinary universal affirmative propositions. If we take the proposition *All S is P*, it is clear that we cannot write it $S = P$, since the class P , instead of being coextensive with the class S , may include it and a good deal more besides. Since, however, by the law of identity *All S is S*, it follows from *All S is P* that *All S is SP*. We can also pass back from the latter of these propositions to the former. Hence the two propositions are equivalent. But *All S is SP* may at once be reduced to the equational form $S = SP$. For this breaks up into the two propositions *All S is SP* and *All SP is S*, and since the second of these is a mere formal proposition based on the law of identity, the equation must necessarily hold good if *All S is SP* is given. To take a concrete example, the proposition *All men are mortal* becomes equationally $Men = mortal\ men$.

Limited identities are of the form $VS = VP$, which may be interpreted "Within the sphere of the class V , all S is P and all P is S ," or "The S 's and P 's, which are V 's, are identical." So far as V represents a determinate class, there is little difference between these limited identities and simple identities. This is shewn by the fact that Jevons himself gives *Equilateral triangles = equiangular triangles* as an instance of a simple identity, whereas it is clear that its proper place in his classification is amongst the limited identities, for its interpretation is that "*within the sphere of triangles*—all the equilaterals are all the equiangulars."

The equation $VS = VP$ is, however, used by Boole—and also by Jevons subsequently—as the expression equationally of the particular proposition, and if it can really suffice for this, its recognition as a distinct type is of course justified. If we take the proposition *Some S is P*, we find that the classes S and P are affirmed to have some part in common, but no indication is given whereby this part can be identified. Boole, however, indicates it by the arbitrary symbol V . It is then clear that *All VS is VP* and also that *All VP is VS*, and we have the above equation.

It is no part of our present purpose to discuss systems of

symbolic logic, but it may be briefly pointed out that the above representation of the particular proposition is far from satisfactory. In order to justify it, limitations have to be placed upon the interpretation of V which altogether differentiate it from other class-symbols. Thus, the equation $VS = VP$ is consistent with *No S is P* (and, therefore, cannot be equivalent to *Some S is P*) provided that no V is either S or P , for in this case we have $VS = 0$ and $VP = 0$. V must, therefore, be limited by the antecedent condition that it represents an existing class and a class that contains either S or P , and it is in this condition quite as much as in the equation itself that the real force of the particular proposition is expressed¹.

If particular propositions are true contradictories of universal propositions, then it would seem to follow that in a system in which universals are expressed as equalities, particulars should be expressed as inequalities. This would mean the introduction of the symbols $>$ and $<$, related to the corresponding mathematical symbols in just the same way as the logical symbol of equality is related to the mathematical symbol of equality; that is to say, $S > SP$ would imply logically more than mere numerical inequality, it would imply that the class S includes the whole of the class SP and more besides. Thus interpreted, $S > SP$ expresses the particular negative proposition, *Some S is not P*². If we further introduce the symbol 0 as expressing nonentity, *No S is P* may be written $SP = 0$, and its contradictory, i.e., *Some S is P*, may be written $SP > 0$. We shall then have the following scheme (where $p = \text{not-}P$):

<i>All S is P</i>		expressed by $S = SP$ or by $Sp = 0$;
<i>Some S is not P</i>	„ „	$S > SP$ „ $Sp > 0$;
<i>No S is P</i>	„ „	$SP = 0$ „ $S = Sp$;
<i>Some S is P</i>	„ „	$SP > 0$ „ $S > Sp$.

This scheme, it will be observed, is based on the assumption

¹ Compare Venn, *Symbolic Logic*, pp. 161, 2.

² Similarly $X > Y$ expresses the two statements "All Y is X but Some X is not Y ", just as $X = Y$ expresses the two statements "All Y is X and All X is Y ."

that particulars are existentially affirmative while universals are existentially negative. This introduces a question which will be discussed in detail in the following chapter. The object of the present section is merely to illustrate the expression of propositions equationally, and the symbolism involved has, therefore, been treated as briefly as has seemed compatible with a clear explanation of its purport. Any more detailed treatment would involve a discussion of problems belonging to symbolic logic.

101. *The expression of Propositions as Equations.*—There are rare cases in which propositions fall naturally into what is practically an equational form; for example, *Civilization and Christianity are co-extensive*. But, speaking generally, the equational relation, as implicated in ordinary propositions, is not one that is spontaneously, or even easily, grasped by the mind. Hence as a psychological account of the process of judgment the equational rendering may be rejected. It is, moreover, not desirable that equations should supersede the generally recognised propositional forms in ordinary logical doctrine, for such doctrine should not depart more than can be helped from the forms of ordinary speech. But, on the other hand, the equational treatment of propositions must not be simply put on one side as erroneous or unworkable. It has been shewn in the preceding section that it is at any rate possible to reduce all categorical propositions to a form in which they express equalities or inequalities; and such reduction is of the greatest importance in systems of symbolic logic. Even for purposes of ordinary logical doctrine, the enquiry how far propositions may be expressed equationally serves to afford a more complete insight into their full import, or at any rate their full implication. Hence while ordinary formal logic should not be entirely based upon an equational reading of propositions, it cannot afford altogether to neglect this way of regarding them.

We may pass on to consider in somewhat more detail a special equational or semi-equational system—open also to special criticisms—by which Hamilton and others sought to revolutionise ordinary logical doctrine.

102. *The eight propositional forms resulting from the explicit Quantification of the Predicate.*—We have seen that in the ordinary fourfold schedule of propositions, the quantity of the predicate is determined by the quality of the proposition, negatives distributing their predicates, while affirmatives do not. It seems a plausible view, however, that by explicit quantification the quantity of the predicate may be made independent of the quality of the proposition, and Sir William Hamilton was thus led to recognise eight distinct propositional forms instead of the customary four:—

<i>All S is all P,</i>	U.
<i>All S is some P,</i>	A.
<i>Some S is all P,</i>	Y.
<i>Some S is some P,</i>	I.
<i>No S is any P,</i>	E.
<i>No S is some P,</i>	η .
<i>Some S is not any P,</i>	O.
<i>Some S is not some P.</i>	ω .

The symbols attached to the different propositions in the above schedule are those employed by Archbishop Thomson¹, and they are the ones now commonly adopted so far as the quantification of the predicate is recognised in modern textbooks.

The symbols used by Hamilton were *Afa, Afi, Ifa, Ifi, Ana, Ani, Ina, Ini*. Here *f* indicates an affirmative proposition, *n* a negative; *a* means that the corresponding term is distributed, *i* that it is undistributed.

Spalding's symbols (*Logic*, p. 83) are $A^2, A, I^2, I, E, \frac{1}{2}E, O, \frac{1}{2}O$. Mr Carveth Read (*Theory of Logic*, p. 193) suggests $A^2, A, I^2, I, E, E_2, O, O_2$.

The equivalence of these various symbols is shewn in the following table:—

¹ Thomson himself, however, ultimately rejects the forms η and ω .

	Thomson.	Hamilton.	Spalding.	Carveth Read.
<i>All S is all P</i>	U	<i>Afa</i>	A^2	A^2
<i>All S is some P</i>	A	<i>Afi</i>	A	A
<i>Some S is all P</i>	Y	<i>Ifa</i>	I^2	I^2
<i>Some S is some P</i>	I	<i>Ifi</i>	I	I
<i>No S is any P</i>	E	<i>Ana</i>	E	E
<i>No S is some P</i>	η	<i>Ani</i>	$\frac{1}{2}E$	E_2
<i>Some S is not any P</i>	O	<i>Ina</i>	O	O
<i>Some S is not some P</i>	ω	<i>Ini</i>	$\frac{1}{2}O$	O_2

For the new forms we might also use the symbols SuP , SyP , $S\eta P$, $S\omega P$, on the principle explained in section 40.

103. *Sir William Hamilton's fundamental Postulate of Logic.*—The fundamental postulate of logic, according to Sir William Hamilton, is “that we be allowed to state explicitly in language all that is implicitly contained in thought”; and it will be well to consider very briefly the meaning to be attached to this postulate before going on to discuss the use that is made of it in connexion with the doctrine of the quantification of the predicate.

Giving the natural interpretation to the phrase “implicitly contained in thought,” the postulate might at first sight appear to be a broad statement of the general principle underlying the logician’s treatment of formal inferences. In all such inferences the conclusion is implicitly contained in the premisses; and since logic has to determine what inferences follow legitimately from given premisses, it may in this sense be said to be part of the function of logic to make *explicit in language* what is *implicitly contained in thought*.

It seems clear, however, from the use made of the postulate by Hamilton and his school that he is not thinking of this, and indeed that he is not intending any reference to *discursive*

thought at all. His meaning rather is that we should make explicit in language not what is implicit in thought but what is explicit in thought, or, as it may be otherwise expressed, that we should make explicit in language all that is really present in thought in the act of judgment.

Adopting this interpretation, we may come to the conclusion that the postulate is very obscurely expressed, but we can have no hesitation in admitting its validity. It is obviously of importance to the logician to clear up all ambiguities and ellipses of language. For this reason it is, amongst other things, desirable that we should as far as possible avoid in logic condensed and elliptical modes of expression. But whether Hamilton's postulate, as now interpreted, supports the doctrine of the quantification of the predicate is another question. This point will be considered in the next two sections.

104. *Advantages claimed for the Quantification of the Predicate.*—Hamilton maintains that "in thought the predicate is always quantified," and hence he makes it follow immediately from the postulate discussed in the preceding section that "in logic, the quantity of the predicate must be expressed, on demand, in language." "The quantity of the predicate," says Dr Baynes in the authorised exposition of Hamilton's doctrine contained in his *New Analytic of Logical Forms*, "is not expressed in common language because common language is elliptical." Whatever is not really necessary to the clear comprehension of what is contained in thought, is usually elided in expression. But we must distinguish between the ends which are sought by common language and logic respectively. Whilst the former seeks to exhibit with clearness the matter of thought, the latter seeks to exhibit with exactness the form of thought. Therefore in logic the predicate must always be quantified." It is further maintained that the quantification of the predicate is necessary for intelligible predication. "Predication is nothing more or less than the expression of the relation of quantity in which a notion stands to an individual, or two notions to each other. If this relation were indeterminate—if we were uncertain whether it was of part, or whole, or none—there could be no predication."

Amongst the practical advantages said to result from quantifying the predicate are the reduction of all species of the conversion of propositions to one, namely, simple conversion; and the simplification of the laws of syllogism. As regards the first of these points, it may be observed that if we adopt the doctrine of the quantification of the predicate, the distinction between subject and predicate resolves itself into a difference in order of statement alone. Each propositional form can without any alteration in meaning be read either forwards or backwards, and every proposition may, therefore, rightly be said to be simply convertible.

It is further argued that the new propositional forms resulting from the quantification of the predicate are required in order to express relations that cannot otherwise be so simply expressed. Thus, U alone serves to express the fact that two classes are co-extensive; and even ω is said to be needed in logical divisions, since if we divide (say) Europeans into Englishmen, Frenchmen, &c., this requires us to think that some Europeans are not some Europeans (*e.g.*, Englishmen are not Frenchmen).

105. *Objections urged against the Quantification of the Predicate.*—Those who reject Hamilton's doctrine of the quantification of the predicate deny at the outset the fundamental premiss upon which it is based, namely, that the predicate of a proposition is always thought of as a determinate quantity. They go further and deny that it is as a rule thought of as a quantity, that is, as an aggregate of objects, at all. We have already in section 97 indicated grounds for the view that in the great majority of instances the subject of a proposition is in ordinary thought naturally interpreted in denotation, but the predicate in connotation. This psychological argument is valid against Hamilton, inasmuch as he really bases his doctrine upon a psychological consideration; and it seems unanswerable.

Mill (in his *Examination of Hamilton*, pp. 495-7) puts the point as follows: "I repeat the appeal which I have already made to every reader's consciousness: Does he, when he judges that all oxen ruminates, advert even in the minutest degree to the question, whether there is anything else which ruminates?"

Is this consideration at all in his thoughts, any more than any other consideration foreign to the immediate subject? One person may know that there are other ruminating animals, another may think that there are none, a third may be without any opinion on the subject: but if they all know what is meant by ruminating, they all, when they judge that every ox ruminates, mean exactly the same thing. The mental process they go through, as far as that one judgment is concerned, is precisely identical; though some of them may go on further, and add other judgments to it. The fact, that the proposition 'Every A is B ' only means 'Every A is *some* B ,' so far from being always present in thought, is not at first seized without some difficulty by the tyro in logic. It requires a certain effort of thought to perceive that when we say, 'All A 's are B 's,' we only identify A with a portion of the class B . When the learner is first told that the proposition 'All A 's are B 's' can only be converted in the form 'Some B 's are A 's,' I apprehend that this strikes him as a new idea; and that the truth of the statement is not quite obvious to him, until verified by a particular example in which he already knows that the simple converse would be false, such as, 'All men are animals, therefore, all animals are men.' So far is it from being true that the proposition 'All A 's are B 's' is spontaneously quantified in thought as 'All A is some B .'"

A word may be added in reply to the argument that if the quantity of the predicate were indeterminate—if we were uncertain whether the reference was to the whole or part or none—there could be no predication. This is perfectly true so long as we are left with all three of these alternatives; but we may have predication which involves the elimination of only one of them, so that there is still indeterminateness as regards the other two. To argue that unless we are definitely limited to one of the three we are left with all of them is practically to confuse contradictory with contrary opposition.

A further objection that is raised to the doctrine of the quantification of the predicate is that some of the quantified forms are composite not simple predications. Thus *All S is all P* is a condensed mode of expression, which may be analysed

into the two propositions *All S is P* and *All P is S*. Similarly, if we interpret *some* as exclusive of *all*, a point to which we shall presently return, *All S is some P* is an exponible proposition resolvable into *All S is P* and *Some P is not S*. But, as Professor Fowler observes, "it is the object of logic not to state our thoughts in a condensed form, but to analyse them into their simplest elements" (*Deductive Logic*, p. 32). As a rule, the use of exponible forms tends to make the detection of fallacy the more difficult, and this general consideration applies with undoubted force to the particular case of the quantification of the predicate. The bearing of the quantification doctrine upon the syllogism will be briefly touched upon subsequently, and it will be found that the problem of discriminating between valid and invalid moods is rendered more complex and difficult. It may indeed be doubted whether any logical problem, with the one exception of conversion, is really simplified by the introduction of quantified predicates.

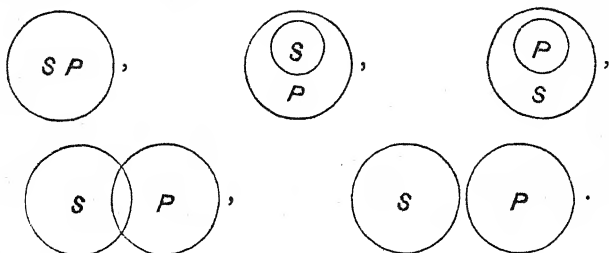
Even apart from the above objections, the Hamiltonian doctrine of quantification is sufficiently condemned by its want of internal consistency. Its unphilosophical character in this respect will be shewn in the following sections.

106. *The meaning to be attached to the word "some" in the eight propositional forms recognised by Sir William Hamilton.*—Professor Baynes, in his authorised exposition of Sir William Hamilton's new doctrine, would at the outset lead us to suppose that we have no longer to do with the indeterminate *some* of the Aristotelian Logic, but that this word is now to be used in the more definite sense of *some, but not all*. He argues, as we have seen, that intelligible predication requires an absolutely determinate relation in respect of quantity between subject and predicate, and that this ought to be clearly expressed in language. Thus, "if the objects comprised under the subject be some part, but not the whole, of those comprised under the predicate, we write *All X is some Y*, and similarly with other forms."

But if it is true that we know definitely the relative extent of subject and predicate, and if *some* is used strictly in the sense of *some but not all*, we should have but *five* propositional forms

instead of eight, namely,—*All S is all P, All S is some P, Some S is all P, Some S is some P¹, No S is any P.*

We have already seen (in section 89) that the only possible relations between two terms in respect of their extension are given by the five diagrams—



These correspond respectively to the above five propositions²; and it is clear that on the view indicated by Dr Baynes the eight forms are redundant³.

It is altogether doubtful whether writers who have adopted the eightfold scheme have themselves recognised the pitfalls surrounding the use of the word *some*. Many passages might be quoted in which they distinctly adopt the meaning—*some but not all*. Thus, Thomson (*Laws of Thought*, p. 150) makes **U** and **A** inconsistent. Bowen (*Logic*, pp. 169, 170) would pass from **I** to **O** by immediate inference⁴. Hamilton himself agrees with Thomson and Bowen on these points; but he is curiously indecisive on the general question here raised. He remarks (*Logic*, II. p. 282) that *some* "is held to be a definite *some* when the other term is definite," i.e., in **A** and **Y**, **η** and **O**: but "on the other hand, when both terms are indefinite or particular, the *some* of each is left wholly indefinite," i.e., in **I** and **ω**⁵.

¹ Using *some* in the sense here indicated, *Some S is some P* necessarily implies *Some S is not any P* and *No S is some P*.

² Namely **U**, **A**, **Y**, **I**, **E**. **O** and **η** cannot be interpreted as giving precisely determinate information; **O** allows an alternative between **Y** and **I**, and **η** between **A** and **I**. For the interpretation of **ω** see note 2 on p. 178.

³ Cf. Venn, *Symbolic Logic*, chapter 1.

⁴ "This sort of Inference," he remarks, "Hamilton would call *Integration*, as its effect is, after determining one part, to reconstitute the whole by bringing into view the remaining part."

⁵ Compare Veitch, *Institutes of Logic*, pp. 307 to 310, and 367, 8. "Hamilton would introduce *some only* into the theory of propositions, without, however,

This is very confusing, and it would be most difficult to apply the distinction consistently. Hamilton himself certainly does not so apply it. For example, on his view it should no longer be the case that two affirmative premisses necessitate an affirmative conclusion; or that two negative premisses invalidate a syllogism¹. Thus, the following should be regarded as valid:—

All P is some M,
All M is some S,
 therefore, *Some S is not any P.*

No M is any P,
Some S is not any M,
 therefore, *Some or all S is not any P.*

Such syllogisms as these, however, are not admitted by Hamilton and Thomson; and, on the other hand, Thomson admits as valid certain combinations which on the above interpretation are not valid. Hamilton's supreme canon of the categorical syllogism is:—"What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related; that relation subsists between the two terms themselves" (*Logic*, II. p. 357). This clearly provides that one premiss at least shall be affirmative, and that an affirmative conclusion shall follow from two affirmative premisses. Thomson (*Laws of Thought*, p. 165) explicitly lays down the same rules; and his table of valid moods (given on p. 188) is (with the exception of one obvious misprint) correct and correct only if *some* means "some, it may be all."

discarding the meaning of *some at least*. It is not correct to say that Hamilton discarded the ordinary logical meaning of *some*. He simply supplemented it by introducing into the propositional forms that of *some only*." "*Some*, according to Hamilton, is always thought as semi-definite (*some only*) where the other term of the judgment is universal." Mr Lindsay, however, in expounding Hamilton's doctrine (*Appendix to Ueberweg's System of Logic*, p. 580) says more decisively—"Since the subject must be equal to the predicate, vagueness in the predesignations must be as far as possible removed. *Some* is taken as equivalent to *some but not all*." Spalding (*Logic*, p. 184) definitely chooses the other alternative. He remarks that in his own treatise "the received interpretation *some at least* is steadily adhered to."

¹ The anticipation of syllogistic doctrine which follows is necessary in order to illustrate the point which we are just now discussing.

107. *The use of "some" in the sense of "some only."*—Jevons, in reply to the question, "What results would follow if we were to interpret 'Some *A*'s are *B*'s' as implying that 'Some other *A*'s are not *B*'s'?" writes, "The proposition 'Some *A*'s are *B*'s' is in the form **I**, and according to the table of opposition **I** is true if **A** is true; but **A** is the contradictory of **O**, which would be the form of 'Some other *A*'s are not *B*'s.' Under such circumstances **A** could never be true at all, because its truth would involve the truth of its own contradictory, which is absurd" (*Studies in Deductive Logic*, p. 151). It is not, however, the case that we necessarily involve ourselves in self-contradiction if we use *some* in the sense of *some only*. What should be pointed out is that if we use the word in this sense, the truth of **I** no longer follows from the truth of **A**; but on the other hand these two propositions are inconsistent with each other.

Taking the five propositional forms which are obtained by attaching this meaning to *some*, namely, *All S is all P*, *All S is some P*, *Some S is all P*, *Some S is some P*, *No S is P*, it should be observed that each one of these propositions is inconsistent with each of the others, whilst at the same time no one is the contradictory of any one of the others. If, for example, on this scheme we wish to express the contradictory of **U**, we can do so only by affirming an alternative between **Y**, **A**, **I**, and **E**. Nothing of all this appears to have been noticed by the Hamiltonian writers. Thus, Thomson (*Laws of Thought*, p. 149) gives a scheme of opposition in which **I** and **E** appear as contradictories, but **A** and **O** as contraries.

One of the strongest arguments against the use of *some* in the sense of *some only* is very well put by Professor Veitch, himself a disciple of Sir William Hamilton. *Some only*, he remarks, is not so fundamental as *some at least*. The former implies the latter; but I can speak of *some at least* without advancing to the more definite stage of *some only*. "Before I can speak of *some only*, must I not have formed two judgments—the one that *some are*, the other that others of the same class *are not*?.....The *some only* would thus appear as the composite of two propositions already formed.....It seems to me that we must, first of all, work out logical principles on the

indefinite meaning of *some at least*.....*Some only* is a secondary and derivative judgment" (*Institutes of Logic*, p. 308).

If *some* is used in the sense of *some only*, the further difficulty arises how we are to express any knowledge that we may happen to possess about a part of a class when we are in ignorance in regard to the remainder. Supposing, for example, that all the *S*'s of which I happen to have had experience are *P*'s, I am not justified in saying either that *all S's are P's* or that *some S's are P's*. The only solution of the difficulty is to say that *all or some S's are P's*. The complexity that this would introduce is obvious.*

108. *The interpretation of the eight Hamiltonian forms of proposition, "some" being used in its ordinary logical sense*¹.—Taking the five possible relations between two terms, as illustrated by the Eulerian diagrams, and denoting them respectively by α , β , γ , δ , ϵ , as in section 89, we may write against each of the propositional forms the relations which are compatible with it, on the supposition that *some* is used in its ordinary logical sense, that is, as exclusive of *none* but not of *all*²:—

U	α
A	α, β
Y	α, γ
I	$\alpha, \beta, \gamma, \delta$
E	ϵ
η	β, δ, ϵ
O	γ, δ, ϵ
ω	$\alpha, \beta, \gamma, \delta, \epsilon$

¹ The corresponding interpretation when *some* is used in the sense of *some only* is given in note 2, page 172, and in note 2, p. 178.

² If the Hamiltonian writers had attempted to illustrate their doctrine by means of the Eulerian diagrams, they would I think either have found it to be unworkable, or they would have worked it out to a more distinct and consistent issue.

We have then the following pairs of contradictories—**A, O**; **Y, η** ; **I, E**. The contradictory of **U** is obtained by affirming an alternative between η and **O**.

Without the use of quantified predicates, the same information may be expressed as follows :—

$$\mathbf{U} = SaP, PaS;$$

$$\mathbf{A} = SaP;$$

$$\mathbf{Y} = PaS;$$

$$\mathbf{I} = SiP;$$

$$\mathbf{E} = SeP;$$

$$\eta = PoS;$$

$$\mathbf{O} = SoP.$$

What information, if any, is given by ω will be discussed in section 111.

109. *The propositions U and Y.*—It must be admitted that these propositions are met with in ordinary discourse. We may not indeed find propositions which are actually written in the form *All S is all P*; but we have to all intents and purposes **U**, whenever there is an unmistakeable affirmation that the subject and the predicate of a proposition are co-extensive. Thus, all definitions are practically **U** propositions; so are all affirmative propositions of which both the subject and the predicate are singular terms¹. Take also such propositions as the following: Christianity and civilization are coextensive; Europe, Asia, Africa, America, and Australia are all the continents²; The three whom I have mentioned are all who have ever ascended the mountain by that route; Common salt is the same thing as sodium chloride³.

¹ Take the proposition, "Mr Gladstone is the present Prime Minister." If any one denies that this is **U**, then he must deny that the proposition, "Mr Gladstone is an Englishman" is **A**. We have at an earlier stage discussed the question how far singular propositions may rightly be regarded as constituting a sub-class of universals.

² In this and the example that follows the predicate is clearly quantified universally; so that if these are not **U** propositions, they must be **Y** propositions. But it is equally clear that the subject denotes the whole of a certain class, however limited that class may be.

³ These are all examples of what Jevons would call *simple identities* as distinguished from *partial identities*. Compare section 100.

Again, any exclusive proposition (as defined in section 48) may be given as an example of *Y*; e.g., *Only S is P*; Graduates alone are eligible for the appointment; Some passengers are the only survivors. These propositions are respectively equivalent to the following: *Some S is all P*; Some graduates are all who are eligible for the appointment; Some passengers are all the survivors¹. Moreover, as was shewn in section 48, this is the only way of treating the propositions which will enable us to retain the original subjects and predicates.

We cannot then agree with Professor Fowler that the additional forms "are not merely unusual, but are such as we never do use" (*Deductive Logic*, p. 31). **U** and **Y**² ought certainly to receive some recognition in logic. Still in treating the syllogism &c. on the traditional lines, it is better to retain the traditional schedule of propositions. The addition of **U** and **Y** does not tend towards simplification, but the reverse; and their full force can be expressed in other ways. On this view, when we meet with a **U** proposition, *All S is all P*, we may resolve it into the two **A** propositions, *All S is P* and *All P is S*, which taken together are equivalent to it; and when we meet with a **Y** proposition, *Some S is all P* or *S alone is P*, we may replace it by the **A** proposition *All P is S*, which it yields by conversion.

110. *The proposition η .*—This proposition in the form *No S is some P* is not I think ever found in ordinary use. We may, however, recognise its possibility; and it must be pointed out that a form of proposition which we do meet with, namely, *Not only S is P* or *Not S alone is P*, is practically η , provided that we do not regard this proposition as implying that any *S* is certainly *P*.

Archbishop Thomson remarks that η "has the semblance only, and not the power of a denial. True though it is, it does not prevent our making another judgment of the affirmative kind, from the same terms" (*Laws of Thought*, § 79). This is erroneous; for although **A** and η may be true together, **U** and

¹ In these propositions, *some* is of course to be interpreted in the indefinite sense, and not as exclusive of *all*.

² The *some* of the subject being interpreted as meaning merely *some at least*.

η cannot, and \mathbf{Y} and η are strictly contradictories¹. The relation of contradiction in which \mathbf{Y} and η stand to each other is perhaps brought out more clearly if they are written in the forms *Only S is P*, *Not only S is P*, or *S alone is P*, *Not S alone is P*. It will be observed, moreover, that η is the converse of \mathbf{O} , and *vice versa*. If, therefore, η has no power of denial, the same will be true of \mathbf{O} also. But it certainly is not true of \mathbf{O} .

111. *The proposition ω .*—The proposition ω , *Some S is not some P*, is not inconsistent with any of the other propositional forms, not even with \mathbf{U} , *All S is all P*. For example, granting that “all equilateral triangles are all equiangular triangles,” still “this equilateral triangle is not that equiangular triangle,” which is all that ω asserts. *Some S is not some P* is indeed always true except when both the subject and the predicate are the name of an individual and the same individual². De Morgan³ (*Syllabus*, p. 24) points out that its contradictory is—“*S* and *P* are singular and identical ; there is but one *S*, there is but one *P*, and *S* is *P*.”⁴ It may be said without hesitation that the proposition ω is of absolutely no logical importance.

¹ We are again interpreting *some* as indefinite. If it means *some at most*, then the power of denial possessed by η is increased.

² *Some* being again interpreted in its ordinary logical sense. Mr*Johnson points out that if *some* means *some but not all*, we are led to the paradoxical conclusion that ω is equivalent to \mathbf{U} . *Some but not all S is not some but not all P* informs us that certain *S*'s are not to be found amongst a certain portion of the *P*'s but that they are to be found amongst the remainder of the *P*'s, while the remaining *S*'s are to be found amongst the first set of *P*'s. Hence *all S is P*; and it follows similarly that *all P is S*. *Some but not all S is not some but not all P* is, therefore, equivalent to *All S is all P*.

³ De Morgan in several passages criticizes with great acuteness the Hamiltonian scheme of propositions.

⁴ Professor Veitch remarks that in ω “we assert parts, and that these can be divided, or that there are parts and parts. If we deny this statement, we assert that the thing spoken of is indivisible or a unity.....We may say that there are men and men. We say, as we do every day, there are politicians and politicians, there are ecclesiastics and ecclesiastics, there are sermons and sermons. These are but covert forms of the *some are not some*..... ‘Some vivisection is not some vivisection’ is true and important; for the one may be with an anæsthetic, the other without it” (*Institutes of Logic*, pp. 320, 1). It will be observed that the proposition *There are politicians and politicians* is here given as a typical example of ω . The appropriateness of this is denied by Mr Monck. “Again, can it be said that the proposition *There are patriots and*

112. *Sixfold schedule of propositions obtained by recognising Y and η , in addition to A, E, I, O¹.*—The schedule of propositions obtained by adding Y and η to the ordinary schedule presents many interesting features, and is worthy of incidental recognition and discussion². It has been shewn in section 66 that in the ordinary scheme there are six and only six independent propositions connecting any two terms, namely, *SaP, PaS, SeP (= PeS), SiP (= PiS), PoS, SoP*. If we write the second and the last but one of these in such a form as to keep *S* and *P* as subject and predicate throughout, we have the schedule which we are now considering, namely,

$$\begin{aligned} SaP &= \text{All } S \text{ is } P; \\ SyP &= \text{Only } S \text{ is } P; \\ SeP &= \text{No } S \text{ is } P; \\ SiP &= \text{Some } S \text{ is } P; \\ S\eta P &= \text{Not only } S \text{ is } P; \\ SoP &= \text{Some } S \text{ is not } P. \end{aligned}$$

It will be observed that the pair of propositions, *SyP* and *S η P*, are contradictories; so that we now have three pairs of contradictories. There are of course other additions to the traditional table of opposition, and some new relations will need to be recognised, *e.g.*, between *SaP* and *SyP*. With the help, however, of the discussion contained in section 73, the reader will have no difficulty in working out the required hexagon of opposition for himself.

As regards immediate inferences, we cannot in this scheme

patriots is adequately rendered by *Some patriots are not some patriots*? The latter proposition simply asserts non-identity: the former is intended to imply also a certain degree of dissimilarity [*i.e.*, in the characteristics or consequences of the patriotism of different individuals]. But two non-identical objects may be perfectly alike" (*Introduction to Logic*, p. xiv).

¹ In this schedule *some* is interpreted throughout in its ordinary logical sense. *U* is omitted on account of its composite character; its inclusion would also destroy the symmetry of the scheme.

² It is not of course intended that this sixfold schedule should supersede the fourfold schedule in the main body of logical doctrine. It is, however, most important to remember that the selection of any one schedule is more or less arbitrary, and that no schedule should be set up as authoritative to the exclusion of all others.

obtain any satisfactory obverse of either **Y** or η , the reason being that they have quantified predicates, and that, therefore, the negation cannot in these propositions be simply attached to the predicate. We have, however, the following interesting table of other immediate inferences¹:—

	Converse.	Contrapositive.	Inverse.
SaP	$= PyS$	$= P'aS'$	$= S'yP'$
SyP	$= PaS$	$= P'yS'$	$= S'aP'$
SeP	$= PeS$	$= P'yS$	$= S'yP$
SiP	$= PiS$	$= P'\eta S$	$= S'\eta P$
$S\eta P$	$= PoS$	$= P'\eta S'$	$= S'oP'$
SoP	$= P\eta S$	$= P'oS'$	$= S'\eta P'$

The main points to notice here are (1) that each proposition now admits of conversion, contraposition, and inversion; and (2) that the inferred proposition is in every case *equivalent* to the original proposition, so that there is not in any of the inferences any loss of logical force. In other words, we obtain in each case a *simple* converse, a *simple* contrapositive, and a *simple* inverse.

EXERCISES.

113. Explain precisely how it is that **O** admits of ordinary conversion if the principle of the quantification of the predicate is adopted, although not otherwise. [K.]

114. Draw out a table, corresponding to the ordinary Aristotelian table of opposition, for the six propositions, **A**, **Y**, **E**, **I**, η , **O** (some being interpreted in the sense of *some at least*). [K.]

¹ It will be observed that the impracticability of obverting **Y** and η leads to a certain want of symmetry in the third and fourth columns.

CHAPTER VII.

a grand + joyful!

THE EXISTENTIAL IMPORT OF CATEGORICAL PROPOSITIONS¹.

115. *Existence and the Universe of Discourse.*—The discussion of "existence" upon which we are about to enter is no kind of metaphysical enquiry. By existence is merely meant membership of the universe of discourse, whatever that may happen to be; and the object of our enquiry is to determine the extent, if any, to which membership of the universe of discourse is implied in categorical propositions, and the ways in which ordinary logical doctrines are affected by the different answers that may be given to this question.

Experience shews that much difficulty is found in grasping the conception of a universe of discourse, and in attaching the intended meaning to membership of that universe. It will, therefore, be desirable to discuss this point in some little detail.

One source of misunderstanding may be cleared out of the way by raising the question at the outset whether there is any sense in which the existence of objects must necessarily be postulated in all predication. This question must certainly be answered in the affirmative, inasmuch as something corresponding to every term that can be used in an intelligible sense must exist as an object of thought, even if it exist in no other sphere than this. Unless the terms of a proposition satisfy this condition, we cannot properly speaking be said to have a proposition at all, that is, the expression in words of an actual judgment; for we cannot have more than a mere unmeaning jumble of words. It may be noted further that the contradictories of our terms must also exist as objects of thought, since by the law of relativity we can only know a thing as distinguished from some-

¹ It will be advisable for students, on a first reading, to omit this chapter.

thing else. Let it be understood then that existence in this sense—"logical" existence, as it has sometimes been called—is not in question in the discussion which follows. Something more than mere existence in the sphere of thought is meant when we talk about existence in the universe of discourse.

It will be admitted that whatever else is included in the full implication of a universal proposition, it at least denies the existence of a certain class of objects. *No S is P* denies the existence of objects that are both *S* and *P*; *All S is P* denies the existence of objects that are *S* without also being *P*. But it is clear that in these propositions we do not, at any rate generally speaking¹, intend to deny the *logical* existence of *SP* (or *SP'*) in the sense above indicated; that is to say, we do not intend to deny the existence of *SP* (or *SP'*) as objects of thought. For example, in the proposition *No roses are blue* it is not our intention to deny that we can form an idea of *blue roses*; nor in the proposition *All ruminant animals are cloven-hoofed* is it our intention to deny that *ruminant animals without cloven hoofs* can exist as objects of thought. These illustrations will assist the reader in distinguishing between mere logical existence and existence in the universe of discourse. *The universe of discourse in the case of the proposition No S is P is the universe (whatever it may be) in which the existence of SP is denied.* The universe of discourse in the case of a universal affirmative proposition may be defined similarly. As regards particulars it may be best to seek an interpretation through the universals by which the particulars are contradicted. Thus, the universe of discourse in the case of the proposition *Some S is P* may be defined as the universe (whatever it may be) in which the existence of *SP* would be understood to be denied in the corresponding universal negative. The proposition *Some S is not P* may be dealt with similarly.

The question whether the assertion of a categorical proposition necessarily implies that its terms are the names of actually existing things may then be interpreted as follows: *Given a categorical proposition with S and P as subject and predicate, is*

¹ The only exception will be where the universe of discourse coincides with the universe of the conceivable.

the existence of S or of P necessarily implied in that sphere (whatever it may be) in which the existence of SP (or SP') is denied by the proposition (or by its contradictory)?

It is clear from the above account that the universe of discourse will vary in different cases. It may be the whole universe of things, using the word "thing" in its very widest signification; but more usually the reference is to some limited universe¹. It is specially important to bear in mind that the universe of discourse is by no means necessarily identical with the region of what we ordinarily call "fact"; it may be the universe of dreams, or of imagination, or of some particular realm of imagination, *e.g.*, modern fiction, or fairy land, or the world of the Homeric poems.

The problem of determining what is actually the universe of discourse in any particular case belongs to the matter and not to the form of thought. All that formal logic is concerned with is that the universe of discourse shall remain the same throughout any given argument. No one, however, who is at all acquainted with the subject-matter is likely to be in doubt as regards the universe of discourse in the case of any given concrete proposition. Thus, in the propositions *No roses are blue*, *All men are mortal*, *All ruminant animals are cloven-hoofed*, the reference clearly is to the actual physical universe; in *The wrath of the Olympian Gods is very terrible* to the universe of the Greek mythology; in *Fairies are able to assume different forms* to the universe of folk-lore; in *Two straight lines cannot enclose a space* to the universe of the imaginable.

116. Formal Logic and the Existential Import of Propositions.—A fundamental objection may perhaps be raised at this point. Formal logic, it may be said, cannot possibly from its very nature have any concern with questions relating to existence in any other sense than mere "logical" existence. The function of the formal logician is to distinguish between

¹ "The universe of discourse is sometimes limited to a small portion of the actual universe of things, and is sometimes co-extensive with that universe" (Boole, *Laws of Thought*, p. 166). On the conception of a limited universe of discourse, compare also De Morgan, *Syllabus of a Proposed System of Logic*, §§ 122, 3, and *Formal Logic*, p. 55; Venn, *Symbolic Logic*, pp. 127, 8; and Jevons, *Principles of Science*, chapter 3, § 4.

that which is self-consistent and that which is self-contradictory. Putting the same thing in another form, it is his business to distinguish between what can and what cannot exist in the world of thought. But beyond this he cannot go. The conditions of empirical existence are material, and therefore beyond the scope of formal logic.

The above argument may be met by clearly defining our position. It is of course no function of formal logic to determine whether or not certain classes actually exist in any given universe of discourse, any more than it is the function of formal logic to determine whether given propositions are true or false. But it does not follow that formal logic has, therefore, no concern with any questions relating to empirical existence. For, just as certain propositions being given true, formal logic determines what other propositions will as a consequence also be true, so given an assertion or a set of assertions to the effect that certain combinations do or do not exist in a given universe of discourse, it can determine what other assertions about existence in the same universe of discourse follow therefrom¹. As a matter of fact, the premisses in any argument necessarily contain certain implications in regard to existence in the particular universe of discourse to which reference is made, and the same is true of the conclusion; it is accordingly essential that the logician should make sure that the latter implications are clearly warranted by the former.

Without at present going into any detail we may very briefly indicate one or two questions relating to empirical existence, which cannot be altogether excluded from consideration in formal logic. Universal propositions, as we have already seen, assert non-existence in some sphere other than that of thought; and the logician cannot properly bring out their full import without calling attention to this fact. Again, the proposition *All S is P* at least involves that if there are any *S*'s

¹ The latter part of this statement is indeed nothing more than a repetition of the former part from a rather different point of view. The doctrine that the conclusions reached by the aid of formal logic can never do more than relate to what is merely conceivable is a very mischievous error. The material truth of the conclusion of a formal reasoning is only limited by the material truth of the premisses.

in the universe of discourse, there must also be some P 's, while it does not seem necessarily to involve that if there are any P 's there must be some S 's¹. But now convert the proposition. The result is *Some P is S* , and this does involve that if there are any P 's there must be some S 's. How then can the process of conversion be shewn to be valid without some assumption which will serve to justify this latter implication? Similarly, in passing from *All S is P* to *Some not- S is not- P* , it must at least be assumed that if S does not constitute the entire universe of discourse, neither does P do so.

Jevons remarks that he does not see how there can be in deductive logic any question about existence, and observes, with reference to the opposite view taken by De Morgan, that "this is one of the few points in which it is possible to suspect him of unsoundness" (*Studies in Deductive Logic*, p. 141). It is, however, impossible to attach any meaning to Jevons's own "Criterion of Consistency," unless it has some reference to "existence." "It is assumed as a necessary law that every term must have its negative. Thence arises what I propose to call the *Criterion of Consistency*, stated as follows:—*Any two or more propositions are contradictory when, and only when, after all possible substitutions are made, they occasion the total disappearance of any term, positive or negative, from the Logical Alphabet*" (p. 181). What can this mean but that although we may deny the existence of the combination AB , we cannot without contradiction deny the existence of A itself, or *not- A* , or B , or *not- B* ? This assumption regarding the existential implication of propositions runs through the whole of Jevons's equational logic. The following passage, for example, is taken quite at random: "There remain four combinations, ABC , aBC , abC , and abc . But these do not stand on the same logical

¹ And the same is true of the proposition *Some S is P* . Hence we cannot agree with those who regard *Some S is P* and *S may be P* as equivalent forms. ("The particular judgment *Some S is P* is the same as the judgment *S may be P* ."—Bradley, *Principles of Logic*, p. 197.) For the former at least implies that if there are any S 's there are also P 's, while the latter does not imply this. Or we may perhaps put it in this way, that the reference is not in each case to the same universe of discourse; in the former case the reference is to an actual universe of some kind, in the latter to a conceivable universe of some kind.

footing, because if we were to remove ABC , there would be no such thing as A left; and if we were to remove abc there would be no such thing as c left. Now it is the Criterion or condition of logical consistency that every separate term and its negative shall remain. Hence there must exist some things which are described by ABC , and other things described by abc " (p. 216).

117. *Various Suppositions concerning the Existential Import of Categorical Propositions.*—Several different views may be taken as to what implication with regard to existence, if any, is involved in categorical propositions. The following may be formulated for special discussion¹:—

(1) It may be held that every categorical proposition implies the existence both of objects denoted by the terms directly involved and also of objects denoted by their contradictories; that, for example, *All S is P* implies the existence of *S*, *not-S*, *P*, *not-P*. This view is implied in Jevons's Criterion of Consistency mentioned in the preceding section. It is also practically adopted by De Morgan².

¹ The suppositions that follow are not intended to be exhaustive. We might, for instance, regard propositions as implying the existence both of their subjects and predicates, but not of the contradictories of these; or we might regard universals as always implying the existence of their subjects, but particulars as not necessarily implying the existence of theirs (see note 2 on p. 206); or affirmatives as always implying the existence of their subjects, but negatives as not necessarily implying the existence of theirs. This last supposition represents the view of Ueberweg. Still another view is taken by Lewis Carroll, who regards all categorical propositions, except universal negatives, as implying the existence of their subjects. "In every proposition beginning with *some* or *all*, the *actual existence* of the subject is asserted. If, for instance, I say 'all misers are selfish,' I mean that misers *actually exist*. If I wished to avoid making this assertion, and merely to state the *law* that miserliness necessarily involves *selfishness*, I should say 'no misers are unselfish,' which does not assert that any misers exist at all, but merely that, if any *did* exist, they *would* be selfish" (*Game of Logic*, p. 19). It would take too much space, however, to give a separate discussion to suppositions other than those mentioned in the text.

² "By the *universe* (of a proposition) is meant the collection of all objects which are contemplated as objects about which assertion or denial may take place. *Let every name which belongs to the whole universe be excluded as needless*: this must be particularly remembered. Let every object which has not the name X (of which there are always some) be conceived as therefore marked with the name x meaning not- X " (*Syllabus*, pp. 12, 13). Compare, also, *Formal Logic*, p. 55.

(2) It may be held that every proposition implies simply the existence of its subject. This is Mill's view (as regards real propositions); for he holds that we cannot give information about a non-existent subject¹. This is no doubt the view that, at any rate on a first consideration of the subject, appears to be at once the most reasonable and the most simple.

(3) It may be held that in formal logic we should not regard propositions as necessarily implying the existence either of their subjects or of their predicates. On this view, the full implication of *All S is P* may be expressed by saying that it denies the existence of anything that is at the same time *S* and *not-P*. Similarly *No S is P* implies the existence neither of *S* nor of *P*, but merely denies the existence of anything that is both *S* and *P*. *Some S is P* (or *is not P*) may be read *Some S, if there is any S, is P* (or *is not P*). Here we neither affirm nor deny the existence of any class absolutely²; the sum total of what we affirm is that *if any S exists*, then something which is both *S* and *P* (or *S* and *not-P*) also exists. On this interpretation, therefore, particular propositions have a hypothetical and not a purely categorical character.

(4) It may be held that universal propositions should not be regarded as implying the existence of their subjects, but that particular propositions should be regarded as doing so³. On this view *All S is P* merely denies the existence of anything that is both *S* and *not-P*; *No S is P* denies the existence of anything that is both *S* and *P*; *Some S is P* affirms the existence of something that is both *S* and *P*; *Some S is not P* affirms the existence of something that is both *S* and *not-P*. Thus, *universals* are interpreted as having existentially a *nega-*

¹ "An accidental or non-essential affirmation does imply the real existence of the subject, because in the case of a non-existent subject there is nothing for the proposition to assert" (*Logic*, i., chapter 6, § 2).

² Jevons lays down the dictum that "we cannot make any statement except a truism without implying that certain combinations of terms are contradictory and excluded from thought" (*Principles of Science*, 2nd edition, p. 32). This is true of universals (though somewhat loosely expressed), but it does not seem to be true of particular propositions, whatever view may be taken of them.

³ Dr Venn advocates this doctrine with special reference to the operations of symbolic logic; but there is no reason why it should not be extended to ordinary formal logic.

tive force, while *particulars* have an *affirmative* force. This hypothesis will be found to lead to certain paradoxical results, but it will also be shewn to be the only one which renders possible a completely scientific and symmetrical treatment of logical problems¹.

118. *Immediate Inferences and the Existential Import of Propositions.*—Before coming to any decision in regard to the existential import of propositions, it will be well to enquire how certain logical doctrines are affected by the different existential assumptions upon which we may proceed. This discussion will as far as possible be kept distinct from the enquiry as to which of the assumptions ought normally to be adopted by the logician. The latter question is a highly controversial one, but the logical consequences of the various suppositions ought to be capable of demonstration, so as to leave no room for differences of opinion.

We shall in the present section enquire how far different hypotheses regarding the existential import of propositions affect the validity of obversion and conversion and the other immediate inferences based upon these. In the next section we shall consider inferences connected with the square of opposition.

We may take in order the suppositions explained in the preceding section.

(1) *Let every proposition imply the existence of both subject and predicate and their contradictories.*

It is clear that on this hypothesis the validity of conversion, obversion, contraposition, and inversion will not be affected by existential considerations. The terms of the original proposition together with their contradictories being in each case identical with the terms of the inferred proposition together with their contradictories, the latter cannot possibly contain any existential implication that is not already contained in the original proposition².

¹ The hypothesis in question has been already provisionally adopted in the scheme of logical equivalences given in section 74, and also in the symbolic scheme of propositions given on p. 164.

² The reader may be reminded that in our original working out of these immediate inferences we provisionally adopted the supposition in question.

(2) *Let every proposition imply simply the existence of its subject.*

(a) The validity of obversion is not affected.

(b) The conversion of **A** is valid, and also that of **I**. If *All S is P* and *Some S is P* imply directly the existence of *S*, then they clearly imply indirectly the existence of *P*; and this is all that is required in order that their conversion may be legitimate. The conversion of **E** is not valid; for *No S is P* implies neither directly nor indirectly the existence of *P*, whilst its converse does imply this.

(c) The contraposition of **E** is valid, and also that of **O**. *No S is P* and *Some S is not P* both imply on our present supposition the existence of *S*, and since by the law of excluded middle every *S* is either *P* or *not-P*, it follows that they imply indirectly the existence of *not-P*. The contraposition of **A** is not valid; for it involves the conversion of **E**, which we have already seen not to be valid¹.

(d) The process of inversion is not valid; for it involves in the case of both **A** and **E** the conversion of an **E** proposition².

If along with an **E** proposition we are specially given the information that *P* exists, or if this is implied in some other proposition given us at the same time, then the **E** proposition may of course be converted. Under corresponding circumstances the contraposition and inversion of **A** and the inversion of **E** may be valid³. Or again, given simply *No S is P*, we may infer *Either P is non-existent or no P is S*⁴; and similarly in other cases.

¹ Or we might argue directly that the contraposition of **A** is not valid, since *All S is P* does not imply the existence of *not-P*, whilst its contrapositive does imply this.

² Or again we might argue directly from the fact that neither *All S is P* nor *No S is P* implies the existence of *not-S*.

³ For example, given (a) *No S is P*, (β) *All R is P*, we may under our present supposition convert (a), since (β) implies indirectly the existence of *P*; and we may contraposit (β), since, as shewn above, (a) implies indirectly the existence of *not-P*. It will also be found that given these two propositions together, they both admit of inversion.

⁴ This conclusion may also be written in the hypothetical form, *If P exists then no P is S*.

(3) *Let no proposition imply the existence either of its subject or of its predicate.*

Having now got rid of the implication of the existence either of subject or predicate in the case of all propositions, we might naturally suppose that in no case in which we make an immediate inference need we trouble ourselves with any question of existence at all. As already indicated, however, this conclusion would be erroneous.

(a) The process of obversion is still valid. Take, for example, the obversion of *No S is P*. The obverse *All S is not-P* implies that if there is any *S* there is also some *not-P*. But this is necessarily implied in the proposition *No S is P* itself. If there is any *S* it is by the law of excluded middle either *P* or *not-P*; therefore, given that *No S is P*, it follows immediately that if there is any *S* there is some *not-P*.

(b) The conversion of **E** is valid. Since *No S is P* denies the existence of anything that is both *S* and *P*, it implies that if there is any *S* there is some *not-P* and that if there is any *P* there is some *not-S*; and these are the only implications with regard to existence involved in its converse. The conversion of **A**, however, is not valid; nor is that of **I**. For *Some P is S* implies that if there is any *P* there is also some *S*; but this is not implied either in *All S is P* or in *Some S is P*.

(c) That the contraposition of **A** is valid follows from the fact that the obversion of **A** and the conversion of **E** are both valid¹. That the contraposition of **E** and that of **O** are invalid follows from the fact that the conversion of **A** and that of **I** are both invalid.

(d) That inversion is invalid follows similarly.

On our present supposition then the following are valid: the obversion and contraposition of **A**, the obversion of **I**, the obversion and conversion of **E**, the obversion of **O**; the following are invalid: the conversion and inversion of **A**, the conver-

¹ Or we might argue directly as follows: since the proposition *All S is P* denies the existence of anything that is both *S* and *not-P*, it implies that if there is any *S* there is some *P* and that if there is any *not-P* there is some *not-S*; and these are the only implications with regard to existence involved in its contrapositive.

sion of **I**, the contraposition and inversion of **E**, the contraposition of **O**. We find, therefore, that inferences to universals remain in all cases valid; inferences to particulars are rendered invalid except in the case of obversion¹.

(4) *Let particulars imply, while universals do not imply, the existence of their subjects.*

(a) The validity of obversion is again obviously unaffected².

(b) The conversion of **E** is valid, and also that of **I**, but not that of **A**³.

(c) The contraposition of **A** is valid, and also that of **O**, but not that of **E**.

(d) The process of inversion is not valid.

These results are obvious; and the final outcome is—as might have been anticipated—that we may infer a universal from a universal, or a particular from a particular, but not a particular from a universal⁴.

An important point to notice is that in the immediate inferences which remain valid on this supposition (namely,

¹ We are not in this section considering the process of subalternation.

² Obversion thus remains valid on all the suppositions which have been specially discussed above. If, however, affirmatives are regarded as implying the existence of their subjects while negatives are not so regarded, then of course we cannot pass by obversion from **E** to **A**, or from **O** to **I**.

³ But of course from the two propositions, *All S is P*, *Some R is S*, we can infer *Some P is S*; and similarly in other cases.

⁴ On the assumption, however, that the universe of discourse can never be entirely emptied of content, *Something is P* may be inferred from *Everything is P*, and *Something is not P* may be inferred from *Nothing is P*. Again, as is shewn by Dr Venn (*Symbolic Logic*, pp. 142 to 9), the three universals *All S is P*, *No not-S is P*, *All not-S is P*, together establish the particular *Some S is P*. Any universe of discourse contains *a priori* four classes—(1) *SP*, (2) *S not-P*, (3) *not-S P*, (4) *not-S not-P*. *All S is P* negatives (2); *No not-S is P* negatives (3); *All not-S is P* negatives (4). Given these three propositions, therefore, we are able to infer that there is some *SP*, for this is all that we have left in the universe of discourse. The assumption that the universe of discourse can never be entirely emptied of content was in previous editions rejected as not being formally legitimate. But I am now inclined to take the opposite view and to regard it as a necessary assumption on the ground that it is an essential condition of a significant judgment that it relate to reality. If the universe of discourse is entirely emptied of content we must either fail to satisfy this condition, or else unconsciously transcend the assumed universe of discourse and refer to some other and wider one in which the former is affirmed not to exist.

obversion, simple conversion, and simple contraposition) there is no loss of logical force; while at the best the reverse would be the case in those that are no longer valid (namely, conversion *per accidens*, contraposition *per accidens*, and inversion).

119. *The Doctrine of Opposition and the Existential Import of Propositions.*—The ordinary doctrine of opposition is as follows: (a) The truth of *Some S is P* follows from that of *All S is P*, and the truth of *Some S is not P* from that of *No S is P* (doctrine of subalternation); (b) *All S is P* and *Some S is not P* cannot both be true and they cannot both be false, similarly for *Some S is P* and *No S is P* (doctrine of contradiction); (c) *All S is P* and *No S is P* cannot both be true but they may both be false (doctrine of contrariety); (d) *Some S is P* and *Some S is not P* may both be true but they cannot both be false (doctrine of sub-contrariety). We will now examine how far these several doctrines hold good under various suppositions with regard to the existential import of propositions.

(1) *Let every proposition imply the existence of both subject and predicate and their contradictories.*

On this supposition, if either the subject or the predicate of a proposition is the name of a class which is unrepresented in the universe of discourse or which exhausts that universe, then that proposition is false¹; for it implies what is inconsistent with fact. It follows that a pair of contradictories as usually stated, and also a pair of sub-contraries, may both be false. For example, *All S is P* and *Some S is not P* both imply the existence of *S* in the universe of discourse. In the case then in which *S* does not exist in that universe, these propositions would both be false². We must not of course say that

¹ The hypothesis that no proposition *can* contain such a term is obviously arbitrary. Compare, however, the following note.

² Mr Welton writes as follows in criticism of this result: "But, if our propositions are real predications, and not mere shams, it is impossible that they should be made about the absolutely non-existent; for of that it is impossible to make any predication whatever. Of course, we may put together an unmeaning jumble of words, and call it a proposition; but the term is a misnomer. A true logical proposition is the verbal expression of a real judgment, and a judgment cannot be made about that which is without logical existence. Indeed the mere fact that an idea becomes the subject, or nucleus, of a logical judgment necessitates that it exists in the appropriate sphere—whether of physical

under our present supposition true contradictories cannot be found; for this is always possible. The true contradictory of *All S is P* is *Either some S is not P, or else either S, or not-S, or P, or not-P is non-existent*. Similarly in other cases. The ordinary doctrines of subalternation and contrariety remain unaffected.

(2) *Let every proposition imply simply the existence of its subject.*

For reasons similar to those stated above, the ordinary doctrines of contradiction and sub-contrariety again fail to hold good. The true contradictory of *All S is P* now becomes *Either some S is not P, or S is non-existent*. The ordinary doctrines of subalternation and contrariety again remain unaffected.

reality or of imagination and thought. It seems impossible to conceive two contradictory propositions made about the absolutely non-existent, or about that which does not exist in the universe to which the proposition refers; and Mr Keynes, unfortunately, does not give an example. Hence, we conclude that the doctrine of opposition holds; for it is supposed to refer only to really significant propositions" (*Logic*, p. 276). This passage is unfortunately based on a misunderstanding of what is meant by existence in the universe of discourse. As already explained, it is not simply a question of "logical" existence or of existence in the sphere of thought; but Mr Welton does not appear to distinguish between "the absolutely non-existent, that which is without logical existence," and "that which does not exist in the universe to which a given proposition refers." It will be necessary to return later on to Mr Welton's fundamental position that a proposition whose subject has no existence in the universe of discourse is necessarily without meaning. At this stage it may suffice to give a concrete illustration of the particular point raised in the text. Take the propositions, *No physically incapacitated Frenchmen are bound to perform military service*, *Some physically incapacitated Frenchmen are bound to perform military service*, and assume that every proposition includes as part of its implication the actual existence of its subject in the universe of discourse. Then our position is that if anyone declares that no Frenchmen are physically incapacitated, he cannot admit the truth of either of the above propositions. The universe of discourse to which our propositions refer is clearly the actual physical universe. No one in denying that any Frenchmen are physically incapacitated would mean that we cannot even conceive them to be so; and the denial in the sense intended does not render the given propositions meaningless, it only renders them false, their full import being assumed to be respectively *There are physically incapacitated Frenchmen but none of them are bound to perform military service*, *There are physically incapacitated Frenchmen and some of them are bound to perform military service*.

(3) *Let no proposition imply the existence either of its subject or of its predicate.*

(a) The ordinary doctrine of subalternation holds good.

(b) The ordinary doctrine of contradiction does not hold good. *All S is P*, for example, merely denies the existence of any *S*'s that are not *P*'s; *Some S is not P* merely asserts that if there are any *S*'s, some of them are not *P*'s. In the case in which *S* does not exist in the universe of discourse we cannot affirm the falsity of either of these propositions.

(c) The ordinary doctrine of contrariety does not hold good. For if there is no implication of the existence of the subject in universal propositions we are not actually precluded from asserting together two propositions that are ordinarily given as contraries. *All S is P* merely denies that there are any *S not-P*'s, *No S is P* that there are any *SP*'s. We may, therefore, without inconsistency affirm both *All S is P* and *No S is P*; but this is virtually to deny the existence of *S*¹.

(d) The ordinary doctrine of sub-contrariety remains unaffected.

(4) *Let particulars imply, while universals do not imply, the existence of their subjects.*

(a) The ordinary doctrine of subalternation does not hold good. *Some S is P*, for example, implies the existence of *S*, while this is not implied by *All S is P*.

¹ Of course on the view under consideration it is somewhat misleading to continue to speak of these two propositions as contraries. Thus, Mr Bradley, in reference to Dr Venn's treatment of this question, remarks on "the extraordinary assertion that contrary judgments, such as 'All *x* is *y*' and 'No *x* is *y*,' can be compatible." He continues, "It is not worth while to enter into a discussion of this matter. They are of course compatible if you allow yourself to play on their ambiguity; but how in that case they can be said to be contrary I have no conception. 'The interesting and unexpected application' is to me, I confess, not anything beyond a confused example of a well-known doctrine concerning the relations of possibility and existence. But I confess besides that I have never been much used 'to discuss the question in a perfectly matter of fact way'" (*Principles of Logic*, p. 154). This is scornful, after Mr Bradley's wont, but the position taken by Dr Venn is apparently not understood. Granted that universals do not imply the existence of their subjects, we get the following result: if by contraries are meant propositions that are incompatible, then *all x is y* and *no x is y* are not contraries; if by contraries are meant (as in the ordinary square of opposition) the two propositions *all x is y* and *no x is*

(b) The ordinary doctrine of contradiction holds good. *All S is P* denies that there is any *S* that is *not-P*; *Some S is not P* affirms that there is some *S* that is *not-P*. It is clear that these propositions cannot both be true; it is also clear that they cannot both be false. Similarly for *No S is P* and *Some S is P*.

(c) The ordinary doctrine of contrariety does not hold good. *All S is P* and *No S is P* are not inconsistent with one another, but the force of asserting both of them is to deny that there are any *S*'s¹. This follows just as in the case of our third supposition².

y, then so-called contraries are not necessarily incompatible. Here there is nothing extraordinary, though there may be something unexpected; nor is there anything confused. It is really Mr Bradley who is "playing upon an ambiguity."

¹ But of course given *No S is P* and *Some S is R*, we are able to infer that *All S is P* is false. The second of these propositions affirms the existence of *S*, and therefore destroys the hypothesis on which alone the first and third can be treated as compatible.

² The above doctrine has been criticized on the ground that it practically amounts to saying that neither of the given propositions has any meaning whatever, but that each is a mere sham and pretence of predication; and a request is made for concrete examples. In working out the various questions raised in this and the preceding sections, I have intentionally relied mainly on symbolic examples, because they have the great advantage of avoiding false issues, while they are at the same time quite sufficient for their purpose. It is only by an abstract method of treatment that we are likely to reach a completely consistent logical scheme. At the same time it is not unreasonable to ask that symbolic examples should be supplemented by concrete examples. The following may perhaps suffice to illustrate the particular point now at issue: "An honest miller has a golden thumb"; "Well, I am sure that no miller, honest or otherwise, has a golden thumb." These two propositions are in the form of what would ordinarily be called contraries; but taken together they may quite naturally be interpreted as meaning that no such person can be found as an honest miller. The first of them would indeed probably be intended to be supplemented by the second, and so to carry inferentially the denial of the existence of its subject. Another good example is contained in the following quotation from Mrs Ladd Franklin: "*All x is y*, *No x is y*, assert together that *x* is neither *y* nor *not-y*, and hence that there is no *x*. It is common among logicians to say that two such propositions are incompatible; but that is not true, they are simply together incompatible with the existence of *x*. When the schoolboy has proved that the meeting point of two lines is not on the right of a certain transversal and that it is not on the left of it, we do not tell him that his propositions are incompatible and that one or other of them must be false, but we allow him to draw the natural conclusion that there is no meeting point, or that the lines are parallel" (*Mind*, 1890, p. 77 n.).

(d) The ordinary doctrine of sub-contrariety does not hold good. *Some S is P* and *Some S is not P* are both false in the case in which *S* does not exist¹.

The relation between contradictories is by far the most important relation with which we are concerned in dealing with the opposition of propositions, and it will be observed that the last of the above suppositions is the only one under which the ordinary doctrine of contradiction holds good.

The results obtained in this and the preceding section are interesting and valuable, whatever view may ultimately be taken in regard to the existential import of propositions. The connexion between existential import and the validity of syllogistic reasonings will be considered subsequently. The whole question has hitherto failed to receive the amount of attention it deserves in consequence of the prevalence of the notion that formal logic cannot possibly have any concern with considerations of existence².

120. *The relation between the propositions All S is P and All not-S is P.*—This is an interesting case to notice in connexion with the discussion raised in the preceding sections.

$$\begin{aligned} SaP &= SeP' = P'eS; \\ S'aP &= S'eP' = P'eS' = P'aS. \end{aligned}$$

The given propositions come out, therefore, as contraries.

On the view that we ought not to enter into any discussion concerning existence in connexion with immediate inference, we must, I suppose, rest content with this statement of the

¹ The objection is again urged that if our propositions are really significant predications their subjects necessarily have an existence in the appropriate sphere. But take the propositions, *Some of the answers to the question shewed originality*, *Some of the answers to the question did not shew originality*. On the existential assumption with which we are now dealing, these propositions are equivalent to the following: *There were answers to the question and some of them were original*, *There were answers to the question and some of them were not original*. If, as a matter of fact, there were no answers to the question, these propositions are not meaningless, but they are both false.

² Dr Venn discusses the question fully in its relation to symbolic logic. He does not profess to deal with it fully in its relation to ordinary formal logic; but the germs of a complete treatment from this point of view are contained in what he says.

case. It seems, however, sufficiently curious to demand further investigation and explanation. We may as before take different suppositions with regard to the existential import of propositions.

(1) If every proposition implies the existence of both subject and predicate and their contradictories, then it is at once clear that the two propositions cannot both be true together; for between them they deny the existence of *not-P*.

(2) On the view that propositions imply simply the existence of their subjects, it has been shewn in section 118, that we are not justified in passing from *All not-S is P* to *All not-P is S* unless we are given independently the existence of *not-P*. But it will be observed that in the case before us, the given propositions make this impossible. Since *all S is P* and *all not-S is P*, and everything is either *S* or *not-S* by the law of excluded middle, it follows that nothing is *not-P*. In order, therefore, to reduce the given propositions to such a form that they appear as contraries (and consequently¹ as inconsistent with each other) we have to assume the very thing that taken together they really deny.

(3) and (4). On the view that at any rate universal propositions do not imply the existence of their subjects, we have found in the preceding section that the propositions *No not-P is S*, *All not-P is S*, are not necessarily inconsistent, for they may express the fact that *P* constitutes the entire universe of discourse. But this fact is just what is given us by the propositions in their original form.

Under each hypothesis, then, the result obtained is satisfactorily accounted for and explained.

121. Jevons's Criterion of Consistency.—In passing to the explicit discussion of the existential import of categorical propositions, we may consider first the Criterion of Consistency, which is laid down by Jevons (following De Morgan):—Any two or more propositions are contradictory when, and only when, after all possible substitutions are made, they occasion the total disappearance of any term, positive or negative, from the

¹ It will be remembered that under suppositions (1) and (2) the ordinary doctrine of contrariety holds good.

Logical Alphabet. The criterion amounts to this, that every proposition must be understood to imply the existence of things denoted by every simple term contained in it, and also of things denoted by the contradictories of such terms. If, for example, we have the proposition *All S is P*, this implies that among the members of the universe of discourse are to be found *S*'s and *P*'s, *not-S*'s and *not-P*'s. In defence of this doctrine Jevons appears to rely mainly upon the psychological law of relativity, namely, that we cannot think at all without separating what we think about from other things. Hence if either a term or its contradictory represents nonentity, that term cannot be either subject or predicate in a significant proposition¹. It is clear, however, that this psychological argument falls away as soon as it is allowed that we may be confining ourselves to a limited universe of discourse, or indeed if we confine ourselves to any universe less extensive than that which covers the whole realm of the conceivable. Of course the more limited the universe to which our proposition is supposed to relate the more easily may *S* or *P* either exhaust it or be absent from it; but with very complex subjects and predicates the contradictory of one or both of our terms may easily exhaust even an extended universe. Take, for example, the proposition, *No satisfactory solution of the problem of squaring the circle has ever been published by Mr A.* Here the subject is non-existent; and it may happen also that Mr A. has never published anything at all². Further, if I am not allowed to negative *X*, why should I be allowed to negative *AB*? There is nothing to prevent *X* from representing a class formed by taking the part common to two other classes. In certain

¹ This point is put somewhat tentatively in a passage in Jevons's *Principles of Science* (chapter 5, § 5) where he remarks: "If *A* were identical with '*B* or not-*B*,' its negative not-*A* would be non-existent. This result would generally be an absurd one, and I see much reason to think that in a strictly logical point of view it would always be absurd. In all probability we ought to assume as a fundamental logical axiom that every term has its negative in thought. We cannot think at all without separating what we think about from other things, and these things necessarily form the negative notion. If so, it follows that any term of the form '*B* or not-*B*' is just as self-contradictory as one of the form '*B* and not-*B*.'"

² Other examples will be given in the following section.

combinations indeed it may be convenient to substitute X for AB , or *vice versâ*. It would appear then that what is contradictory when we use a certain set of symbols may not be contradictory when we use another set of symbols. This argument has a special bearing on the complex propositions which are usually relegated to symbolic logic, but to which Jevons's criterion is intended specially to apply.

No doubt Jevons's criterion is sometimes a convenient assumption to make; provisionally, for example, in working out the doctrine of immediate inferences on the traditional lines. But it is an assumption that should always be explicitly referred to when made; and it ought not to be regarded as having an axiomatic and binding force, so as to make it necessary to base the whole of logic upon it.

122. *The Existential Import of General Categorical Propositions.*—We may now turn to the more limited problem whether general categorical propositions should be regarded as logically implying the existence of their subjects¹. Two questions may be distinguished, the first relating to popular usage, the second to logical convention. We might indeed regard the second of these questions as practically decided by the answer to the first, if popular usage were absolutely consistent and unvarying; but we shall find that this is not the case.

In the discussion which follows it is most important to bear in mind precisely what is meant by the universe of discourse, as already explained. In particular, the universe of discourse must not be formally identified with the sphere of thought, although the two spheres may happen in special cases to be co-extensive. Usually the existence with which we shall be concerned is not *logical* existence, but *empirical* existence (which, however, is not necessarily the same thing as ordinary physical existence²).

This consideration may enable us to dispose of a very summary solution of our problem, which, if it were correct,

¹ The case of singular propositions will be discussed in the following section.

² Thus, under empirical existence we include existence in the universe of mythology, or heraldry, or modern fiction, all of which are clearly to be distinguished from logical existence.

would render any further discussion needless. How, it may be asked, can we possibly speak about anything and at the same time exclude it from the universe of discourse? This question suggests a certain ambiguity which may attach to the phrase *universe of discourse*, but which can hardly remain an ambiguity after the explanations already given. The answer is that we can certainly speak about a thing *with reference to* a given universe of discourse without implying its existence in that universe. Suppose, for example, that I say there are no such things as unicorns. If this statement is to be accepted, the universe of discourse must be taken to be the actual material universe; for unicorns do exist both in imagination and in the universe of fiction. I speak then of unicorns with reference to the actual material universe, but deny that such creatures are to be found (or exist) in it.

There is another possible source of misunderstanding which it is desirable to clear out of the way before proceeding further. If we decide that categorical propositions may in certain cases be regarded as not logically implying the existence of their subjects, it will not be meant that such propositions contain no affirmation relating to reality. The universe of discourse may be regarded as the ultimate reality to which any given proposition refers, and it is for this reason that we have arrived at the conclusion that the universe of discourse can never itself be a nonentity. In what follows, then, we have no intention of denying that "in every proposition, an analysis of the meaning will find a reality of which something else is affirmed or denied."¹ For example, if *All S is P* is regarded as not implying the existence of *S*, but merely as denying the existence of *SP'*, this means that the universe of discourse does not contain any *SP'*s, and the universe of discourse is the reality about which something is affirmed. In this sense the universe of discourse may be called the ultimate subject of the proposition,

¹ Bradley, *Principles of Logic*, p. 41. Similarly Mr Johnson observes that "the 'existence' of a subject is a presupposition of significant judgment" (*Mind*, 1892, p. 24). But he does not here mean by "subject" what is ordinarily regarded as the logical subject of a proposition; and his position is not opposed to the general view taken in this chapter.

but it is not the subject in the ordinary logical sense, and the position here taken does not really affect the particular problem discussed in this section.

The question which we have to raise in regard to popular usage is whether in ordinary speech we always intend to imply the existence of the subjects of our propositions. We shall find that in answering this question a distinction must be drawn between universals and particulars. There can be no doubt that in the great majority of cases the categorical propositions of ordinary discourse would be interpreted as implying the existence of their subjects. "The practical exigencies of life," as Dr Venn remarks, "confine most of our discussions to what does exist, rather than to what might exist" (*Symbolic Logic*, p. 131). Moreover, when we are doubtful of the existence of our subject, we often make this explicit by the addition of a hypothetical clause, so that our proposition takes the form *All S's, if indeed there be any such, are P's*. Sometimes, however, the simple categorical form is still retained when this doubt is nevertheless felt; and, so far as universals are concerned, exceptions to what has been above laid down as the general rule can without difficulty be found, especially where the subject of the proposition is a complex term. In the case of particulars, on the other hand, it seems practically impossible to find any exception at all.

The following may be given as examples of universal propositions, which need not be regarded as implying the existence of their subjects: No unicorns have ever been seen; All candidates arriving five minutes late are fined one shilling; All candidates who stammer are excused reading aloud; Who steals my purse steals trash; No ghosts have troubled me; An honest miller has a golden thumb; All the carts that come to Crowland are shod with silver¹; Every body, not compelled by impressed forces to change its state, continues in a state of

¹ This proposition and the preceding one may be naturally interpreted as containing an indirect denial of the existence of their subjects. "Crowland is situated in such moorish rotten ground in the Fens, that scarce a horse, much less a cart, can come to it" (Bohn's *Handbook of Proverbs*, p. 211). It would appear, however, that this proverb has now lost its force, inasmuch as "since the draining, in summer time, carts may go thither."

rest or of uniform motion in a straight line; A planet moving in a hyperbolic orbit can never return to any position it once occupied¹.

We may make the first of the above assertions without intending to imply that unicorns exist unseen²; the second does not commit us to the prophecy that any candidates will arrive five minutes late; and similarly for the remaining propositions.

If it is argued that in such cases as these, the propositions ought properly to be written in the conditional or hypothetical and not in the categorical form³, the reply is that this is to misunderstand the point just now at issue, which is whether

¹ This example is taken from Dixon, *Essay on Reasoning*, p. 62. Other examples are given in Venn's *Symbolic Logic*. "Assertions about the future do not carry any such positive presumption with them, though the logician would commonly throw them into precisely the same 'All X is Y' type of categorical assertion. 'Those who pass this examination are lucky men' would certainly be tacitly supplemented by the clause 'if any such there be.' So too when we are clearly talking of an ideal. 'Perfectly conscientious men think but little of law and rule' has a sense without implying that there are any such men to be found" (p. 132). "As an instance of a possibly non-existent subject of a negative proposition, take the following: 'No person condemned for witchcraft in the reign of Queen Anne was executed'" (p. 132).

² The universe of discourse must here be regarded as the actual material universe. With reference to this example, however, a critic writes, "But surely the universe of imagination is the only one applicable; for unicorns have long been known not to belong to the actual material universe." The universe of imagination may be required in order to sustain the position that the subject of the proposition exists in the universe of discourse; but any person making the statement would certainly not be referring to the world of imagination or the universe of heraldry, for the simple reason that in either of these cases the proposition would obviously not be true. On the other hand, we can quite well suppose the statement made with reference to the actual material universe: "Whether unicorns exist or not, at any rate they have never been seen." Again, to take another example of a similar kind where the reference is also to the phenomenal universe, we can quite well suppose the statement made: "Whether there are ghosts or not, at any rate none have ever troubled me." In order to avoid misapprehension, it is particularly important to distinguish the above examples from such propositions as the following: "The wrath of the Homeric gods is very terrible," "Fairies are able to assume different forms." In each of these cases, the subject of the proposition exists in the particular universe to which reference is obviously made.

³ For example, If any candidate arrives five minutes late, that candidate is fined one shilling; If any one were to steal my purse, he would steal trash; &c.

we ever meet with propositions in ordinary discourse which are categorical in form and yet are hypothetical so far as the existence of their subjects is concerned. That this point must be decided in the affirmative seems quite clear. The question whether logically we should interpret categorical propositions in such a way that the propositions above given must no longer be expressed as categoricals is a different one. We shall consider this question very shortly, but the point which we are just now discussing relates only to popular usage.

In the case of particular propositions, it seems no longer possible to give examples, such as might be met with in ordinary discourse, in which there is no implication of the existence of the subjects of the propositions. The cases are at any rate exceedingly rare in which in ordinary speech we predicate anything of a non-existent subject without doing so universally. The main reason for this is, as Dr Venn points out, that "an assertion confined to 'some' of a class generally rests upon observation or testimony rather than on reasoning or imagination, and therefore almost necessarily postulates existent data, though the nature of this observation and consequent existence is, as already remarked, a perfectly open question. 'Some twining plants turn from left to right,' 'Some griffins have long claws,' both imply that we have looked in the right quarters to assure ourselves of the fact. In one case I may have observed in my own garden, and in the other on crests or in the works of the poets, but according to the appropriate tests of verification, we are in each case talking of what *is*."¹ If we look at the question from the other side, we find that when our primary object is to affirm the existence of a class of objects, our assertion very naturally takes the form of a particular proposition. If, for example, we desire to affirm the existence of black swans, we say *Some swans are black*. The existential implication of a proposition of this kind in ordinary discourse is one of its most fundamental characteristics.

¹ *Symbolic Logic*, p. 181. Again, in such a proposition as "Some sea-serpents are not half a mile long" (meaning *your so-called* sea-serpents), the subject of the proposition exists in the universe to which reference is made, namely, the universe which may be described as the universe of travellers' tales.

Passing now to the question what existential import should logically be assigned to categorical propositions, there appear to be strong grounds for giving a final preference to the last of the four alternatives formulated in section 117. It is not meant that we may not provisionally work on a different hypothesis, and we have as a matter of fact done so in some of our earlier chapters, mainly in order not to depart too soon from the traditional lines. But we are compelled to come to the conclusion that no thoroughly satisfactory development of logical doctrines is possible except on the supposition that particulars imply, while universals do not imply, the existence of their subjects¹.

(1) It may, in the first place, be observed that this solution of the problem is fully justified by popular usage. We have seen that the implication of the existence of the subject does not admit of any exception in the case of particular propositions, whereas, although there may usually be a similar implication in the case of universals, there are also exceptions to this rule. But if it is granted that in ordinary thought the existence of the subject of a universal proposition sometimes is and sometimes is not implied, it follows that, since the logician cannot discriminate between these cases, he had better, unless there are very strong reasons to the contrary, content himself with leaving the question open, that is, he should regard such existence as not necessarily or logically implied².

(2) A consideration of the manner in which the validity of immediate inferences is affected by the existential import of propositions affords distinct reasons for the adoption of the view here advocated³. The most important immediate infer-

¹ The treatment of complex propositions in Part iv. is based throughout upon this supposition.

² On this view whenever it is desired specially to affirm the existence in the universe of discourse of the subject of a universal proposition, a separate statement to this effect must be made. For example, *There are S's, and all of them are P's*. If, on the other hand, it is ever desired to affirm a particular proposition *without* implying the existence of the subject, then recourse must be had to the hypothetical or conditional form of statement. Thus, if we do not intend to imply the existence of S, instead of writing *Some S's are P's*, we must write, *If there are any S's, then in some such cases they are also P's*.

³ It has been objected that to base our view of the existential import of pro-

ences are simple conversion (*i.e.*, the conversion of **E** and of **I**) and simple contraposition (*i.e.*, the contraposition of **A** and of **O**). If, however, universals are regarded as implying the existence of their subjects, then, as shewn in section 118, neither the conversion of **E** nor the contraposition of **A** is valid, irrespective of some further assumption; whereas, if universals are not regarded as implying the existence of their subjects, then both these operations are legitimate without qualification. On the other hand, the conversion of **I** and the contraposition of **O** are valid only if particulars *do* imply the existence of their subjects¹.

Turning to immediate inferences of another kind, it is clear that if universal propositions necessarily imply the existence of their subjects, we cannot legitimately pass from *All X is Y* to *All AX is Y*². For it is possible that there may be *X*'s and yet no *AX*'s, and in this case the former proposition may be true, while the latter will certainly be false. Again, given that *A is X, B is Y, C is Z*, we cannot infer that *ABC is XYZ*. Such restrictions as these would constitute an almost insur-

positions upon the validity or invalidity of immediate inferences is to argue in a circle. "Whether," it is said, "the immediate inferences are valid or not must be a consequence of the view taken of the existential import of the proposition, and should not, therefore, be made a portion of the ground on which that view is based." This objection involves a confusion between different points of view from which the problem of the relation between the existential import of propositions and the validity of logical operations may be regarded. In section 118 the logical consequences of various assumptions were worked out without any attempt being made to decide between these assumptions. Our point of view is now different; we are investigating the grounds on which one of the assumptions may be preferred to the others, and there is no reason why the consequences previously deduced should not form part of our data for deciding this question. The argument contains nothing which is of the nature of a *circulus in probando*.

¹ Thus, the very important table of equivalences given in section 72 is valid on the supposition with which we are now dealing. The dependence of the table given in section 74 upon the same supposition is still more obvious. It has been already pointed out that the remaining immediate inferences based on conversion and obversion are of much less importance; see pp. 191, 2.

² It will be observed further that upon the same assumption we cannot even affirm the formal validity of the proposition *All X is X*, unless indeed the universe of discourse coincides with the whole realm of the conceivable. In any narrower universe *X* might be non-existent, and the proposition would then be false.

mountable bar to progress in inference as soon as we have to do with complex propositions¹.

(3) We may next consider the existential import of propositions with reference to the doctrine of opposition. It has been shewn in section 119 that if particulars are interpreted as implying the existence of their subjects, while universals are not so interpreted, then **A** and **O**, **E** and **I**, are true contradictories; but that this is not the case under any of the other suppositions discussed in the same section². There can, however, be no doubt that the most important function of particular propositions is to contradict the universal propositions of opposite quality; and hence we have the strongest possible argument in favour of a view of the existential import of propositions which will leave the ordinary doctrine of contradiction unaffected³.

As regards the doctrines of subalternation, contrariety, and subcontrariety, our results (namely, that **I** does not follow from **A** or **O** from **E**, that **A** and **E** may both be true, and that **I** and **O** may both be false) are no doubt paradoxical. But this objection is far more than counterbalanced by the fact that the doctrine of contradiction is saved. For as compared with the

¹ Hence Mrs Ladd Franklin is led to the conclusion that "no consistent logic of universal propositions is possible except with the convention that they do not imply the existence of their terms" (*Mind*, 1890, p. 88).

² **A** and **O**, **E** and **I**, will also be true contradictories if universals are interpreted as implying the existence of their subjects, while particulars are not so interpreted. It would be interesting, if space permitted, to work out the results of this supposition in detail. If the student does this for himself, he will find that this is the *only* supposition, under which the ordinary doctrine of opposition holds good throughout. All other considerations, however, are opposed to its adoption. It altogether conflicts with popular usage; it renders the processes of simple conversion and simple contraposition illegitimate; and whilst making universals double judgments, it destroys the categorical character of particulars altogether. In regard to this last point, see p. 187.

³ The position that particulars, but not universals, imply the existence of their subjects has been described as having the air of a compromise, inasmuch as we do not adopt the same solution for all propositions irrespective of their quantity. Having regard, however, to the doctrine of contradiction, it follows that whatever our solution as regards universals, we ought to adopt a different solution as regards particulars. Thus, given that *All S is P* simply negatives *SP'*, and that *No S is P* simply negatives *SP*, then if particulars are to retain their function of contradicting the universals of opposite quality, we have no alternative but to interpret *Some S is P* as positing *SP*, and *Some S is not P* as positing *SP'*.

relation between contradictories, these other relations are of little importance. We may specially consider the relation between **A** and **I**. *Some S is P* cannot now without qualification be inferred from *All S is P*, since the former of these propositions implies the existence of *S*, while the latter does not. But as a matter of fact this is an inference which we never have occasion to make. If their existential import is the same why should we ever lay down a particular proposition when the corresponding universal is at our service? On the other hand, the view which we are advocating gives *Some S is P* a status relatively to *All S is P* as well as relatively to *No S is P* which it could not otherwise possess; and similarly for *Some S is not P*. Our result as regards the relation between *SaP* and *SiP* has been described as equivalent to saying "that a statement of partial knowledge carries more real information than a statement of full knowledge; since if we only possess limited information, and so can only assert *SiP*, we thereby affirm the existence of *S*; but if we have sufficient knowledge to speak of *all S* (*S* remaining the same) the statement of that full knowledge immediately casts a doubt upon that existence." This way of putting it is, however, misleading if not positively erroneous. On the view in question it is incorrect to say simply that *SiP* and *SaP* give "partial" and "full" knowledge respectively, for *SiP* while giving less knowledge than *SaP* in one direction gives more in another. In other words, the knowledge which is "full" relatively to *SiP* is not expressed by *SaP* by itself, but by *SaP* together with the statement that there are such things as *S*¹.

¹ The position taken above in regard to subalternation is very well expressed by Mrs Ladd Franklin. "Nothing of course is now illogical that was ever logical before. It is merely a question of what convention in regard to the existence of terms we adopt before we admit the warm-blooded sentences of real life into the iron moulds of logical manipulation. With the old convention (which was never explicitly stated) subalternation ran thus: *No x's are y's* (and we hereby mean to imply that there are *x's*, whatever *x* may be), therefore, *Some x's are non-y's*. With the new convention the requirement is simply that if it is known that there are *x's* (as it is known, of course, in by far the greater number of sentences that it interests us to form) that fact must be expressly stated. The argument then is: *No x's are y's*, *There are x's*, therefore, *There are x's which are non-y's*."

(4) There is one further point of importance to be taken into account, and that is, that the interpretation of **A**, **E**, **I**, **O** propositions under consideration is the only interpretation according to which each one of these propositions is resolved into a *single categorical statement*. For if **A** and **E** imply the existence of their subjects they express *double*, not single, judgments, being equivalent respectively to the statements: *There are S's, but there are no SP's*; *There are S's, but there are no SP's*; whereas on our interpretation they simply express respectively the single judgments: *There are no SP's*; *There are no SP's*. On the other hand, if **I** and **O** do not imply the existence of their subjects, instead of expressing categorical judgments, they express somewhat complex hypothetical ones, being equivalent respectively to the statements: *If there are any S's then there are some SP's*; *If there are any S's then there are some SP's*¹; whereas on our interpretation they express respectively the categorical judgments: *There are SP's*; *There are SP's*.

On the whole, the cumulative argument seems almost overwhelming in favour of interpreting particulars, but not universals, as implying the existence of their subjects². This

¹ Compare section 117.

² We may briefly discuss in a note one or two objections to this view which have not yet been explicitly referred to.

(a) Mill argues that a synthetical proposition necessarily implies "the real existence of the subject, because in the case of a non-existent subject there is nothing for the proposition to assert" (*Logic*, Book i. chapter 6, § 2). In answer to this it is sufficient to point out that a non-existent thing will be described as possessing attributes which are separately attributes of existing things, although that particular combination of them may not anywhere be found, and if we know (as we may do) that certain of these attributes are always accompanied by other attributes we may predicate the latter of the non-existent thing, thereby obtaining a real proposition which does not involve the actual existence of its subject. As an argument *ad hominem* it may further be pointed out that Mill inclines to deny the existence of perfect straight lines or perfect circles. Would he therefore affirm that we can make no real assertions about such things?

(b) Mr Welford repeats several times that a proposition which relates to a non-existent subject must be a mere jumble of words, a predication in appearance only. "That the meaning of a universal proposition can be expressed as a denial is true, but this is not its primary import. And this denial itself must rest upon what the proposition affirms. Unless *SaP* implies the existence of *S*, and asserts that it possesses *P*, we have no data for denying the existence of *SP*'. For if *S* is non-existent the denial that it is non-*P* can have no intelli-

solution may be regarded as partly of the nature of a convention; and it is not meant that other hypotheses may not from time to time be usefully adopted for special purposes. We arrive, however, at the conclusion that no other solution can equally well suffice as the basis of a complete and scientific treatment of the problems of formal logic.

123. *The Existential Import of Singular Propositions.*—

The result arrived at in the preceding section with regard to universals cannot be applied to singulars in any case in which the subject is a proper name. For a purely denotative name must necessarily be the name of an object which exists or is supposed to exist in the universe to which reference is made. The implication of existence is also, as a rule, very clear, even when the subject is a general name individualised by an indi-

gible meaning" (*Logic*, p. 241). The examples which we have already given are sufficient to dispose of this objection; but it may be worth while to add a further argument. According to Mr Welton, an E proposition implies the existence of its subject but not of its predicate. We cannot then infer *PeS* from *SeP* because we have no assurance of the existence of *P*. But in accordance with the position taken by Mr Welton, we ought to go further and say that *PeS* must be a mere jumble of words unless we are assured of the existence of *P*. It is impossible, however, to regard *PeS* as a mere unmeaning jumble of words, a predication in appearance only, when *SeP* is a significant and true proposition. *PeS* may be false, or it may be an unnatural form of statement, but it cannot be meaningless if *SeP* has a meaning. Take, for example, the propositions—*No woman is now hanged for theft in England*, *No person now hanged for theft in England is a woman*. The second of these propositions is false if it is taken to imply that there are at the present time persons who are hanged for theft in England, but how it can possibly be regarded as meaningless I cannot understand.

(c) Miss Jones argues that if *some* carries with it an implication of existence, when used with a subject-term, it must do so equally when used with a predicate-term; but the predicate of an A proposition being undistributed is practically qualified by *some*; hence, if *Some S is P* implies the existence of *S* and therefore of *P*, *All S is P* must imply the existence of *P* and therefore of *S*. In reply to this argument it may be pointed out, first, that a distinction may fairly be drawn without any risk of confusion between a term explicitly quantified by the word *some* and a term which we can shew to be undistributed but which is not explicitly quantified at all; and, secondly, that the position which we have taken is based upon a consideration of the import of propositions as a whole, not upon the force of signs of quantity considered in the abstract. The irrelevancy of the argument will be apparent if it is taken in connexion with the reasons which we have urged for holding that particulars should be regarded as implying the existence of their subjects.

vidualising prefix. Take, for example, such propositions as the following: This hat is an old one; The garden belonging to that house leads down to the river; The Lord Mayor of London this year is a Roman Catholic. In order, therefore, to avoid too great a divergence from common thought, it will be well to differentiate singulars from general universals, and to regard them as implying the existence of their subjects in all cases, and not merely when the subject is a proper name¹.

But here it is necessary to add a word with regard to the opposition of singulars. *Socrates lived in Greece* and *Socrates did not live in Greece* cannot on the above view be regarded as true contradictories, since they would both be false in case Socrates turned out to be a myth. The true contradictory of a singular proposition will now take the form of a hypothetical; thus, the contradictory of the first of the above propositions will be, *If there ever was such a man as Socrates, he did not live in Greece.*

EXERCISES.

124. Assign precisely the meaning of the assertion that it is false to say that some English soldiers did not behave discredibly in South Africa. [L.]

125. "The king of Utopia died on Tuesday last." Examine carefully the meaning to be attached to the denial of this proposition. [K.]

126. On the assumption that particulars imply while universals do not imply the existence of their subjects in the universe of discourse, examine (stating your reasons) the validity of the following inferences: *All S is P* and *Some R is not S*, therefore, *Some not-S is not P*; *All S is P* and *Some R is not P*, therefore, *Some not-S is not P*; *All S is P* and *Some R is S*, it is, therefore, false that *No P is S*; *All S is P* and *Some R is P*, it is, therefore, false that *No P is S*. [K.]

¹ Exceptional cases in which singular propositions cannot be regarded as implying the existence of their subjects must on this view be written logically in the hypothetical form. Such a proposition as *The first man to ascend Mount Everest will be famous* is a case in point. It must, therefore, be written for logical purposes in some such form as the following: *If any one ever succeeds in ascending Mount Everest, the first man who does so will be famous.*

CHAPTER VIII.

CONDITIONAL AND HYPOTHETICAL PROPOSITIONS.

127. *The distinction between Conditional Propositions and Hypothetical Propositions*¹.—Propositions commonly written in the form *If A is B, C is D* belong to two very different types. *A being B* and *C being D* may be two events or two combinations of properties, concerning which it is affirmed that whenever or wherever the first occurs the second will occur also. For example, *If an import duty is a source of revenue, it does not afford protection; If a child is spoilt, his parents suffer; If a straight line falling upon two other straight lines make the alternate angles equal to one another, the two straight lines shall be parallel to one another; Where the carcass is, there shall the eagles be gathered together*. What is affirmed in all such cases as these is a connexion between phenomena; it may be either a co-inherence of attributes in a common subject, or a relation in time or space between certain occurrences. Propositions belonging to this type may be called distinctively *conditional*.

But again, *A is B* and *C is D* may be two propositions of independent import, the relation between which cannot be resolved into any time or space relation or into an affirmation of the co-inherence of attributes in a common subject. In other words, a relation may be affirmed between two propositions as holding good once and for all without distinction

¹ For the distinction indicated in the present section I was in the first instance indebted to an essay, written in 1884, by Mr W. E. Johnson. This essay has not been published in its original form; but the substance of it has been included in some papers on *The Logical Calculus* by Mr Johnson which appeared in *Mind* in 1892.

of place or time or circumstance. For example, *If it be a sin to covet honour, I am the most offending soul alive; If patience is a virtue, there are painful virtues; If there is a righteous God, the wicked will not escape their just punishment; If virtue is involuntary, so is vice.* Propositions belonging to this type may be called *hypothetical* as distinguished from conditional, or they may be spoken of still more distinctively as *true hypotheticals* or *pure hypotheticals*¹.

The parts of the conditional and also of the true hypothetical are called the *antecedent* and the *consequent*. Thus, in the proposition *If A is B, C is D*, the antecedent is *A is B*, the consequent is *C is D*.

It is impossible formally to distinguish between conditionals and hypotheticals so long as we keep to the expression *If A is B, C is D*, since this may be either the one or the other. The following forms, however, are unmistakeably conditional: *Whenever A is B, C is D; In all cases in which A is B, C is D; If any P is Q then that P is R.*² The form *If A is true then C is true* is, on the other hand, distinctively hypothetical. *A* and *C* here stand for *propositions*, not terms, and the words "is true" are introduced in order to make this explicit. It is quite sufficient, however, to write the true hypothetical in the form *If A then C*.

Since a conditional proposition usually contains a reference to some concurrence in time or space, the *if* of the antecedent may as a rule be replaced either by *when* or by *where*, as the case may be, without any change in the significance of the proposition; but the same cannot be said in the case of the

¹ The above distinction has been adopted in some recent treatises on Logic, but it must be borne in mind that most logicians use the terms *conditional* and *hypothetical* as synonymous or else draw a distinction between them different from the above. Compare the note on p. 59.

² Conditionals can always be reduced to the last of these three forms, and such reduction is sometimes useful. A consideration of the concrete examples already given will, however, shew that a good deal of manipulation may sometimes be required in order to effect the reduction. The following are examples: *If any child is spoilt, then that child will have suffering parents; If any two straight lines are such that another straight line falling upon them makes the alternate angles equal to one another, then those two straight lines are parallel to one another.*

true hypothetical. This consideration will often suffice to resolve any doubt that may arise in concrete cases as to the particular type to which any given proposition belongs. Another and more fundamental criterion may be found in the answer to the question whether or not the antecedent and consequent are propositions of independent import, whose meaning will not be impaired if they are considered apart from one another. If the answer is in the affirmative, then the proposition is hypothetical. Thus, taking examples of hypotheticals already given, we find that the antecedents *It is a sin to covet honour*, *Patience is a virtue*, *Virtue is involuntary*, and the consequents *I am the most offending soul alive*, *There are painful virtues*, *Vice is involuntary*, all retain their full meaning though separated from one another. If, on the other hand, the consequent necessarily refers us back to the antecedent in order that it may be fully intelligible, then the proposition is conditional. Thus, taking by itself the consequent in the first conditional given on p. 211, namely, *it does not afford protection*, we are at once led to ask what is here meant by *it*. The answer is—that *import duty*. But *what import duty*? An adequate answer can be given only by introducing into the consequent the whole of the antecedent—an *import duty which is a source of revenue does not afford protection*. We now have the full force of our original conditional proposition in the form of a single categorical. It will be found that if other conditionals are treated in the same way, they resolve themselves similarly into categoricals of the form *All PQ is R*. The problem of the reduction of conditionals and hypotheticals to categorical form will be considered in more detail later on in this chapter, and it will be shewn that whilst such reduction is always possible, and generally simple and natural, in the case of conditionals, it is not possible at all (with terms corresponding to the original antecedent and consequent) in the case of hypotheticals.

The distinction between conditionals and hypotheticals may also be expressed by saying that the former are complex propositions, whereas the latter are compound propositions (as defined in section 48). This follows, indeed, from our original

way of putting the distinction; a conditional expresses a relation between two phenomena, a hypothetical between two propositions of independent import. The former, therefore, in the last resort connects two terms, the latter two propositions.

128. *The Import of Conditional Propositions.*—It is sometimes held that the real *differentia* of all propositions of the form *If A is B, C is D* is “to express human doubt.” Clearly, however, there is no intention to express doubt as regards the relation between the antecedent and the consequent; and the doubt must, therefore, be supposed to relate to the actual occurrence of the antecedent. But so far at any rate as *conditionals* are concerned, the doubt which they may thus imply must be considered incidental rather than the fundamental or differentiating characteristic belonging to them. The *if* of the conditional may, as we have seen, usually be replaced by *when* without altering the significance of the proposition, and in this case the element of doubt is no more prominent than in the categorical proposition. From the *material* standpoint, conditionals may or may not imply the actual occurrence of their antecedents. Whenever the connexion between the antecedent and the consequent can be inferred from the nature of the antecedent independently of specific experience (and this may be the more usual case), then the actual happening of the antecedent is not in any sense involved; but if our knowledge of the connexion does depend on specific experience (as it sometimes may), and could not have been otherwise obtained, then such actual happening is materially involved. For example, the statement, “If we descend into the earth, the temperature increases at a nearly uniform rate of 1° Fahr. for every fifty feet of descent down to almost a mile,” requires that actual descents into the earth should have been made, for otherwise the truth of the statement could not have been known.

The enquiry into the import of the conditional proposition resolves itself mainly into the question whether the conditional and the categorical forms can be regarded as mutually interchangeable; and in dealing with this question the two principal points which arise are, *first*, whether the relation between the

antecedent and the consequent of a conditional proposition is precisely analogous to that between the subject and the predicate of a categorical proposition; *secondly*, whether we are to interpret the conditional with regard to the occurrence of its antecedent in the same way as we interpret the categorical with regard to the existence of its subject.

Miss Jones (*General Logic*, p. 42) includes both conditionals and hypotheticals in a wider class which she terms *inferential* propositions. On this view, the relation between the antecedent and the consequent of a conditional is held to be that the consequent is an *inference* from the antecedent, and there is accordingly said to be a loss of force when the conditional is reduced to categorical form. Mr Welton, taking the same position, writes, "The categorical merely states a fact, whilst the conditional not only expresses that fact, but implies that the nature of things renders it a necessary one. 'If any person suffers from an infectious disease, he ought to be isolated' may be written in the categorical form, 'Every person who is suffering from an infectious disease ought to be isolated'; but the former gives the antecedent as the *ground* or *reason* for asserting the consequent, whilst the latter merely asserts a fact for which it offers no justification" (*Logic*, p. 205). The distinction here implied appears to be quite illusory. Neither of the two forms gives explicitly the justification for passing from antecedent to consequent (or from subject to predicate); in both cases the right to do so is merely *asserted*, not justified. Compare, again, the propositions, "All isosceles triangles have the angles at their base equal to one another," "If the angles at the base of a triangle are equal to one another, that triangle is isosceles." These propositions fall naturally into the categorical and conditional forms respectively, simply because there happens to be no single adjective (like "isosceles") which connotes "having two equal angles." But surely it cannot be said that the second proposition implies a relation rendered necessary by the nature of things, while the former contains no such implication. It is held that a distinction ought to be drawn between universal conditionals and particular conditionals, inasmuch as the latter "imply no necessary connexion of attributes. They

simply express facts empirically observed, and indeed though conditional in form are really categorical in meaning." Universal conditionals, however, may also express facts empirically observed, as in the example already given, "If we descend into the earth, the temperature increases at a nearly uniform rate of 1° Fahr. for every fifty feet of descent down to almost a mile." It cannot then be allowed that conditionals suffer any loss of force when expressed in categorical form¹.

Independently of the above erroneous distinction, it is argued that, starting from the categorical form, we cannot pass to the conditional, if the subject of the proposition is a simple term. The basis of this argument is that the antecedent of a conditional requires *two* terms, and that in the case supposed these are not provided by the categorical. Thus, Miss Jones (*Elements of Logic*, p. 112), takes the example "All lions are quadrupeds." It will not do, she says, to reduce this to the form, "If any creatures are lions, they are quadrupeds," since this involves the introduction of a new term, and passing back again to the categorical form, we should have "All creatures which are lions are quadrupeds," a proposition not equivalent to our original proposition. If, however, "creature" is regarded as part of the connotation of "lion," there is no reason for refusing to allow that the two propositions are equivalent to one another. Similarly, in any concrete instance, by taking some part of the connotation of the subject of our categorical proposition, we can obtain the additional term required for its reduction to the conditional form. Where we are dealing with purely symbolic expressions, and this particular solution of the difficulty is not open to us, we may have recourse to the all-embracing term "anything," such a proposition as *All S is P* being reduced to the form *If anything is S it is P*. Thus, starting from the categorical, we are again led to regard the two forms as mutually interchangeable.

¹ If it were so, it would of course always be formally illegitimate to pass back from the categorical form to the conditional. Miss Jones, however, seems to regard the reduction of categoricals to conditionals as *generally* valid, and Mr Welton holds that universal categoricals of the form *SM is P* can always be reduced to conditionals. This is obviously inconsistent with the view that the conditional expresses something more than is contained in the categorical.

The question, however, remains whether we are to interpret the conditional with regard to the occurrence of its antecedent in the same way as we interpret the categorical with regard to the existence of its subject.

So far as *universals* are concerned, it seems clear that a conditional proposition does not necessarily imply the actual occurrence of its antecedent; and, therefore, if the view is taken that a universal categorical proposition does necessarily imply the actual existence of its subject, we have a marked distinction between the two kinds of propositions¹. *If any A is B then that A is C* cannot be resolved into *All AB is C*, since the latter implies the existence of *AB* while the former does not.

If, on the other hand, the view advocated in the preceding chapter is adopted and universal categorical propositions are not regarded as logically implying the existence of their subjects, then universal conditionals and universal categoricals may be resolved into one another. We may say indifferently *All B is C* or *If anything is B then it is C*; *If ever A is B then on all such occasions C is D* or *All occasions of A being B are occasions of C being D*.

Particular conditionals are distinguished from universals in that they are almost without exception based upon specific experience. Hence they imply the occurrence of their antecedents, as, for example, in the proposition, "Sometimes when Parliament meets, it is opened by the Queen in person." We may, therefore, assimilate particular as well as universal conditionals with categoricals as regards their existential import; and the two forms will again be mutually interchangeable.

The conclusion then at which we arrive is that there is no vital distinction between conditionals and categoricals. Some conditional propositions are indeed so obviously equivalent to categoricals that they seem hardly to require a separate consideration². At the same time, some statements will fall more

¹ This is Ueberweg's view. "The categorical judgment, in distinction from the hypothetical, always includes the pre-supposition of the existence of the subject" (*Logic*, § 122).

² The examples given at the commencement of the preceding section are reducible to the following categoricals: Import duties, which are sources of revenue, do not afford protection; All spoilt children have suffering parents; All

naturally into the one form and some into the other. The more complex the real subject-term the greater is the probability that the natural form of the proposition will be conditional.

129. *The Opposition of Conditional Propositions.*—The ordinary distinctions both of quality and of quantity can be applied to conditional propositions. We may consider that the quality of a conditional is determined by the quality of its consequent. Thus, the proposition *If any P is Q then that P is not R* may be treated as negative¹. As regards quantity, conditionals are to be regarded as universal or particular, according as the consequent is affirmed to accompany the antecedent in all or merely in some cases.

We have then the four types included in the ordinary four-fold schedule:—

If any P is Q, it is also R. A.

Sometimes if a P is Q, it is also R. I.

If any P is Q, it is not also R. E.

Sometimes if a P is Q, it is not also R. O.

These propositions constitute the ordinary square of opposition, and if conditionals are assimilated with categoricals so far as their existential import is concerned, then the opposition of conditionals seems to require no separate discussion. It may, however, be pointed out that there is more danger of contradictions being confused with contraries in the case of conditionals than in the case of categoricals. *If A is B then C is not D* is very liable to be given as the contradictory of *If A is B then C is D*. But it is clear on consideration that both these propositions may be false. For example, the two statements—If the Times says one thing, the Pall Mall says another; If the Times says one thing, the Pall Mall says the same, *i.e.*, does not say another—are both false; the two papers are sometimes in agreement and sometimes not.

pairs of straight lines which are such that another straight line falling upon them makes the alternate angles equal to one another are parallel; Any place where there is a carcass is a place where the eagles will gather together.

¹ The negative force of this proposition would be more clearly brought out if it were written in the form *If any P is Q then it is not the case that it is also R*.

130. Immediate Inferences from Conditional Propositions.—In a conditional proposition the antecedent and the consequent correspond respectively to the subject and the predicate of a categorical proposition. In conversion therefore the old consequent must be the new antecedent, and in contraposition the negation of the old consequent must be the new antecedent. Thus, taking the **E** proposition *If any P is Q then it is not R*, we have for its converse *If any P is R then it is not Q*; and taking the **A** proposition *If any P is Q then it is R*, we have for its contrapositive *If any P is not R then it is not Q*. In obversion we may confine ourselves to the consequent; thus, *If any P is Q then it is R* obverts to *If any P is Q then it is not not-R*.

The analogy with categoricals is so close that it is unnecessary to treat immediate inferences from conditionals in any detail. It will suffice if, in addition to what has been said above, we give all the ordinary inferences in the case of a concrete example. We may take the **A** proposition, *If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines shall be parallel*. This proposition yields the following:—

Obverse, If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines will not fail to be parallel;

Converse, In some cases in which two straight lines are parallel, a straight line falling upon them will make the alternate angles equal to one another;

Contrapositive, If two straight lines are not parallel, then a straight line falling upon them shall not make the alternate angles equal to one another;

Inverse, In some cases in which a straight line falling upon two other straight lines does not make the alternate angles equal to one another, these two straight lines shall not be parallel.

Amongst immediate inferences from a conditional proposition we must of course also include its reduction to categorical form. This is a case of what is sometimes called *change of relation*, which may be defined as an immediate inference in which we pass from a given proposition to another which

belongs to a different category in the division of propositions according to relation (see sections 37, 38). The more convenient term *transversion* is used by Miss Jones for this process. Thus, we pass by transversion from the above conditional proposition to the categorical proposition, Any pair of straight lines, which are such that a third straight line falling upon them makes the alternate angles equal to one another, are parallel.

131. *The Import of Hypothetical Propositions.*—The pure hypothetical may be written symbolically in the form *If A is true then C is true*, or more briefly, *If A then C*, where *A* and *C* stand for propositions of independent import. It is clear that this proposition affirms nothing as regards the truth or falsity of either *A* or *C* taken separately. A certain conjunction is, however, denied; namely, the conjunction of A true with C false. So much must be granted; but it is a disputed question whether this denial exhausts the import of the hypothetical. It is held by some writers that the hypothetical proposition affirms the consequent to be an inference from the antecedent; and on this view it does not merely deny the conjunction of *A* true with *C* false, but its import is that *to assert the conjunction of A true with C false involves logical inconsistency*¹.

The contrast between the two interpretations may be illustrated by considering a proposition, hypothetical in form, in which both the antecedent and the consequent are manifestly true, but the latter is not logically dependent upon the former. For

¹ Thus Whately considers a hypothetical proposition to be "an assertion of the *validity* of a certain argument; since to assert that an argument is valid is to assert that the conclusion necessarily results from the premisses, whether these premisses be true or not" (*Logic*, p. 103). Miss Jones similarly defines a hypothetical proposition as one "in which two (expressed or indicated) categoricals (or combinations of categoricals) are combined in such a way as to express that one (the consequent) is an inference from the other (the antecedent)" (*General Logic*, p. 44). Hypotheticals are then divided into *Formal or Self-contained Hypotheticals* and *Referential Hypotheticals*. In the former, "the consequent is an inference from the antecedent alone"; in the latter, "the consequent is inferred not from the antecedent alone, but from the antecedent taken in conjunction with some other unexpressed proposition or propositions" (p. 45). It may be observed in passing that there would be more reason for accepting the view of hypotheticals taken by Miss Jones if *all* hypotheticals were *self-contained*.

example, "If Cromwell was an Englishman, he was a Usurper." This proposition is, according to Whately, "false, or at least absurd." He ought, however, to limit himself to the first of these alternatives. On his view, the form of the proposition implies that the consequent is an inference from the antecedent, but this is certainly not the case. On the other view of the import of the hypothetical, however, such a proposition as the above may be defended on the ground that if the antecedent is a truth which no one would think of denying, this may be an emphatic way of declaring the truth of the consequent. The proposition now denies the conjunction, *Cromwell was an Englishman but not a Usurper*; and if the first element of this conjunction is admittedly true, the proposition amounts to the simple denial of the second element. In the same way we may deny a proposition emphatically by combining it as antecedent with a manifestly false consequent; e.g., "If what you say is true, I'm a Dutchman"; "If that boy comes back, I'll eat my head" (*vide Oliver Twist*). On the view that a hypothetical proposition merely denies the conjunction of the truth of its antecedent with the falsity of its consequent, these propositions respectively deny the conjunctions—What you say is true and *I am not a Dutchman*; That boy will come back and *I shall not eat my head*. Here the elements of the conjunctions which are in italics being admittedly true, the force of the propositions is to deny the truth of the other elements.

We shall see more clearly what the difference of view really amounts to by enquiring how the hypothetical is to be contradicted on the two interpretations respectively. In the one case, the contradictory will affirm the *actual* truth of the conjunction of *A* true with *C* false; in the other case, it will simply affirm that to assert this conjunction involves no logical inconsistency, *i.e.*, it will affirm the *possible* truth of the conjunction. In other words, the contradictory will be in the one case *A is true but C is false*; in the other *If A is true then C may be false*.

• Hence the difference between the two interpretations is, as Mr Johnson shews, really one of modality. Mr Johnson's grounds for preferring the first interpretation are given in the

following passage: "The proposition, *If A then C*, I propose simply to regard as an assertoric hypothetical, not as a modal hypothetical. In other words, it is taken to *assert a relation* of disjunction between *A* and *not-C*, not to *assert the obligation to assert this relation*. This interpretation is only in conformity with that of the simple proposition, *A is true*, which is regarded as an *assertoric* categorical, not a *modal* categorical; it asserts *A*, it does not assert *the obligation to assert A*; it is contradicted by *A is false*, not by *A may be false*. In justification of my interpretation it is only necessary to urge that the ordinary use of *if* must at least *include* the affirmation of the disjunctive. Of course a speaker must have some grounds for his statement. But it is one thing to dispute the validity of his grounds and quite another thing to dispute his statement itself. Where the speaker intends primarily to assert his right to affirm the disjunction—not to assert the disjunction itself—this meaning has only to be made explicit, and the symbolist will be able to deal with it. But the change of meaning involves a reference to new *sorts of terms*, which cannot without confusion be mixed up with the old terms."¹ The argument, as here stated, appears conclusive in favour of the assertoric interpretation of the proposition, *If A then C*.² This solution of the question is in accordance with the general principle that, whenever any doubtful point arises, logical forms should be interpreted so as to minimise rather than maximise their import³.

The argument may be strengthened by considering the relation between the hypothetical *If A is true then C is true* and what is ordinarily regarded as its disjunctive equivalent *Either C is true or A is not true*⁴. If we adopt the modal interpretation of the hypothetical, we must either interpret the disjunctive as also expressing an inferential relation, or else we

¹ *Mind*, 1892, pp. 18, 19. The symbols have been altered so as to agree with those used throughout this section.

² The modal hypothetical may of course still be recognised in the form, *If A is true then C must be true*.

³ Compare the interpretation given to the sign of quantity *some*. A further illustration of the same principle will be found in the interpretation of the disjunctive (or alternative) proposition in the following chapter.

⁴ This equivalence will be considered further in section 141.

must regard the import of the disjunctive as falling short of that of the hypothetical. But both these alternatives seem to yield paradoxical results. Taking the first, it follows that the propositions *P or Q is true* and *P and Q are both false* are not contradictories. Taking the second, it follows that the propositions *P or Q is true* and *If P is not true then Q is true* are not equivalents¹.

It has been already pointed out that a true hypothetical proposition expresses a relation between two other propositions of independent import, not between two terms; and it must now be added that this relation is not, either on the assertoric

¹ Miss Jones appears to accept the first of the above alternatives, Mr McColl explicitly accepts the second. Mr McColl writes (using the symbolism, $a : b$ for *If a then b*, $a + b$ for *a or b*, a' for the denial of a)—"The expression $a : b$ may be read *a implies b* or *If a is true, b must be true*. The statement $a : b$ implies $a' + b$. But it may be asked are not the two statements really equivalent; ought we not therefore to write $a : b = a' + b$? Now if the two statements are really equivalent their denials will also be equivalent. Let us see if this will be the case, taking as concrete examples: 'If he persists in his extravagance he will be ruined'; 'He will either discontinue his extravagance or he will be ruined.' The denial of $a : b$ is $(a : b)'$ and this denial may be read—'He may persist in his extravagance without necessarily being ruined.' The denial of $a' + b$ is ab' , which may be read—'He will persist in his extravagance and he will not be ruined.' Now it is quite evident that the second denial is a much stronger and more positive statement than the first. The first only asserts the *possibility* of the combination ab' ; the second asserts the *certainty* of the same combination. The denials of the statements $a : b$ and $a' + b$ having thus been proved to be not equivalent, it follows that the statements $a : b$ and $a' + b$ are themselves not equivalent, and that, though $a' + b$ is a necessary consequence of $a : b$, yet $a : b$ is not a necessary consequence of $a' + b$ " (see *Mind*, 1880, pp. 50 to 54; one or two slight verbal changes have been made in this quotation). Mr Welton accepts Miss Jones's view up to a certain point, but apparently does not recognise all that it involves and hence obtains inconsistent results. He regards (1) *A or B* and (2) *If not-A then B* as equivalents. For the contradictory of (1) he gives (3) *Neither A nor B*, and he considers that (2) yields as its contradictory (4) *If not-A then not-B*, this again being equivalent in his view to (5) *A or not-B*. But (3) and (5) are obviously not equivalents. We may take Mr Welton's concrete example (*Logic*, p. 281). The propositions (a) "This pen is either cross-nibbed or corroded by the ink," and (b) "This pen is neither cross-nibbed nor corroded by the ink," are given as contradictories. But (a) is regarded as equivalent to (c) "If this pen is not cross-nibbed it is corroded by the ink"; and for the contradictory of (c) Mr Welton would give (d) "If this pen is not cross-nibbed, it is not corroded by the ink." But (b) and (d) are clearly not equivalent to one another.

or on the modal interpretation of the hypothetical, completely analogous to the relation between terms expressed in a categorical proposition. Hence a pure hypothetical cannot be reduced to a categorical, the subject and predicate of which correspond precisely to the original antecedent and consequent. It is true that the proposition *If A then C* may, according to our interpretation of it, be expressed in one or other of the following categorical forms: *A is a proposition the truth of which is incompatible with the falsity of C*; *A is a proposition from which C may be inferred*. But in neither of these propositions is the predicate equivalent to the consequent of the hypothetical¹.

132. The Opposition of Hypothetical Propositions.—Distinctions of quality may be applied to hypotheticals according as the truth of some proposition is affirmed or denied in the consequent. Thus, the following may be regarded as respectively affirmative and negative: *If A is true then C is true*; *If A is true then C is not true*.

But whilst distinctions of *quality* can be recognised in the case of hypotheticals, the same is not true of distinctions of *quantity*. The antecedent of a hypothetical is not an event which may recur an indefinite number of times, but a proposition which is simply true or false. The same proposition cannot be sometimes true and sometimes false, for propositions referring to different times are different propositions².

If required to characterise hypotheticals in respect of their

¹ Amongst other differences it will be observed that the contrapositives of both these propositions differ from the contrapositive of the hypothetical. For, on either interpretation of the hypothetical, its contrapositive is *If C is not true then A is not true*, whilst the contrapositives of the above categorical propositions are respectively—*No proposition whose truth is compatible with the falsity of C is A*, *No proposition from which C cannot be inferred is A*.

² It is pointed out by Mr Johnson that the above statements must be taken in connexion with the recognition of propositions involving *multiple quantification*. "Thus we may indicate a series of propositions involving single, double, triple...quantification, which may reach any order of multiplicity: (1) 'All luxuries are taxed'; (2) 'In some countries all luxuries are taxed'; (3) 'At some periods it is true that in all countries all luxuries are taxed'.... With respect to each of the types of proposition (1), (2), (3)...I contend that, when made explicit with respect to time or place, &c., it is absurd to speak of them as sometimes true and sometimes false" (*Mind*, 1892, p. 30 n.).

quantity we should have to say that they are all singular. This might, however, suggest a misleading analogy with singular categoricals¹; and on the whole it is better to regard hypotheticals as not subject to distinctions of quantity at all².

The opposition of hypotheticals has been incidentally referred to in the preceding section. It follows from what has just been said in regard to quantity, that we cannot apply to hypotheticals, as we can to conditionals, the ordinary square of opposition. Every proposition, however, has a contradictory, and we have accordingly to enquire what is the contradictory of the proposition *If A then C*.

If we regard this proposition as merely an assertoric hypothetical then its contradictory is *A but not C*. We may look at it in this way. Let *AC* denote the truth of both *A* and *C*, *AC'* the truth of *A* and the falsity of *C*, and so on. Then there are four possibilities, namely, *AC*, *AC'*, *A'C*, *A'C'*, one or other of which must hold good, but any pair of which are mutually inconsistent. The proposition *If A then C* merely excludes *AC'*, and still leaves *AC*, *A'C*, *A'C'*, as possible

¹ That the analogy breaks down is shewn by the following consideration. An affirmative categorical with a singular term as subject will have for its predicate either another singular term (*e.g.*, *Tully is Cicero*) or a general term (*e.g.*, *Tully is wise*). In the former case the proposition has a simple converse (*Cicero is Tully*); in the latter case it has a general contrapositive (*No person who is not wise is Tully*). But, as will be shewn in the following section, an affirmative hypothetical is not simply convertible, and its contrapositive is another hypothetical and, therefore, not general.

² If we take the symbolic form *If any S is P then some Q is R* there is a certain ambiguity. We may say definitely that this proposition is not particular (although the antecedent and the consequent, taken separately, are both particular); but it may be either a universal conditional, or a true hypothetical and therefore not subject to distinctions of quantity. Compare, for example, the following: If any phenomenon is out of the common, some wiseacre will proclaim it to be ominous; If any phenomena are supernatural, some philosophers are mistaken. The first of these may be written—Whenever any phenomenon is out of the common, some wiseacre will proclaim it to be ominous. This proposition is conditional and obviously universal. The second proposition may be written—If it is true that any phenomena are supernatural, then it is true that some philosophers are mistaken. This proposition is a pure hypothetical. There is no reference to distinctions of time, and the antecedent is simply either true or false. We cannot with any meaning talk of its being true in all cases or in some cases that some phenomena are supernatural.

alternatives. In denying it, therefore, we must definitely affirm AC' , and exclude the three other alternatives. Hence the contradictory as above stated.

If, on the other hand, we regard *If A then C* as a modal hypothetical, its contradictory may be written *If A is true then C may be false*. We contradict the statement that *C is a legitimate inference from A* by the statement that *C is not a legitimate inference from A*¹. On this interpretation our original proposition affirms more than on the other interpretation, and its contradictory, therefore, affirms less.

It will be observed that on neither interpretation are *If A then C* and *If A then C'* true contradictories. If these are assertoric hypotheticals, it cannot be said that either of them is false supposing that *A* happens not to be true. For *If A then C* merely excludes AC' , and *If A then C'* merely excludes AC . Hence two possibilities are left, namely, $A'C$ and $A'C'$, neither of which is inconsistent with either proposition².

If, on the other hand, we interpret the propositions *If A then C*, *If A then not-C* as modal hypotheticals, they are still not true contradictories. For they may now both be false. It may be that neither *C* nor *not-C* is a legitimate inference from *A*³.

¹ Compare Miss Jones, *Elements of Logic*, pp. 153, 4.

² The validity of the above result will perhaps be more clearly seen by substituting for the hypotheticals their disjunctive equivalents, namely, *Either A is not true or C is true*, *Either A is not true or C is not true*. As a concrete example we may take the propositions, "If this pen is not cross-nibbed, it is corroded by the ink," "If this pen is not cross-nibbed, it is not corroded by the ink." Supposing that the pen happens to be cross-nibbed, we cannot regard either of these propositions as false. It will be observed that their disjunctive equivalents are, "This pen is either cross-nibbed or corroded by the ink," "This pen is either cross-nibbed or not corroded by the ink." Take again the propositions, "If the sun moves round the earth, some astronomers are fallible," "If the sun moves round the earth, all astronomers are infallible." The truth of the first of these propositions will not be denied, and on the interpretation of hypotheticals with which we are here concerned the second certainly cannot be said to be false. It may be taken as an emphatic way of denying that the sun does move round the earth.

³ This is clearly recognised by Miss Jones, *Elements of Logic*, pp. 154, 5. It is, however, sometimes argued that *If A then C* must have for its contradictory *If A then not C*, since the consequence must either follow or not follow from the antecedent. On this view, the proposition *If these books are not moved*,

133. *Immediate Inferences from Hypothetical Propositions.*—

The most important immediate inference from the proposition *If A then C* is *If C' then A'*. This inference is precisely analogous to *contraposition* in the case of categoricals, and it may without any risk of confusion be called by the same name. We may accordingly define the term *contraposition* as applied to hypotheticals as *a process of immediate inference by which we obtain a new hypothetical having for its antecedent the contradictory of the old consequent, and for its consequent the contradictory of the old antecedent.* This process is valid in the case of affirmative hypotheticals only. If having obtained the contrapositive of a proposition we apply the process over again, the result is that we regain the original proposition. Hence an affirmative hypothetical and its contrapositive are equivalents¹. The following are examples: "If patience is a virtue, there are painful virtues" = "If there are no painful virtues, patience is not a virtue"; "If there is a righteous God, the wicked will not escape their just punishment" = "If the wicked escape their just punishment, there is no righteous God."

From the negative hypothetical *If A is true then C is not true* we can infer *If C is true then A is not true*. This is analogous to *conversion* in the case of categoricals. The process of conversion is not, however, valid for affirmative hypotheticals: on the assertoric interpretation *If A then C* merely negatives *AC'*, while *If C then A* merely negatives *A'C*, and hence it is clear that neither of these propositions involves the other; on the modal interpretation the result is the same, for *C* may be a legitimate inference from *A*, although *A* is not a legitimate inference from *C*, and *vice versa*. The passage from

some of them will be injured is contradicted by the proposition *If these books are not moved, none of them will be injured*. But to say that *C* does not follow from *A* is obviously not the same thing as to say that *not-C* follows from *A*. As regards the above example, it may be that injury to the books will not necessarily follow from their non-removal, but that the question of their being injured or not will be quite unaffected by their removal or non-removal.

¹ This holds good whether we adopt the assertoric or the modal interpretation of hypotheticals. On the former interpretation, the import of both the propositions *If A then C* and *If C' then A'* is to negative *AC'*; on the latter interpretation, the import of both is to assert that the conjunction *AC'* involves logical inconsistency.

If A then C to If C then A is perhaps the most dangerous fallacy to be guarded against in the use of hypotheticals¹.

The process of obversion is of course applicable to hypotheticals. For example, If A is true then C is true = If A is true then C' is not true. It is nearly always more natural and more convenient to take hypotheticals in their affirmative rather than in their negative form; and hence in the case of hypotheticals more importance attaches to the process of contraposition than to that of conversion. There is no valid inference corresponding to inversion. For, given a hypothetical with A as antecedent, we cannot in any case infer a new hypothetical with A' as antecedent. The fallacy of arguing from If A then C to If A' then C' , which is equivalent to the fallacy mentioned above of arguing from If A then C to If C then A , may be described as the fallacy of confusing a hypothetical with its complementary.

Reference has been already made to the disjunctive equivalent of a hypothetical, and this equivalence will be further considered from the disjunctive point of view in the following chapter. The inference of a disjunctive from a hypothetical, or *vice versé*, is a case of *transversion*.

EXERCISES.

134. Give the contrapositive of the following proposition: If either no P is R or no Q is R , then nothing that is both P and Q is R . [κ.]

135. Assuming that rain never falls in Upper Egypt, are the following genuine pairs of contradictories?

(α) The occurrence of rain in Upper Egypt is always succeeded by an earthquake; the occurrence of rain in Upper Egypt is sometimes not succeeded by an earthquake.

¹ A consideration of immediate inferences enables us to shew from another point of view that If A then C and If A then C' are not true contradictories. For the contrapositives If A then C' , If C then A' , are equivalent to one another; and whenever two propositions are equivalent, their contradictories must also be equivalent. But If A then C is not equivalent to If C then A .

(b) If it is true that it rained in Upper Egypt on the 1st of July, it is also true that an earthquake followed on the same day; if it is true that it rained in Upper Egypt on the 1st of July, it is not also true that an earthquake followed on the same day.

If the above are not true contradictories, suggest what should be substituted. [B.]

136. Give the contrapositive and the contradictory of each of the following :

(1) If any nation prospers under a Protective System, its citizens reject all arguments in favour of free-trade ;

(2) If any nation prospers under a Protective System, we ought to reject all arguments in favour of free-trade. [J.]

137. Examine the logical relation between the two following propositions ; and enquire whether it is logically possible to hold (a) that both are true, (b) that both are false : (i) If volitions are undetermined, then punishments cannot rightly be inflicted ; (ii) If punishments can rightly be inflicted, then volitions are undetermined.

[J.]

CHAPTER IX.

DISJUNCTIVE (OR ALTERNATIVE) PROPOSITIONS.

138. *The terms Disjunctive and Alternative as applied to Propositions.*—Propositions of the form *Either X or Y is true* are ordinarily called *disjunctive*. It has been pointed out, however, that two propositions are really *disjoined* when it is denied that they are both true rather than when it is asserted that one or other of them is true; and the term *alternative*, as suggested by Miss Jones (*Elements of Logic*, p. 115), is obviously appropriate to express the latter assertion. We should then use the terms *conjunctive*, *disjunctive*, *alternative*, for the three following combinations respectively: *X and Y are both true*, *X and Y are not both true*, *Either X or Y is true*¹.

Whilst, however, the name *alternative* is preferable to *disjunctive* for the proposition *Either X or Y is true*, the latter name has such an established position in logical nomenclature that it seems better to continue to use it, so far as we use it at all, in its old sense. It may be pointed out further that an *alternative* contains a veiled disjunction (namely, between *not-X* and *not-Y*) even in the stricter sense; for the statement that *Either X or Y is true* is equivalent to the statement that *Not-X and not-Y are not both true*. Hence, although generally using the term *alternative*, I shall not altogether discard the term *disjunctive* as synonymous with it.

139. *Two types of Alternative Propositions.*—In the case of

¹ Some writers indeed regard the proposition *Either X or Y is true* as expressing a relation between *X* and *Y* which is disjunctive in the above sense as well as alternative; but the disjunctive character of this proposition as regards *X* and *Y* is at any rate open to dispute, whilst its alternative character is unquestionable (see section 140).

propositions which are ordinarily described as simply disjunctive a distinction must be drawn similar to that between conditionals and true hypotheticals. For the alternatives may be events or combinations of properties one or other of which it is affirmed will (always or sometimes) occur, *e.g.*, *Every blood vessel is either a vein or an artery, Every prosperous nation has either abundant natural resources or a good government*; or they may be propositions of independent import whose truth or falsity cannot be affected by varying conditions of time, space, or circumstance, and which must therefore be simply true or false, *e.g.*, *Either there is a future life or many cruelties go unpunished, Either it is no sin to covet honour or I am the most offending soul alive*.

Any proposition belonging to the first of the above types may be brought under the symbolic form *All (or some) S is either P or Q*, and may, therefore, be regarded as a complex categorical proposition with an alternative term as predicate. It is usual and for some reasons convenient to defer the discussion of the import of alternative terms until propositions of this type are being dealt with. Such propositions might otherwise be dismissed after a very brief consideration¹.

Alternative propositions of the second type are in the strict sense compound, not complex. They contain an alternative combination of propositions of independent import; and they have for their typical symbolic form *Either X is true or Y is true*, or more briefly, *Either X or Y*, where *X* and *Y* are symbols representing *propositions* (not terms). So far as it is necessary

¹ It should be particularly observed that although the proposition *Every S is P or Q* may be said to state an alternative, it cannot be resolved into a true alternative combination of *propositions*. Such a resolution is, however, possible if the subject is *singular or particular* (the proposition remaining affirmative and still having an alternative predicate): for example, *This S is P or Q = This S is P or this S is Q*; *Some S is P or Q = Some S is P or some S is Q*. It may be added that a negative categorical proposition with an alternative predicate cannot be said to state an alternative at all, since to deny an alternation is the same thing as to affirm a conjunction. Thus the proposition *No S is either P or Q* can only be resolved into a *conjunctive* synthesis of propositions, namely, *No S is P and no S is Q*. The equivalences and other relations between complex propositions and compound propositions will be considered in more detail in Part IV.

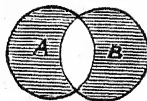
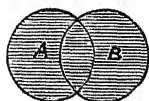
to give them a distinctive name, they have a claim to be called true alternative propositions, since they involve a true alternative synthesis of *propositions*, and not merely an alternative synthesis of terms¹.

140. *The Import of Alternative Propositions.*—The main question in dispute as regards the import of alternative propositions is whether the alternants should necessarily be regarded as mutually exclusive; whether, for example, in the proposition *All S is either A or B* it is necessarily implied that no *S* is both *A* and *B*², and whether in the proposition *X is true or Y is true* it is necessarily implied that *X* and *Y* are not both true. The question here raised may be considered from two different points of view.

(1) In ordinary speech do we always³ intend that the alternants in an alternative proposition should be understood as excluding one another? A very few instances will enable us to decide in the negative. Take, for example, the proposition, "He has either used bad text-books or he has been badly taught." No one would naturally understand this to exclude the possibility of a combination of bad teaching and the use of bad text-books. Or suppose it laid down as a condition of eligibility for some appointment that every candidate must be a member either of the University of Oxford, or of the University of Cambridge, or of the University of London. Would anyone regard this as implying the ineligibility of persons who happened to be members of more than one of these Universities? Jevons (*Pure Logic*, p. 68) instances

¹ Using the same phraseology as in the case of terms, *X* and *Y* may be spoken of as the *alternants* of the alternative proposition *Either X or Y*.

² This is an alternative proposition of the first type, and the same question is raised by asking whether the term *A or B* includes *AB* under its denotation or excludes it; in other words, whether the denotation of *A or B* is represented by the shaded portion of the first or of the second of the following diagrams.



³ There are of course many cases in which as a matter of fact we understand alternants to be mutually exclusive; the only point in dispute is whether we can lay this down as a universal rule.

the following proposition: "A peer is either a duke, or a marquis, or an earl, or a viscount, or a baron." We do not consider this statement incorrect because many peers as a matter of fact possess two or more titles. Take, again, the proposition, "Either the witness is perjured or the prisoner is guilty." The import of this proposition is that the evidence given by the witness is sufficient, supposing it true, to establish the guilt of the prisoner; but clearly there is no implication that the falsity of this particular piece of evidence would suffice to establish the prisoner's innocence.

(2) Still this does not definitely settle the question of the logical import of alternative propositions. Granted that in common speech the alternants may or may not be mutually exclusive, it may nevertheless be maintained that this is only because common speech is elliptical, that in logic we should be more precise, and that an alternative statement should accordingly not be admitted in logic except on the condition that the alternants mutually exclude one another.

This is a question of interpretation or method, in the solution of which dogmatism would be out of place, though it seems clear on which side the balance of advantage lies¹. No adequate reason can be given for diverging from the usage of everyday language, and it should always be remembered that if logic is to be of practical utility, the less logical forms diverge from those of ordinary speech the better. Moreover, condensed forms of expression do not conduce to clearness², or even

¹ The following passage from Mr Bradley's *Principles of Logic* may serve to illustrate the mistaken spirit of dogmatism by which such questions as that now under consideration are sometimes obscured. "Our slovenly habits of expression and thought are no real evidence against the exclusive character of disjunction. '*A is b or c*' does strictly exclude '*A is both b and c.*' When a speaker asserts that a given person is a fool or a rogue, he may not mean to deny that he is both. But, having no interest in shewing that he is both, being perfectly satisfied provided he is one, either *b* or *c*, the speaker has not the possibility *bc* in his mind. Ignoring it as irrelevant, he argues as if it did not exist. And thus he may practically be right in what he says, though formally his statement is downright false: for he has excluded the alternative *bc*" (p. 124).

² Professor Fowler indicates this view in his statement that "it is the object of logic not to state our thoughts in a condensed form but to analyse them into their simplest elements" (*Deductive Logic*, p. 32); though he does not apply it

ultimately to conciseness. For where our information is meagre, a condensed form is likely to express more than we intend, and in order to keep within the mark we must indicate additional alternatives. Were the exclusive interpretation of alternatives to be adopted the manipulation of complex propositions would certainly be rendered thereby very much more complicated.

It is of course always possible to express an alternative statement in such a way that the alternants are *formally* incompatible or exclusive. Thus, not wishing to exclude the case of A being both B and C we may write A is B or bC^1 ; or, wishing to exclude that case, A is Bc or bC^1 . But in neither of these instances can we say that the incompatibility of the alternants is really given by the alternative proposition. It is a merely formal proposition that *No A is both B and bC* , or that *No A is both Bc and bC* . The proposition *Every A is Bc or bC* does, however, tell us that no A is both B and C ; and when from our knowledge of the subject-matter it is obvious that an alternative is intended to be understood in the exclusive sense (and no doubt this is a very frequent case), we have in the above form a means of correctly and unambiguously expressing the fact. Where it is inconvenient to use this form it is open to us to make a separate statement to the effect that *No A is both B and C* . All that is here contended for is that the bare symbolic form A is *either B or C* should not be interpreted as equivalent to A is *either Bc or bC^2* .

to the case before us. Obviously a disjunctive proposition is a more condensed form of expression on the exclusive than on the non-exclusive interpretation. Compare Mansel's *Aldrich*, p. 242, and *Prolegomena Logica*, p. 238. "Let us grant for a moment the opposite view, and allow that the proposition *All C is either A or B* implies as a condition of its truth *No C can be both*. Thus viewed, it is in reality a complex proposition, containing two distinct assertions, each of which may be the ground of two distinct processes of reasoning, governed by two opposite laws. Surely it is essential to all clear thinking that the two should be separated from each other, and not confounded under one form by assuming the Law of Excluded Middle to be, what it is not, a complex of those of Identity and Contradiction" (*Aldrich*, p. 242). It may be added that one paradoxical result of the exclusive interpretation of alternatives is that *not P or Q* is not equivalent to *neither P nor Q* .

¹ Where b = not- B , and c = not- C .

² On the question of the import of alternatives, compare, further, section 245.

141. *The Reduction of Alternative Propositions to the form of Conditionals or Hypotheticals.*—Alternative propositions are reducible to the conditional or the true hypothetical form according to the type to which they belong. Thus, the proposition, "Every blood-vessel is either a vein or an artery," which is a complex proposition with an alternative predicate, yields the conditional, "If any blood-vessel is not a vein then it is an artery"; the true compound alternative proposition, "Either there is a future life or many cruelties go unpunished," yields the true hypothetical, "If there is no future life then many cruelties go unpunished."

It may be asked whether an alternative proposition does not require a conjunction of *two* conditionals or hypotheticals in order fully to express its import. This is not the case, however, on the view that the alternants are not necessarily exclusive. It is true that even on this view an alternative proposition, such as *Either X or Y*, is primarily reducible to two hypotheticals, namely, *If not X then Y* and *If not Y then X*. But these are contrapositives the one of the other, and therefore mutually inferable. Hence the full meaning of the alternative proposition is expressed by means of either of them¹. It is of course assumed here that we interpret hypotheticals and alternatives as either both assertoric or both modal².

¹ On the exclusive interpretation, the alternative proposition given in the text yields primarily four hypotheticals, namely, *If X then not Y* and *If Y then not X* in addition to the above. But these again are logical contrapositives the one of the other. Hence the full import of the alternative proposition will now be expressed by a conjunction of the two hypotheticals, *If X then not Y* and *If not X then Y*.

² Compare section 131. It may be added that there is even less to be said for the modal interpretation of alternatives than for the modal interpretation of hypotheticals. Hence if we decided to differentiate them in this respect, it would be to give the hypothetical the fuller meaning, with the result that we could pass from hypothetical to alternative, but not *vice versa*. Mr Bradley (*Principles of Logic*, p. 121) seems to hold on the contrary that the alternative proposition expresses more than the hypothetical. "Disjunctive judgments," he says, "cannot be reduced to hypotheticals." His view as to the relation between disjunctives and hypotheticals is, however, involved in some obscurity. On p. 130 he distinctly resolves a disjunctive proposition into hypotheticals, but he holds that, although the meaning of disjunctives can "be given hypothetically, we must not go on to argue from this that they are hypothetical" (p. 121);

142. *The Opposition of Alternative Propositions.*—Distinctions of quantity may be applied to complex propositions with an alternative predicate; thus, *All S is P or Q* is universal, while *Some S is P or Q* is particular. We may also have a negative proposition with an alternative predicate, for example, *No S is either P or Q*. Hence starting from the proposition *All S is either P or Q* we may complete the ordinary square of opposition by the propositions *Some S is either P or Q*, *No S is either P or Q*, ~~*Some S is not either P or Q*~~. If a complex alternative proposition is defined simply as a proposition with an alternative predicate, then all four of these propositions are to be described as alternative. It has, however, already been pointed out that the two negative propositions do not in any sense state an alternative, since the denial of an alternative is equivalent to the affirmation of a conjunctive¹. Hence if a complex alternative proposition is defined as a proposition which states an alternative by means of an alternative predicate, then only affirmative propositions can fall into this category².

Distinctions of quantity cannot be applied to true compound alternatives³; nor can distinctions of quality. The contradictory of the compound alternative proposition *X is true or Y is true* is given by the denial of *both* the alternants. Thus, *X is not true and Y is not true*⁴. But this proposition cannot from any point of view be described as alternative.

143. *Immediate Inferences from Alternative Propositions.*—The process of *transversion* whereby we pass from alternatives to conditionals or hypotheticals has been considered in section 141. It may be added that since conditionals are in their

for they "declare a fact without any supposition" (p. 122). It may be pointed out that the same is true of the hypothetical proposition itself, since it affirms without any supposition the connexion between the antecedent and the consequent. The term *hypothetical* as applied to propositions is perhaps not well chosen; Mr Bradley appears to be misled by it.

¹ See the note on p. 231.

² For a further treatment of the opposition of complex propositions see Part IV.

³ The argument here is practically the same as in the case of hypotheticals and need not be repeated.

⁴ Compare, however, section 131. p. 220

turn reducible to categoricals, alternative propositions of the first type may always be expressed in the form of categoricals which contain no alternation in their predicates. Thus, *All S is P or Q = All SP' is Q* (where $P' = \text{not-}P$). But it is of course unnecessary to pass through the conditional form in order to obtain this equivalence¹.

Other immediate inferences from alternative propositions may be briefly considered.

Alternative propositions belonging to the type which may be regarded as complex categoricals with alternative predicates will of course yield all the immediate inferences which are obtainable from ordinary categoricals; but the alternative form will not be retained. Thus, starting from the proposition *All S is either P or Q*, we have for its obverse, *No S is both not-P and not-Q*²; for its converse, *Some things that are either P or Q are S*; for its contrapositive, *Nothing that is both*

¹ It will be shewn in Part IV. that the immediate inference from *All S is P or Q* to *All SP' is Q* may be treated as a form of contraposition. We may notice in passing that, on the exclusive interpretation, the proposition *Every S is P or Q* is equivalent to the identity $SP = SQ'$, i.e., to the \mathbf{U} proposition *All SP is all SQ'*.

² *S is either P or it is not not-Q* has been given as the obverse of *S is either P or Q*. But it is clear that there is here no change in the quality of the proposition taken as a whole, as is required in obversion; and there is moreover no reason why Q should be manipulated any more than P. We must in an alternative regard the order of statement as indifferent; for we form no new judgment when we pass from *All S is P or Q* to *All S is Q or P*. The following curious doctrine is indeed laid down in a recent text-book of Logic: "When we say *S is either P or Q*, attention is concentrated on P rather than on Q; its true force is given by the inferential *If S is not P, it is Q*; where Q is a kind of residuary alternative. But when we say *S is either Q or P*, the stress is transferred from P to Q. For example, 'He is either a knave or a fool' means 'Most likely he is a knave, but, if not, he must be a fool'; but 'He is either a fool or a knave' implies 'He is probably a fool, and if not, he must be a knave'." Having arrived at this distinction, the writer goes on to argue that *S is either P or Q* has for its obverted contrapositive *S is either Q or P*. This result will be considered further in the next note but one. Here it is sufficient to observe that if the doctrine contained in the above extract is correct, then the two propositions which are given as contrapositives are really inconsistent with each other. The former is interpreted to mean that *S is one or the other but more probably P than Q*, while the latter is interpreted to mean that *S is one or the other but more probably Q than P*. Incidentally it seems to follow further that *If S is not P, it is Q* and *If S is not Q, it is P* are not really equivalents.

not-P and not-Q is S; for its inverse, *Some not-S is neither P nor Q*¹.

The compound alternative proposition *Either X or Y is true* has for its obverse *X' and Y' are not both true*. Since, however, there is in a compound alternative nothing that corresponds to the subject or the predicate of an ordinary categorical proposition, the processes of conversion, contraposition, and inversion are out of the question².

EXERCISES.

144. Shew how an alternative proposition in which the alternants are not known to be mutually exclusive (*e.g., Either X or Y or Z is true*) may be reduced to a form in which they necessarily are so. Write the new proposition in as simple a form as possible. [k.]

145. Shew why the following propositions are not contradictions: *Wherever A is present, B is present and either C or D is also present*; *In some cases where A is present, either B or C or D is absent*. How must each of these propositions in turn be amended in order that it may become the true contradictory of the other? [k.]

¹ Immediate inferences from complex propositions will be discussed in detail in Part IV., where however we shall proceed definitely upon the assumption that particulars imply, while universals do not imply, the existence of their subjects in the universe of discourse. On this assumption the processes of conversion and inversion from an A proposition are of course no longer valid.

² We may here briefly consider further the doctrine laid down by some logicians that the contrapositive of an alternative proposition is obtained by simply transposing the two alternants. Thus, (a) *X or Y is true* and (b) *Y or X is true* are given as contrapositives of each other, on the ground that (c) *If not-X then Y* and (d) *If not-Y then X* are respectively the hypothetical equivalents of (a) and (b), and (c) and (d) are contrapositives of each other. There is no valid reason why (c) any more than (d) should be regarded as the hypothetical equivalent of (a), or (d) any more than (c) as the hypothetical equivalent of (b). But, apart from this, it does not follow that two propositions are themselves contrapositives because their respective equivalents are contrapositives. On similar principles any A proposition might be shewn to be its own contrapositive. To give *Y or X is true* as the contrapositive of *X or Y is true* seems really to involve a total misapprehension of the nature of contraposition.

PART III.

SYLLOGISMS.

CHAPTER I.

THE RULES OF THE SYLLOGISM.

146. *The Terms of the Syllogism.*—A reasoning consisting of three categorical propositions (of which one is the conclusion), and containing three and only three terms, is called a *categorical syllogism*¹.

Every categorical syllogism, therefore, contains three and only three terms, of which two appear in the conclusion and also in one or other of the premisses, and one in the premisses only. That which appears as the predicate of the conclusion, and in one of the premisses, is called the *major term*; that which appears as the subject of the conclusion, and in one of the premisses, is called the *minor term*²; and that which appears in both the premisses, but not in the conclusion (being that term by their relations to which the mutual relation of the two other terms is determined), is called the *middle term*³.

¹ It is incorrect to define a syllogism as any combination of two propositions yielding as a conclusion a third proposition. This would include the argument *a fortiori* and other deductive inferences, which as such are never regarded as syllogistic. Compare sections 148, 153, 154, and 259.

² The major and minor terms are also sometimes called the *extremes* of the syllogism.

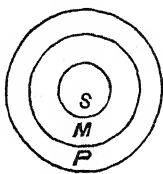
³ The middle term is also called the *argument*. This, says Hamilton (*Discussions*, p. 147), is the strict technical meaning of the term *argumentum* as employed by Cicero, Quintilian, Boethius, &c.

Thus, in the syllogism—

All M is P,
All S is M,
 therefore, *All S is P;*

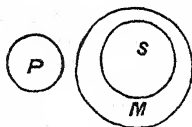
S is the minor term, *M* the middle term, and *P* the major term.

These respective designations of the terms of a syllogism resulted from such a syllogism as that just given being regarded as typical. With the exception of the somewhat rare case in which the terms of a proposition are coextensive, the above syllogism may be represented by the following diagram. Here

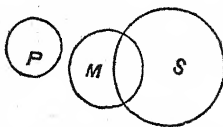


clearly the major term is the largest in extent, and the minor the smallest, while the middle occupies an intermediate position.

But we have no guarantee that the same relation between the terms of a syllogism will hold, when one of the premisses is negative or particular. Thus, the syllogism—*No M is P, All S is M, therefore, No S is P*—yields as one case



where the major term may be the smallest in extent, and the middle the largest. Again, the syllogism—*No M is P, Some S is M, therefore, Some S is not P*—yields as one case



where the major term may be the smallest in extent and the minor the largest.

Whilst, however, the middle term is not always a middle term in extent, it is always a middle term in the sense that by its means the two other terms are connected, and their mutual relation determined.

147. *The Propositions of the Syllogism.*—Every categorical syllogism consists of three propositions. Of these one is the *conclusion*. The premisses are called the *major premiss* and the *minor premiss* according as they contain the major term or the minor term respectively.

Thus, *All M is P*, (major premiss),
All S is M, (minor premiss),
 therefore, *All S is P*, (conclusion).

It is usual (as in the above syllogism) to state the major premiss first and the conclusion last. This is, however, nothing more than a convention. The order of the premisses in no way affects the validity of a syllogism, and has indeed no logical significance, though in certain cases it may be of some rhetorical importance. Professor Jevons (*Principles of Science*, chapter 6, § 14) argues that the cogency of a syllogism is more clearly recognisable when the minor premiss is stated first. But it is doubtful whether any general rule of this kind can be laid down. In favour of the traditional order, it is to be said that in what is usually regarded as the typical syllogism (*All M is P*, *All S is M*, therefore, *All S is P*¹) there is a philosophical ground for stating the major premiss first, since that gives the general rule, of which the minor premiss enables us to make a particular application.

148. *The Rules of the Syllogism.*—The rules of the categorical syllogism as usually stated are as follows:—

- (1) Every syllogism contains three and only three terms.
- (2) Every syllogism consists of three and only three propositions.

These two so-called rules are not properly speaking rules for the validity of an argument. They simply serve to define the syllogism as a particular form of argument. In other words, a reasoning which does not fulfil these conditions may be

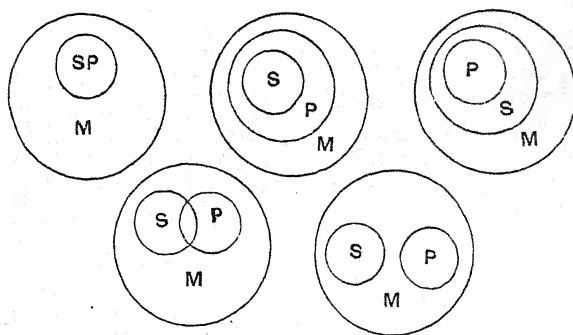
¹ Technically called a syllogism in *Barbara*. See section 203.

formally valid, only we do not call it a syllogism¹. The four rules that follow are really rules in the sense that if, when we have got the reasoning into the form of a syllogism, they are not fulfilled, then the reasoning is invalid².

(3) *No one of the three terms of a syllogism may be used ambiguously; and the middle term must be distributed once at least in the premisses.*

This rule is frequently given in the form: "The middle term must be distributed once at least, and must not be ambiguous." But it is obvious that we have to guard against ambiguous major and ambiguous minor as well as against ambiguous middle. The fallacy resulting from the ambiguity of one of the terms of a syllogism is a case of *quaternio terminorum*, that is, a fallacy of four terms.

The necessity of distributing the middle term may be illustrated by the aid of the Eulerian diagrams. Given, for instance, *All P is M* and *All S is M*, we may have any one of the five following cases:—



¹ For example, *B is greater than C*,
A is greater than B,
 therefore, *A is greater than C*.

Here is a valid reasoning which consists of three propositions. But it contains more than three terms; for the predicate of the second premiss is "greater than B," while the subject of the first premiss is "*B*." It is, therefore, as it stands, not a syllogism. Whether reasonings of this kind admit of being reduced to syllogistic form is a problem which will be discussed subsequently.

² Apparent exceptions to these rules will be shewn in sections 153 and 154 to result from the attempt to apply them to reasonings which have not first been reduced to syllogistic form.

Here all the five relations that are *a priori* possible between *S* and *P* are still possible. We have, therefore, no conclusion.

If in a syllogism the middle term is distributed in neither premiss, we are said to have a fallacy of *undistributed middle*.

(4) No term may be distributed in the conclusion which was not distributed in one of the premisses.

The breach of this rule is called *illicit process of the major*, or *illicit process of the minor*, as the case may be; or, more briefly, *illicit major* or *illicit minor*.

(5) From two negative premisses nothing can be inferred.

This rule may, like rule 3, be very well illustrated by means of the Eulerian diagrams.

(6) If one premiss is negative, the conclusion must be negative; and to prove a negative conclusion, one of the premisses must be negative¹.

149. Corollaries from the Rules of the Syllogism.—From the rules given in the preceding section, three corollaries may be deduced:—

(i) From two particular premisses nothing can be inferred.

Two particular premisses must be either

- (α) both negative,
- or (β) both affirmative,
- or (γ) one negative and one affirmative.

But in case (α), no conclusion follows by rule 5.

In case (β), since no term can be distributed in two particular affirmative propositions, the middle term cannot be distributed, and therefore by rule 3 no conclusion follows.

In case (γ), if any valid conclusion is possible, it must be negative (rule 6). The major term, therefore, will be distributed in the conclusion; and hence we must have two terms distributed in the premisses, namely, the middle and the major (rules 3, 4). But a particular negative proposition and a particular affirmative proposition between them distribute only one term. Therefore, no conclusion can be obtained.

¹ This rule and the second corollary given in the following section are sometimes combined into the one rule, *Conclusio sequitur partem deteriore*; i.e., the conclusion follows the worse or weaker premiss both in quality and

(ii) *If one premiss is particular, the conclusion must be particular.*

We must have either

(α) two negative premisses, but this case is rejected by rule 5;

or (β) two affirmative premisses;

or (γ) one affirmative and one negative.

In case (β) the premisses, being both affirmative and one of them particular, can distribute but one term between them. This must be the middle term by rule 3. The minor term is, therefore, undistributed in the premisses, and the conclusion must be particular by rule 4.

In case (γ) the premisses will between them distribute two and only two terms. These must be the middle by rule 3, and the major by rule 4 (since we have a negative premiss, necessitating a negative conclusion by rule 6, and therefore the distribution of the major term in the conclusion). Again, therefore, the minor cannot be distributed in the premisses, and the conclusion must be particular by rule 4¹.

De Morgan (*Formal Logic*, p. 14) gives the following very ingenious proof of this corollary:—"If two propositions P and Q together prove a third R , it is plain that P and the denial of Q prove the denial of R . For P and Q cannot be true together without R . Now, if possible, let P (a particular) and Q (a universal) prove R (a universal). Then P (particular) and the denial of R (particular) prove the denial of Q . But two particulars can prove nothing."

(iii) *From a particular major and a negative minor nothing can be inferred.*

Since the minor premiss is given negative, the major premiss must by rule 5 be affirmative. But it is also particular, in quantity, a negative being considered weaker than an affirmative and a particular than a universal.

¹ Spalding (*Logic*, p. 209) remarks, "When one of the premisses is particular, the conclusion must be particular. The transgression of this rule is a symptom of illicit process of the minor." It is not, however, the case that if we infer a universal conclusion from a particular premiss we necessarily commit the fallacy of illicit minor. For example, we may have, *Some M is P*, *All S is M*, therefore, *All S is P*; where the fallacy committed is that of undistributed middle.

and it therefore follows that the major term cannot be distributed in it. Hence, by rule 4, it must be undistributed in the conclusion, *i.e.*, the conclusion must be *affirmative*. But also, by rule 6, since we have a negative premiss, it must be *negative*. This contradiction establishes the corollary that under the supposed circumstances no conclusion is possible.

The following mnemonic lines, attributed to Petrus Hispanus, afterwards Pope John XXI., sum up the rules of the syllogism and the first two corollaries :

Distribuas medium : nec quartus terminus adsit :

Utraque nec præmissa negans, nec particularis :

Sectetur partem conclusio deteriolem ;

Et non distribuât, nisi cum præmissa, negetve.

150. Restatement of the Rules of the Syllogism.—It has been already pointed out that the first two of the rules given in section 148 should be regarded as a description of the syllogism rather than as rules for its validity. Again, the part of rule 3 relating to ambiguity may be regarded as contained in the proviso that there shall be only three terms ; for, if one of the terms is ambiguous, there are really four terms, and hence no syllogism according to our definition of syllogism. The rules may, therefore, be reduced to four ; and they may be restated as follows :—

A. Two rules of distribution :—

(1) The middle term must be distributed once at least in the premisses ;

(2) No term may be distributed in the conclusion which was not distributed in one of the premisses ;

B. Two rules of quality :—

(3) From two negative premisses no conclusion follows ;

(4) If one premiss is negative, the conclusion must be negative ; and to prove a negative conclusion, one of the premisses must be negative¹.

¹ The rules of quality might also be stated as follows : To prove an affirmative conclusion, both premisses must be affirmative ; To prove a negative conclusion, one premiss must be affirmative and the other negative.

151. *Dependence of the Rules of the Syllogism upon one another.*—The four rules just given are not ultimately independent of one another. It may be shewn that a breach of the second, or of the third, or of the first part of the fourth involves indirectly a breach of the first; or, again, that a breach of the first, or of the third, or of the first part of the fourth involves indirectly a breach of the second.

(i) *The rule that two negative premisses yield no conclusion may be deduced from the rule that the middle term must be distributed once at least in the premisses.*

This is shewn by De Morgan (*Formal Logic*, p. 13). He takes two universal negative premisses *E*, *E*. In whatever figure they may be, they can be reduced by conversion to

No P is M,
No S is M.

Then by obversion they become (without losing any of their force)—

All P is not-M,
All S is not-M;

and we have undistributed middle. Hence rule 3 is exhibited as a corollary from rule 1. For if any connexion between *S* and *P* can be inferred from the first pair of premisses, it must also be inferable from the second pair.

An objection may perhaps be taken to the above on the ground that the premisses might also be reduced to *All M is not-P*, *All M is not-S*, where the middle term is distributed in both premisses. Here, however, we no longer have a middle term *connecting S and P* at all. We shall return in section 153 to this method of dealing with two negative premisses.

The case in which one of the premisses is particular is dealt with by De Morgan as follows:—"Again, *No Y is X*, *Some Ys are not Zs*, may be converted into

Every X is (a thing which is not Y),
Some (things which are not Zs) are Ys,

in which there is no middle term."

This is not satisfactory, since we may often exhibit a valid

syllogism in such a form that there appear to be four terms; e.g., *All M is P, All S is M*, may be reduced to

All M is P,
No S is not-M,

and there is now no middle term.

The case in question may, however, be disposed of by saying that if we cannot infer anything from two negative premisses both of which are universal, *à fortiori* we cannot from two negative premisses one of which is particular¹.

(ii) *The rules that from two negative premisses nothing can be inferred and that if one premiss is negative the conclusion must be negative are mutually deducible from one another.*

The following proof that the second of these rules is deducible from the first is suggested by De Morgan's deduction of the second corollary as given in section 149. If two propositions *P* and *Q* together prove a third *R*, it is plain that *P* and the denial of *R* prove the denial of *Q*. For *P* and *Q* cannot be true together without *R*. Now if possible let *P* (a negative) and *Q* (an affirmative) prove *R* (an affirmative). Then *P* (a negative) and the denial of *R* (a negative) prove the denial of *Q*. But by hypothesis two negatives prove nothing.

It may be shewn similarly that if we start by assuming the second of the rules then the first is deducible from it.

(iii) *Any syllogism involving directly an illicit process of major or minor involves indirectly a fallacy of undistributed middle, and vice versa*².

Let *P* and *Q* be the premisses and *R* the conclusion of a syllogism involving illicit major or minor, a term *X* which is undistributed in *P* being distributed in *R*. Then the contradictory of *R* combined with *P* must prove the contradictory of *Q*. But any term distributed in a proposition is undistributed in its contradictory. *X* is therefore undistributed in

¹ This argument holds good in the special case under consideration even if we regard particulars, but not universals, as implying the existence of their subjects. For the validity of the above proof that two universal negatives yield no conclusion remains unaffected even if we allow to universals the maximum of existential import.

² For this theorem and its proof I am indebted to Mr Johnson.

the contradictory of *R*, and by hypothesis it is undistributed in *P*. But *X* is the middle term of the new syllogism, which is therefore guilty of the fallacy of undistributed middle. It is thus shewn that any syllogism involving directly a fallacy of illicit major or minor involves indirectly a fallacy of undistributed middle.

Adopting a similar line of argument, we might also proceed in the opposite direction, and exhibit the rule relating to the distribution of the middle term as a corollary from the rule relating to the distribution of the major and minor terms.

152. Statement of the independent Rules of the Syllogism.—

The theorems established in the preceding section shew that the first part of rule 4 (as given in section 150) is a corollary from rule 3, and that rule 3 is in its turn a corollary from rule 1; also that rules 1 and 2 mutually involve one another, so that either one of them may be regarded as a corollary from the other. We are, therefore, left with either rule 1 or rule 2 and also with the second part of rule 4; and the independent rules of the syllogism may accordingly be stated as follows:

(*α*) *Rule of Distribution*:—The middle term must be distributed once at least in the premisses [*or*, as alternative with this, No term may be distributed in the conclusion which was not distributed in one of the premisses];

(*β*) *Rule of Quality*:—To prove a negative conclusion one of the premisses must be negative¹.

It should be clearly understood that it is not meant that every invalid syllogism will offend *directly* against one of these two rules. As a direct test for the detection of invalid syllogisms we must still fall back upon the *four* rules given in section 150². All that we have succeeded in shewing is that

¹ On examination it will be found that the only syllogism rejected by this rule and not also rejected directly or indirectly by the preceding rule is the following:—*All P is M, All M is S, therefore, Some S is not P*. In the technical language explained in the following chapter, this is *AAO* in figure 4. So far, therefore, as the first three figures are concerned, we are left with a single rule, namely, a rule of distribution, which may be stated in either of the alternative forms above given.

² If, for example, for our rule of distribution we select the rule relating to the distribution of the middle term, then the invalid syllogism,

ultimately these four rules are not independent of one another¹.

153. *Two negative premisses may yield a valid conclusion; but not syllogistically.*—Jevons remarks: "The old rules of logic informed us that from two negative premisses no conclusion could be drawn, but it is a fact that the rule in this bare form does not hold universally true; and I am not aware

All M is P,
No S is M,
therefore, *No S is P,*

does not directly involve a breach of either of our two independent rules. But if this syllogism is valid, then must also the following syllogism be valid:

All M is P (original major),
Some S is P (contradictory of original conclusion),
therefore, *Some S is M* (contradictory of original minor);

and here we have undistributed middle. Hence the rule relating to the distribution of the middle term establishes *indirectly* the invalidity of the syllogism in question. The principle involved is the same as that on which we shall find the process of indirect reduction to be based.

Take, again, the syllogism: *PaM, SeM, ∴ SaP*. This does not directly offend against the rules given above; but the reader will find that its validity involves the validity of another syllogism in which a direct transgression of these rules occurs.

¹ I have made several attempts without success to exhibit rule β as a corollary from rule α , and hence to reduce the rules of the syllogism to a single rule of distribution. The attempted proof given below is the result of some correspondence on the subject with Mr Alfred Sidgwick, and it is the most satisfactory that I have been able to obtain. It depends, however, on an assumption which seems no more self-evident than the rule to be dispensed with, namely, "Given a premiss (A) and a conclusion (C)—containing between them three and only three terms—such that C follows from A (though not syllogistically) by the aid of another premiss (X), then it must be possible to construct a valid syllogism of which A and C are components and of which the other premiss (B) is obtainable by some form of immediate inference from X." Granting this assumption, the proof runs as follows:—Let it be possible for two affirmatives to prove syllogistically a negative. Then to avoid undistributed middle, illicit minor, and illicit major, the syllogism must be: *PaM, MaS, ∴ SoP*; and, since *PaM = M'aP'* and *SoP = SiP'*, the premisses *M'aP'* and *MaS* together prove *SiP'*. Now regarding *M'aP'* and *SiP'* as one premiss and the conclusion of a valid syllogism, the other premiss must connect *S* and *M'*, and it must be affirmative by the rule that a negative premiss requires a negative conclusion (which has been already shewn to be a corollary from the rule of distribution). But no affirmative premiss connecting *S* and *M'* is contained in or can be by any process of immediate inference be evolved out of *MaS*. Hence it follows that it is not possible for two affirmatives to prove syllogistically a negative.

that any precise explanation has been given of the conditions under which it is or is not imperative. Consider the following example,—*Whatever is not metallic is not capable of powerful magnetic influence, Carbon is not metallic, therefore, Carbon is not capable of powerful magnetic influence.* Here we have two distinctly negative premisses, and yet they yield a perfectly valid negative conclusion. The syllogistic rule is actually falsified in its bare and general statement" (*Principles of Science*, chapter 4, § 10).

This apparent exception is, however, no real exception. The reasoning (which may be expressed symbolically in the form, *No not-M is P, No S is M, therefore, No S is P*) is certainly valid; but if we regard the premisses as negative it has four terms *S, P, M, and not-M*, and is therefore no syllogism. Reducing it to syllogistic form, the minor becomes by obversion *All S is not-M*, an affirmative proposition¹. It is not the case, therefore, that we have succeeded in finding a valid *syllogism* with two negative premisses. In other words, while we must not say that from two negative premisses nothing follows, it remains true that if a syllogism regularly expressed has two negative premisses it is invalid².

Mr Bradley (*Principles of Logic*, p. 254) returns to the position taken by Professor Jevons. In reference to the example given above, he says, "This argument no doubt has *quaternio terminorum* and is vicious technically, but the fact remains that from two denials you somehow *have* proved a further denial. '*A is not B, what is not B is not C, therefore, A is not C*'; the premisses are surely negative to start with, and it appears pedantic either to urge on one side that '*A is not-B*' is simply positive, or on the other that *B* and not-*B* afford no junction. If from negative premisses I can get my

¹ It may be added that it is in this form that the cogency of the argument is most easily to be recognised. Of course every affirmation involves a denial and *vice versâ*; but it may fairly be said that in Jevons's example the primary force of the minor premiss, considered in connexion with the major premiss, is to affirm that carbon belongs to the class of non-metallic substances, rather than to deny that it belongs to the class of metallic substances.

² This is clearly explained by Jevons himself in his *Elementary Lessons in Logic*, p. 134.

conclusion, it seems idle to object that I have first transformed one premiss; for that objection does not shew that the premisses are not negative, and it does not shew that I have failed to get my conclusion."

This is for the most part irrelevant; and if the points on both sides are clearly stated there appears no room for further controversy. On the one hand, it is implicitly admitted both by Jevons (*Studies in Deductive Logic*, p. 89), and by Mr Bradley, that two negative premisses invalidate a reasoning which is expressed in strict syllogistic form. On the other hand, everyone would allow that from two propositions which may both be regarded as negative, a conclusion may sometimes be obtained; for example, the propositions which constitute the premisses of a syllogism in *Barbara*—*All M is P, All S is M*, therefore, *All S is P*—may be written in a negative form, thus, *No M is not-P, No S is not-M*, and the conclusion *All S is P* still follows¹. We must not, in fact, attach undue importance to the distinction between positive and negative propositions. By availing himself of the process of obversion, the logician may at will regard any given proposition as either the one or the other².

¹ Lotze (*Logic*, § 89) seems to imply that the possibility of writing both premisses in a negative form is confined to reasonings which syllogistically fall into figure 3. The above example shews that this is not the case.

² A case similar to that adduced by Jevons is dealt with in the *Port Royal Logic* (Professor Baynes's translation, p. 211) as follows:—"There are many reasonings, of which all the propositions appear negative, and which are, nevertheless, very good, because there is in them one which is negative only in appearance, and in reality affirmative, as we have already shewn, and as we may still further see by this example: *That which has no parts cannot perish by the dissolution of its parts; The soul has no parts; therefore, The soul cannot perish by the dissolution of its parts*. There are several who advance such syllogisms to shew that we have no right to maintain unconditionally this axiom of logic, *Nothing can be inferred from pure negatives*; but they have not observed that, in sense, the minor of this and such other syllogisms is affirmative, since the middle, which is the subject of the major, is in it the attribute. Now the subject of the major is not that which has parts, but that which has not parts, and thus the sense of the minor is, *The soul is a thing without parts*, which is a proposition affirmative of a negative attribute." Ueberweg also, who himself gives a clear explanation of the case, shews that it was not overlooked by the older logicians; and he thinks it not improbable that the doctrine of qualitative æquipollence between two judgments (i.e., obversion) resulted from the consideration of this very question (*System of Logic*, § 106).

154. *Other apparent exceptions to the Rules of the Syllogism.*—

It is curious that the logicians who have laid so much stress on the case considered in the preceding section do not appear to have observed that as soon as we admit more than three terms, other apparent breaches of the syllogistic rules may occur in what are perfectly valid reasonings. Thus, the premisses *All P is M* and *All S is M*, in which *M* is not distributed, yield the conclusion *Some not-S is not-P*¹; and hence we might argue that undistributed middle does not invalidate an argument. Again, from the premisses *All M is P*, *All not-M is S*, we may infer *Some S is not P*², although there is apparently an illicit process of the major. It is unnecessary after what has been said in the preceding section to give examples of valid reasonings in which we have a negative premiss with an affirmative conclusion, or two affirmative premisses with a negative conclusion, or a particular major with a negative minor. Any valid syllogism which is affirmative throughout will yield the first and, if it has a particular major, also the last of these by the obversion of the minor premiss, and the second by the obversion of the conclusion. The only syllogistic rules, indeed, which still hold good when more than three terms are admitted are the rule providing against illicit minor and the first two corollaries.

¹ By the contraposition of both premisses this reasoning is reduced to the valid syllogistic form, *All not-M is not-P*, *All not-M is not-S*, therefore, *Some not-S is not-P*. Or, we might reason as follows:—Since *S* and *P* are both entirely included in *M*, there must be outside *M* some *not-S* and some *not-P* that are coincident; and this is the same conclusion as before. It would seem that we have here assumed that there is *some not-M*, i.e., that *M* does not constitute the entire universe of discourse; and the question may, therefore, be asked how it is that the necessity of this assumption is not also apparent in our first method of treatment. The truth appears to be that we have an illustration of De Morgan's view (*Formal Logic*, p. 112) that in all syllogisms the existence of the middle term is a *datum*. From the premisses *All B is C*, *All B is A*, we cannot obtain the conclusion *Some A is C* without implicitly assuming the existence of *B*. Take as an example,—All witches ride through the air on broomsticks; All witches are old women; therefore, Some old women ride through the air on broomsticks. This point will be further discussed in section 290.

² By the inversion of the first premiss, this reasoning is reduced to the valid syllogistic form, *Some not-M is not P*, *All not-M is S*, therefore, *Some S is not P*. Compare section 70.

But of course none of the above examples really invalidate the syllogistic rules, for these rules have been formulated solely with reference to reasonings of a certain form, namely, those which contain three and only three terms. In every case the reasoning inevitably conforms to the rule which it appears to violate, as soon as, by the aid of immediate inferences, the superfluous number of terms has been eliminated.

155. *Syllogisms with two singular premisses*.—Professor Bain (*Logic, Deduction*, p. 159) argues that an apparent syllogism with two singular premisses cannot be regarded as a genuine syllogistic or deductive inference; and he illustrates his view by reference to the following syllogism:

Socrates fought at Delium,
Socrates was the master of Plato,
 therefore, *The master of Plato fought at Delium.*

The argument is that “the proposition ‘Socrates was the master of Plato and fought at Delium,’ compounded out of the two premisses, is nothing more than a grammatical abbreviation,” whilst the step hence to the conclusion is a mere omission of something that had previously been said. “Now, we never consider that we have made a real inference, a step in advance, when we repeat less than we are entitled to say, or drop from a complex statement some portion not desired at the moment. Such an operation keeps strictly within the domain of Equivalence or Immediate Inference. In no way, therefore, can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference.”

This argument leads up to some very interesting considerations, but it proves too much. In the following syllogisms the premisses may be similarly compounded together:

All men are mortal, } *All men are mortal and rational;*
All men are rational, }
 therefore, *Some rational beings are mortal.*

All men are mortal, } *All men including kings are mortal;*
All kings are men, }
 therefore, *All kings are mortal*¹.

¹ Compare with the above the following syllogism which has two singular

Do not Dr Bain's criticisms apply to these syllogisms as much as to the syllogism with two singular premisses? The method of treatment adopted is indeed particularly applicable to syllogisms in which the middle term is subject in both premisses¹. But we may always combine the two premisses of a syllogism in a single statement, and it is always true that the conclusion of a syllogism contains a part of, and only a part of, the information contained in the two premisses taken together; hence we may always get Dr Bain's result². In other words, in the conclusion of every syllogism "we repeat less than we are entitled to say," or, if we care to put it so, "drop from a complex statement some portion not desired at the moment."

156. *Charge of incompleteness brought against the ordinary syllogistic conclusion.*—This charge (a consideration of which will appropriately supplement the discussion contained in the preceding section) is brought by Jevons (*Principles of Science*, chapter 4, § 8) against the ordinary syllogistic conclusion. The premisses *Potassium floats on water, Potassium is a metal* yield, according to him, the conclusion *Potassium metal is potassium floating on water*. But "Aristotle would have inferred that *some metals float on water*. Hence Aristotle's conclusion simply leaves out some of the information afforded in the premisses; it even leaves us open to interpret the *some metals* in a wider sense than we are warranted in doing."

In reply to this it may be remarked: first, that the Aristotelian conclusion does not profess to sum up the whole of the information contained in the premisses of the syllogism; secondly, that some in Logic means merely "not none," "one

premisses:—The Lord Chancellor receives a higher salary than the Prime Minister; Lord Herschell is the Lord Chancellor; therefore, Lord Herschell receives a higher salary than the Prime Minister. These premisses would presumably be compounded by Professor Bain into the single proposition, "The Lord Chancellor, Lord Herschell, receives a higher salary than the Prime Minister."

¹ Such syllogisms are said to be in figure 3. Compare section 189.

² It may be pointed out that the general method adopted by Boole in his *Laws of Thought* is to sum up all his given propositions in a single proposition, and then eliminate the terms that are not required. Compare also the methods employed in Part iv.

at least." The conclusion of the above syllogism might perhaps better be written "some metal floats on water," or "some metal or metals &c." Lotze remarks in criticism of Jevons: "His whole procedure is simply a repetition or at the outside an addition of his two premisses; thus it merely adheres to the given facts, and such a process has never been taken for a *Syllogism*, which always means a movement of thought that uses what is given for the purpose of advancing beyond it..... The meaning of the Syllogism, as Aristotle framed it, would in this case be that the occurrence of a floating metal Potassium proves that the property of being so light is not incompatible with the character of metal in general."¹ This criticism is perhaps pushed a little too far. It is hardly a fair description of Jevons's conclusion to say that it is the mere sum of the premisses; for it brings out a relation between two terms which was not immediately apparent in the premisses as they originally stood. Still there can be no doubt that the elimination of the middle term is the very gist of syllogistic reasoning as ordinarily understood.

It may be added, as an *argumentum ad hominem* against Jevons, that his own conclusion *leaves out some of the information afforded in the premisses.* For we cannot pass back from the proposition *Potassium metal is potassium floating on water* to either of the original premisses².

157. *The connexion between the dictum de omni et nullo and the ordinary rules of the syllogism.—The dictum de omni et*

¹ *Logic*, book ii, chapter 3, note. Compare also, Dr Venn in the *Academy*, Oct. 3, 1874, and Professor Croom Robertson in *Mind*, 1876, p. 219. Dr Venn remarks, "Surely, as the old expression 'discursive thought' implies, we designedly pass on from premisses to conclusion, and then drop the premisses from sight. If we want to keep them in sight we can perfectly well retain them as premisses; if not, if all that we want is the final fact, it is no use to burden our minds or paper with premisses as well as conclusion. All reasoning is derived from data which under conceivable circumstances might be useful again, but which we are satisfied to recover when we want them."

² Symbolically, we have in Aristotle's system, *All M is P, All M is S*, therefore, *Some S is P*; and, in Jevons's system, $M=MP$, $M=MS$, therefore, $MS=MP$. Considered as connecting *S* and *P*, Jevons's conclusion is the typical form of a particular proposition; and it is obvious that we cannot pass back to either of the premisses, since they are both universal.

nullo was given by Aristotle as the axiom on which all syllogistic inference is based. It applies directly, however, to those syllogisms only in which the major term is predicate in the major premiss, and the minor term subject in the minor premiss, (*i.e.*, to what are called syllogisms in figure 1). The rules of the syllogism, on the other hand, apply independently of the position of the terms in the premisses. Nevertheless, it is interesting to trace the connexion between them. It will be found that all the rules are involved in the *dictum*, but some of them in a less general form, in consequence of the distinction just pointed out.

The *dictum* may be stated as follows:—“Whatever is predicated, whether affirmatively or negatively, of a term distributed may be predicated in like manner of everything contained under it.”

(1) The *dictum* provides for three and only three terms; namely, (i) a certain term which must be distributed, (ii) something predicated of this term, (iii) something contained under it. These terms are respectively the middle, major, and minor. We may consider the rule relating to the ambiguity of terms also contained here, since if any term is ambiguous we have practically more than three terms.

(2) The *dictum* provides for three and only three propositions; namely, (i) a proposition predicating something of a term distributed, (ii) a proposition declaring something to be contained under this term, (iii) a proposition making the original predication of the contained term. These propositions constitute respectively the major premiss, the minor premiss, and the conclusion of the syllogism.

(3) The *dictum* prescribes not merely that the middle term shall be distributed once at least in the premisses, but more definitely that it shall be distributed in the *major* premiss,—“Whatever is predicated of a term *distributed*.”¹

(4) The proposition declaring that something is contained under the term distributed must necessarily be an affirmative

¹ This is another form of what will be found to be a special rule of figure 1, namely, that the major premiss must be universal. Compare section 190.

proposition. The *dictum* provides, therefore, that the premisses shall not both be negative¹.

(5) The words "in like manner" clearly provide against a breach of the rule, that if one premiss is negative, the conclusion must be negative, and *vice versa*.

(6) Illicit process of the major is provided against indirectly. This fallacy can be committed only when the conclusion is negative; but the words "in like manner" declare that if there is a negative conclusion, the major premiss must also be negative; and since in any syllogism to which the *dictum* directly applies, the major term is predicate of this premiss, it will be distributed in its premiss as well as in the conclusion. Illicit process of the minor is provided against inasmuch as the *dictum* warrants us in making our predication in the conclusion only of what has been shewn in the minor premiss to be contained under the middle term.

EXERCISES².

158. If *P* is a mark of the presence of *Q*, and *R* of that of *S*, and if *P* and *R* are never found together, am I right in inferring that *Q* and *S* sometimes exist separately? [v.]

The premisses may be stated as follows:—

All P is Q,

All R is S,

No P is R;

and in order to establish the desired conclusion we must be able to infer at least one of the following—*Some Q is not S, Some S is not Q.*

But neither of these propositions can be inferred; for they distribute respectively *S* and *Q*, and neither of these terms is distributed in the given premisses. The question is, therefore, to be answered in the negative.

¹ It really provides that the *minor* premiss shall be affirmative, which again is one of the special rules of figure 1.

² The following exercises may be solved without any knowledge beyond what is contained in the preceding chapter, the assumption however being made that if no rule of the syllogism as given in section 148 or section 150 is broken, then the syllogism is valid.

159. If it be known concerning a syllogism in the Aristotelian system that the middle term is distributed in both premisses, what can we infer as to the conclusion? [c.]

If both premisses are affirmative, they can between them distribute only two terms, and by hypothesis the middle term is distributed twice in the premisses; hence the minor term cannot be distributed in the premisses, and it follows that the conclusion must be particular.

If one of the premisses is negative, there may be three distributed terms in the premisses; these must, however, be the middle term twice (by hypothesis) and the major term (since the conclusion must now be negative and the major term will therefore be distributed in it); hence the minor term cannot be distributed in the premisses, and it again follows that the conclusion must be particular.

But either both premisses will be affirmative, or one affirmative and the other negative; in any case, therefore, we can infer that the conclusion will be particular.

160. Shew *directly* in how many ways it is possible to prove the conclusions *SaP*, *SeP*; point out those that conform immediately to the *Dictum de omni et nullo*; and exhibit the equivalence between these and the remainder. [w.]

(1) To prove *All S is P*.

Both premisses must be affirmative, and both must be universal.

S being distributed in the conclusion must be distributed in the minor premiss, which must therefore be *All S is M*.

M not being distributed in the minor must be distributed in the major, which must therefore be *All M is P*.

SaP can therefore be proved in only one way, namely,

All M is P,

All S is M,

therefore, *All S is P*;

and this syllogism conforms immediately to the *Dictum*.

(2) To prove *No S is P*.

Both premisses must be universal, and one must be negative while the other is affirmative; i.e., one premiss must be *E* and the other *A*.

First, let the major be *E*, i.e.,

either *No M is P* or *No P is M*.

In each case the minor must be affirmative and must distribute *S*; therefore, it will be *All S is M*.

Secondly, let the minor be *E*, i.e.,

either *No M is S* or *No S is M*.

In each case the major must be affirmative and must distribute *P*; therefore, it will be *All P is M*.

We can then prove *SeP* in four ways, thus,—

(i) <i>MeP</i> , <i>Sam</i> , <hr/> <i>SeP</i> .	(ii) <i>PeM</i> , <i>Sam</i> , <hr/> <i>SeP</i> .	(iii) <i>PaM</i> , <i>MeS</i> , <hr/> <i>SeP</i> .	(iv) <i>PaM</i> , <i>SeM</i> , <hr/> <i>SeP</i> .
--------------------------------------------------------	---------------------------------------------------------	----------------------------------------------------------	---------------------------------------------------------

Of these, (i) only conforms immediately to the *Dictum*, and we have to shew the equivalence between it and the others.

The only difference between (i) and (ii) is that the major premiss of the one is the simple converse of the major premiss of the other; they are therefore equivalent. Similarly the only difference between (iii) and (iv) is that the minor premiss of the one is the simple converse of the minor premiss of the other; they are therefore equivalent.

Finally, we may shew that (iii) is equivalent to (i) by transposing the premisses and converting the conclusion.

161. Given that the major term is distributed in the premisses and undistributed in the conclusion of a valid syllogism, determine the syllogism. [C.]

Since the major term is undistributed in the conclusion, the conclusion—and, therefore, both premisses—must be affirmative. Hence, in order to distribute *P*, the major premiss must be *PaM*; and, in order to distribute *M* (which is not distributed in the major premiss), the minor premiss must be *MaS*. It follows that the syllogism must be

All P is M,
All M is S,
therefore, *Some S is P*.

162. Prove that if three propositions involving three terms (each of which occurs in two of the propositions) are together incompatible, then (a) each term is distributed at least once, and (b) one and only one of the propositions is negative.

Shew that these rules are equivalent to the rules of the syllogism. [J.]

No two of the propositions can be formally incompatible with one another, since they do not contain the same terms. But each pair must be incompatible with the third, *i.e.*, the contradictory of any one must be deducible from the other two. It follows that we shall have three valid syllogisms, in which the given propositions taken in pairs are the premisses, whilst the contradictory of the third proposition is in each case the conclusion¹.

Then (a) *each term must be distributed once at least*. For if any one of the terms failed to be distributed at least once, we should obviously have undistributed middle in one of our syllogisms; and (since a term undistributed in a proposition is distributed in its contradictory) illicit major or minor in the two others. If, however, the above condition is fulfilled, it is clear that we cannot have either undistributed middle, or illicit major or minor. Hence rule (a) is equivalent to the syllogistic rules relating to the distribution of terms.

Again, (b) *one of the propositions must be negative, but not more than one of them can be negative*. For if all three were affirmative, then (since the contradictory of an affirmative is negative) we should in each of our syllogisms infer a negative from two affirmatives; and if two were negative, we should have two negative premisses in one of our syllogisms, and (since the contradictory of a negative is affirmative) an affirmative conclusion with a negative premiss in each of the others. If, however, the above condition is fulfilled, it is clear that we cannot have either two negative premisses, or two affirmative premisses with a negative conclusion, or a negative premiss with an affirmative conclusion. Hence rule (b) is equivalent to the syllogistic rules relating to quality.

¹ Every syllogism involves two others, in each of which one of the original premisses combined with the contradictory of the conclusion proves the contradictory of the other original premiss. Hence the three syllogisms referred to in the text mutually involve one another.

163. Explain what is meant by a *syllogism*; and put the following argument into syllogistic form:—"We have no right to treat heat as a substance, for it may be transformed into something which is not heat, and is certainly not a substance at all, namely, mechanical work." [N.]

164. Put the following argument into syllogistic form:—How can anyone maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good? [V.]

165. It has been pointed out by Ohm that reasoning to the following effect occurs in some works on mathematics:—"A magnitude required for the solution of a problem must satisfy a particular equation, and as the magnitude x satisfies this equation, it is therefore the magnitude required." Examine the logical validity of this argument. [C.]

166. Obtain a conclusion from the two negative premisses—*No P is M, No S is M.* [K.]

167. If it is false that the attribute *B* is ever found coexisting with *A*, and not less false that the attribute *C* is sometimes found absent from *A*, can you assert anything about *B* in terms of *C*? [C.]

168. Enumerate the cases in which no valid conclusion can be drawn from two premisses. [K.]

169. Give examples (in symbols—taking *S, M, P*, as minor, middle, and major terms, respectively) in which, attempting to infer a universal conclusion where we have a particular premiss, we commit respectively one but one only of the following fallacies—(a) undistributed middle, (b) illicit major, (c) illicit minor. Give also an example in which, making the same attempt, we commit none of the above fallacies. [K.]

170. Can an apparent syllogism break all the rules of the syllogism at once? [K.]

171. Can you give an instance of an invalid syllogism in which the major premiss is universal negative, the minor premiss affirmative, and the conclusion particular negative? If not, why not? [K.]

172. Shew that

(i) If both premisses of a syllogism are affirmative, and one but only one of them universal, they will between them distribute only one term;

(ii) If both premisses are affirmative and both universal, they will between them distribute two terms ;

(iii) If one but only one premiss is negative, and one but only one premiss universal, they will between them distribute two terms ;

(iv) If one but only one premiss is negative, and both premisses are universal, they will between them distribute three terms. [K.]

173. Ascertain how many distributed terms there may be in the premisses of a syllogism more than in the conclusion. [L.]

174. If the minor premiss of a syllogism be negative, what do you know about the position of the terms in the major? [O's.]

175. Prove that, when the minor term is predicate in its premiss, the conclusion cannot be *A*. [L.]

176. If the major term of a syllogism be the predicate of the major premiss, what do we know about the minor premiss? [L.]

177. How much can you tell about a valid syllogism if you know

(1) that only the middle term is distributed ;

(2) that only the middle and minor terms are distributed ;

(3) that all three terms are distributed? [W.]

178. What can be determined respecting a valid syllogism under each of the following conditions: (1) that only one term is distributed, and that only once; (2) that only one term is distributed, and that twice; (3) that two terms only are distributed, each only once; (4) that two terms only are distributed, each twice? [L.]

179. Two propositions are given having a term in common. If they be *I* and *A*, shew that either no conclusion or two can be deduced; but if *I* and *E*, always and only one. [T.]

✓ 180. Find out, from the rules of the syllogism, what are the valid forms of syllogism in which the major premiss is particular affirmative. [J.]

✓ 181. Given that the major premiss of a valid syllogism is particular negative, determine the syllogism. [K.]

✓ 182. Given that the minor premiss of a valid syllogism is particular negative, determine the syllogism. [K.]

✓ 183. Given that the major premiss of a valid syllogism is affirmative, and that the major term is distributed both in premisses and conclusion, while the minor term is undistributed in both, determine the syllogism. [N.]

184. Shew that if the conclusion of a syllogism be a universal proposition, the middle term can be but once distributed in the premisses. [L.]

185. Shew *directly* in how many ways it is possible to prove the conclusions *SiP*, *SoP*. [W.]

186. Shew that if the rule that a negative conclusion requires a negative premiss be omitted from the general rules of the syllogism, the only invalid syllogism thereby admitted is such that, if its conclusion be false whilst its premisses are true, the three terms of the syllogism must be absolutely coextensive. [O'S.]

187. Find, by direct application of the fundamental rules of syllogism, what are the valid forms of syllogism in which neither of the premisses is a universal proposition having the same quality as the conclusion. [J.]

188. In what cases will contradictory major premisses both yield conclusions when combined with the same minor?

How are the conclusions related?

Shew that in no case will contradictory minor premisses both yield conclusions when combined with the same major. [O'S.]

CHAPTER II.

THE FIGURES AND MOODS OF THE SYLLOGISM.

189. *Figure and Mood.*—By the *figure* of a syllogism is meant the position of the terms in the premisses. Denoting the major, middle, and minor terms by the letters *P*, *M*, *S* respectively, and stating the major premiss first, we have four figures of the syllogism as shewn in the following table:—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
<i>M</i> — <i>P</i>	<i>P</i> — <i>M</i>	<i>M</i> — <i>P</i>	<i>P</i> — <i>M</i>
<i>S</i> — <i>M</i>	<i>S</i> — <i>M</i>	<i>M</i> — <i>S</i>	<i>M</i> — <i>S</i>
<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>
<i>S</i> — <i>P</i>	<i>S</i> — <i>P</i>	<i>S</i> — <i>P</i>	<i>S</i> — <i>P</i>

By the *mood* of a syllogism is meant the quantity and quality of the premisses and conclusion. For example, *AAA* is a mood in which both the premisses and also the conclusion are universal affirmatives; *EIO* is a mood in which the major is a universal negative, the minor a particular affirmative, and the conclusion a particular negative. It is clear that if figure and mood are both given, the syllogism is given.

190. *The Special Rules of the Figures; and the Determination of the Legitimate Moods in each Figure*¹.—It may first of all be shewn that certain combinations of premisses are incapable of yielding a valid conclusion in any figure. *A priori*, there are possible the following sixteen different combinations of premisses, the major premiss being always stated first:—*AA, AI, AE, AO, IA, II, IE, IO, EA, EI, EE, EO, OA, OI,*

¹ The method of determination here adopted is only one amongst several possible methods. Another is suggested, for example, in sections 160, 185.

OE, OO. Referring back, however, to the syllogistic rules and corollaries (as given in sections 148, 149), we find that *EE, EO, OE, OO* (being combinations of negative premisses) yield no conclusion by rule 5; that *II, IO, OI* (being combinations of particular premisses) are excluded by corollary i.; and that *IE* is excluded by corollary iii., which tells us that nothing follows from a particular major and a negative minor.

We are left then with the following eight possible combinations:—*AA, AI, AE, AO, IA, EA, EI, OA*; and we may go on to enquire in which figures these will yield conclusions. In pursuing this enquiry, special rules of the various figures may be determined, which, taken together with the three corollaries established in section 149, replace the general rules of distribution. These special rules, supplemented by the general rules of quality and the corollaries¹, will enable the validity of the different moods to be tested by a mere inspection of the form of the propositions of which they consist.

The special rules² and the legitimate moods of Figure 1.

The position of the terms in figure 1 is shewn thus—

$$\begin{array}{c} M - P \\ S - M \\ \hline S - P \end{array}$$

and it can be deduced from the general rules of the syllogism that in this figure:—

(1) *The minor premiss must be affirmative.* For if it were negative, the major premiss would have to be affirmative by rule 5, and the conclusion negative by rule 6. The major term would therefore be distributed in the conclusion, and undistributed in its premiss; and the syllogism would be invalid by rule 4.

(2) *The major premiss must be universal.* For the middle

¹ The general rules of quality and the corollaries, unlike the general rules of distribution, can be directly applied without reference to the position of the terms in the premisses of a syllogism.

² As indicated in section 157, the special rules of figure 1 follow immediately from the *dietum de omni et nullo*.

term, being undistributed in the affirmative minor premiss, must be distributed in the major premiss.

Rule (1) shews that *AE* and *AO*, and rule (2) that *IA* and *OA*, yield no conclusions in this figure. We are accordingly left with only four combinations, namely, *AA*, *AI*, *EA*, *EI*. From the rules that a particular premiss cannot yield a universal conclusion or a negative premiss an affirmative conclusion, while conversely a negative conclusion requires a negative premiss, it follows further that *AA* will justify either of the conclusions *A* or *I*, *EA* either *E* or *O*, *AI* only *I*, *EI* only *O*. There are then six moods in figure 1 which do not offend against any of the rules of the syllogism¹, namely, *AAA*, *AAI*, *AII*, *EAE*, *EAO*, *EIO*.

The actual validity of these moods may be established by shewing that the axiom of the syllogism, the *dictum de omni et nullo*, applies to them; or by taking them severally and shewing that in each case the cogency of the reasoning is self-evident.

The special rules and the legitimate moods of Figure 2.

The position of the terms in figure 2 is shewn thus—

¹ Rule (2) provides against undistributed middle, and rule (1) against illicit major. We cannot have illicit minor, unless we have a universal conclusion with a particular premiss, and this also has been provided against.

Mr Johnson points out that the following symmetrical rules may be laid down for the correct distribution of terms in the different figures; and that these rules (three in each figure) taken together with the *rules of quality* are sufficient to ensure that *no* syllogistic rule is broken.

(i) To avoid undistributed middle: in figure 1, If the minor is affirmative, the major must be universal; in figure 4, If the major is affirmative, the minor must be universal; in figure 2, One premiss must be negative; in figure 3, One premiss must be universal. (The last of these rules is of course superfluous if the corollaries contained in section 149 are supposed given.)

(ii) To avoid illicit major: in figures 1 and 3, If the conclusion is negative, the major must be negative and, therefore, the minor affirmative; in figures 2 and 4, If the conclusion is negative, the major must be universal.

(iii) To avoid illicit minor: in figures 1 and 2, If the minor is particular, the conclusion must be particular; in figures 3 and 4, If the minor is affirmative, the conclusion must be particular. (The first of these two rules is again superfluous as a special rule if the corollaries are supposed given.)

The above rules are substantially identical with those given in the text.

$$P - M$$

$$S - M$$

$$S - P$$

and its special rules (which the reader is recommended to deduce from the general rules of the syllogism for himself) are—

(1) *One premiss must be negative;*

(2) *The major premiss must be universal.*

The application of these rules again leaves six moods, namely, *AEE, AEO, AOO, EAE, EAO, EIO*.

Recourse cannot now be had directly to the *dictum de omni et nullo* in order to shew positively that these moods are legitimate. It may, however, be shewn in each case that the cogency of the reasoning is self-evident. The older logicians did not adopt this course; their method was to shew that, by the aid of immediate inferences, each mood could be reduced to such a form that the *dictum* did apply directly to it. The doctrine of *reduction* resulting from the adoption of this method will be discussed in the following chapter.

The special rules and the legitimate moods of Figure 3.

The position of the terms in this figure is shewn thus—

$$M - P$$

$$M - S$$

$$S - P$$

and its special rules are—

(1) *The minor must be affirmative;*

(2) *The conclusion must be particular.*

Proceeding as before, we are left with six valid moods, namely, *AAI, AII, EAO, EIO, IAI, OAO*.

The special rules and the legitimate moods of Figure 4.

The position of the terms in this figure is shewn thus—

$$P - M$$

$$M - S$$

$$S - P$$

and the following may be given as its special rules—

- (1) *If the major is affirmative, the minor must be universal;*
- (2) *If either premiss is negative, the major must be universal;*
- (3) *If the minor is affirmative, the conclusion must be particular¹.*

The result of the application of these rules is again six valid moods, namely, *AAI, AEE, AEO, EAO, EIO, IAI*.

Our final conclusion then is that there are 24 valid moods, namely, six in each figure.

In Figure 1, *AAA, AAI, EAE, EAO, AII, EIO*.

In Figure 2, *EAE, EAO, AEE, AEO, EIO, AOO*.

In Figure 3, *AAI, IAI, AII, EAO, OAO, EIO*.

In Figure 4, *AAI, AEE, AEO, EAO, IAI, EIO*.

191. *Weakened Conclusions and Subaltern Moods.*—When from premisses that would have justified a universal conclusion we content ourselves with inferring a particular (as, for example, in the syllogism *All M is P, All S is M, therefore, Some S is P*), we are said to have a weakened conclusion, and the syllogism is said to be a weakened syllogism or to be in a subaltern mood (because the conclusion might be obtained by subaltern inference² from the conclusion of the corresponding unweakened mood).

In the preceding section it has been shewn that in each figure there are six moods which do not offend against any of the syllogistic rules; so that in all there are 24 distinct valid moods. Five of these, however, have weakened conclusions; and, since we are not likely to be satisfied with a particular conclusion when the corresponding universal can be obtained from the same premisses, these moods are of no practical importance. Accordingly when the moods of the various figures are enumerated (as in the mnemonic verses)

¹ The special rules of the fourth figure are variously stated. They are given in the above form in the *Port Royal Logic*, pp. 202, 203. See, also, section 200.

² In treating the syllogism on the traditional lines it is assumed that *S, M, P* all represent existing classes. Subaltern inference is, therefore, a valid process.

they are usually omitted. Still, their recognition gives a completeness to the theory of the syllogism, which it cannot otherwise possess. There is also a symmetry in the result of their recognition as yielding exactly six legitimate moods in each figure¹.

The subaltern moods are—

In Figure 1, *AAI*, *EA O*;

In Figure 2, *EA O*, *AEO*;

In Figure 4, *AEO*.

It is obvious that there can be no weakened conclusion in Figure 3, since in no case is it possible to infer more than a particular conclusion in this figure.

AAI in Figure 4 is sometimes spoken of as a subaltern mood. But this is a mistake. With the premisses *All P is M*, *All M is S*, the conclusion *Some S is P* is certainly in one sense weaker than the premisses would warrant since the universal conclusion *All P is S* might have been inferred. But *All P is S* is not the universal corresponding to *Some S is P*. The subjects of these two propositions are different; and we infer all that we possibly can about *S* when we say that *some S is P*. In other words, regarded as a mood of figure 4, this mood is not a subaltern. *AAI* in figure 4 is thus differentiated from *AAI* in figure 1, and its recognition in the mnemonic verses justified.

192. *Strengthened Syllogisms.*—If in a syllogism the same conclusion can still be obtained although for one of the premisses we substitute its subaltern, the syllogism is said to be a strengthened syllogism. A strengthened syllogism is thus a syllogism with an unnecessarily strengthened premiss².

For example, the conclusion of the syllogism—

All M is P,
All M is S,
therefore, *Some S is P,*

¹ It has been remarked that 19 being a prime number at once suggests incompleteness or artificiality in the common enumeration.

² Compare De Morgan, *Formal Logic*, pp. 91, 130. De Morgan calls a syllogism *fundamental*, when neither of its premisses is stronger than is necessary to produce the conclusion (*Formal Logic*, p. 77)

could equally be obtained from the premisses *All M is P, Some M is S*; or from the premisses *Some M is P, All M is S*.

By trial we may find that every syllogism in which there are two universal premisses with a particular conclusion is a strengthened syllogism, with the single exception of AEO in the fourth figure¹.

In a full enumeration there are two strengthened syllogisms in each figure:—

In Figure 1, *AAI, EAO*;

In Figure 2, *AEO, EAO*;

In Figure 3, *AAI, EAO*;

In Figure 4, *AAI, EAO*.

The distinction between a strengthened syllogism (that is, a syllogism with a strengthened premiss) and a weakened syllogism (that is, a syllogism with a weakened conclusion) should be carefully noted.

It will be observed that in figures 1 and 2, a syllogism having a strengthened premiss may also be regarded as a syllogism having a weakened conclusion, and *vice versa*; but that in figures 3 and 4, the contrary holds in both cases. The only syllogism with a weakened conclusion in either of these figures is *AEO* in figure 4; but it will be found that in this mood no conclusion is obtainable if either of the premisses is replaced by its subaltern.

193. *The peculiarities and uses of each of the four figures of the syllogism.*—*Figure 1.* In this figure it is possible to prove conclusions of all the forms *A, E, I, O*; and it is the only figure in which a universal affirmative conclusion can be proved. This alone makes it by far the most useful and important of the syllogistic figures. All deductive science, the object of which is to establish universal affirmatives, tends to work in *AAA* in this figure.

Another point to notice is that only in this figure is it the case that both the subject of the conclusion is subject in the premisses, and the predicate of the conclusion predicate in the premisses; in figure 2 the predicate of the conclusion is

¹ A general proof of this proposition will be given in section 299.

subject in the major premiss; in figure 3 the subject of the conclusion is predicate in the minor premiss; and in figure 4 there is a double inversion¹. This no doubt partly accounts for the fact that reasoning in figure 1 so often seems more natural than the same reasoning expressed in any other figure².

Figure 2. In this figure, only negatives can be proved; and therefore it is chiefly used for purposes of disproof. For example, *Every real natural poem is naïve; those poems of Ossian which Macpherson pretended to discover are not naïve (but sentimental); hence they are not real natural poems* (Ueberweg, *System of Logic*, § 113). It has been called the exclusive figure; because by means of it we may go on excluding various suppositions as to the nature of something under investigation, whose real character we wish to ascertain (a process called *abscissio infiniti*). For example, *Such and such an order has such and such properties, This plant has not those properties; therefore, It does not belong to that order*. A syllogism of this kind may be repeated with a number of different orders till the enquiry is so narrowed down that the place of the plant is easily determined. Whately (*Elements of Logic*, p. 92) gives an example from the diagnosis of a disease.

Figure 3. In this figure, only particulars can be proved. It is frequently useful when we wish to take objection to a universal proposition laid down by an opponent by establishing an instance in which such universal proposition does not hold good.

It is the natural figure when the middle term is a singular term, especially if the other terms are general. It has been already shewn that if one and only one term of an affirmative proposition is singular, that term is almost necessarily the subject. For example, such a reasoning as *Socrates is wise, Socrates is a philosopher, therefore, Some philosophers are wise*, can only with great awkwardness be expressed in any figure other than figure 3.

¹ The double inversion in figure 4 is one of the reasons given by Thomson for rejecting that figure altogether. Compare section 209.

² Compare Solly, *Syllabus of Logic*, pp. 130 to 132.

Figure 4. This figure is seldom used, and some logicians have altogether refused to recognise it. We shall return to a discussion of it subsequently. See section 209.

Lambert in his *Neues Organon* expresses the uses of the different syllogistic figures as follows: "The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinctions between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

De Morgan (*Syllabus*, p. 30) thus characterizes the different figures: "The first figure may be called the figure of *direct transition*; the fourth, which is nothing but the first with a *converted conclusion*¹, the figure of *inverted transition*; the second, the figure of *reference to* (the middle term); the third, the figure of *reference from* (the middle term)."

EXERCISES.

194. Why is *IE* an inadmissible, while *EI* is an admissible, mood in every figure of the syllogism? [L.]

195. Which of the following conjunctions of propositions make valid syllogisms? In the case of those which you regard as invalid, give your reasons for so treating them.

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.	
<i>AAE</i>	<i>AAA</i>	<i>AOE</i>	<i>AII</i>	
<i>AOO</i>	<i>AOE</i>	<i>AEO</i>	<i>AOO</i>	[K.]

196. What moods are good in the first figure and faulty in the second, and *vice versâ*? Why are they excluded in one figure and not in the other? [O.]

197. Shew that *O* cannot stand as premiss in figure 1, as major in figure 2, as minor in figure 3, as premiss in figure 4. [C.]

198. Shew that it is impossible to have the conclusion in *A* in any figure but the first. What fallacies would be committed if there were such a conclusion to a reasoning in any other figure? [C.]

¹ Compare, however, section 209.

199. Find in what figures the following moods respectively are valid:—*AAI*, *AII*, *IAI*. [K.]

200. Shew that the following *two* rules would suffice as the special rules for the fourth figure: (i) The conclusion and major cannot have the same form unless it be particular affirmative; (ii) The conclusion and minor cannot have the same form unless it be universal negative¹. [J.]

¹ It must be remembered that the special rules of the figures are intended to provide against breaches of the general rules of distribution only, and that for this purpose they are to be taken in conjunction with the three corollaries established in section 149. The general rules of quality and the corollaries may, therefore, be assumed in working out this problem.

CHAPTER III.

THE REDUCTION OF SYLLOGISMS.

201. *The Problem of Reduction.*—By reduction is meant a process whereby the reasoning contained in a given syllogism is expressed in some other mood or figure. Unless an explicit statement is made to the contrary, reduction is supposed to be to figure 1.

The following syllogism in figure 3 may be taken as an example:

All M is P,
Some M is S,
therefore, *Some S is P.*

It will be seen that by simply converting the minor premiss, we have precisely the same reasoning in figure 1.

This is an example of direct or ostensive reduction.

202. *Indirect Reduction.*—A proposition is established *indirectly* when its contradictory is proved false; and this is effected if it can be shewn that an ultimate consequence of the truth of its contradictory is self-contradiction.

The method of indirect proof is in several cases adopted by Euclid; and it is sometimes employed in the reduction of syllogisms from one mood to another. Thus, AOO in figure 2 is usually reduced in this manner. The argument may be stated as follows:—

From the premisses—*All P is M,*
Some S is not M,
it follows that *Some S is not P;*

for if this conclusion is not true, then, by the law of excluded middle, its contradictory (namely, *All S is P*) must be so, and

the premisses being given true we shall have true together the three propositions—*All P is M*,

Some S is not M,

All S is P.

But combining the first and the third of these we have a syllogism in figure 1, namely,

All P is M,

All S is P,

yielding the conclusion *All S is M*.

Some S is not M and *All S is M* are, therefore, true together; but, by the law of contradiction, this is absurd, since they are contradictories.

Hence it has been shewn that the consequence of supposing *Some S is not P* false is a self-contradiction; and we may accordingly infer that it is true.

It will be observed that the only syllogism made use of in the above argument is in figure 1; and the process is, therefore, regarded as a reduction of the reasoning to figure 1.

This method of reduction is called *Reductio ad impossibile*, or *Reductio per impossibile*¹, or *Deductio ad impossibile*, or *Deductio ad absurdum*. It is the only way of reducing *AOO* in figure 2 or *OAO* in figure 3 to figure 1, unless negative terms are used (as in obversion and contraposition); and it was adopted by the old writers in consequence of their objection to negative terms².

The process of indirect reduction may be connected with the contraposition of hypotheticals. Let *P* and *Q* be the given premisses, and *R* the conclusion which has to be proved; and let *P'* denote the contradictory of *P*, and so on. The argument may then be stated as follows:—*If P and R then Q'* (by a

¹ Compare Mansel's *Aldrich*, pp. 88, 89.

² The reasoning involved in indirect reduction is somewhat complex, and since the two moods to which it is generally applied can also be reduced directly (as will be shewn in section 204), some modern logicians are inclined to banish it entirely from their treatment of the syllogism. It will, however, be found that the direct reduction of the moods in question is also complex; and, at any rate as applied to these particular moods, the indirect process seems as natural as the direct. Compare Ueberweg, *Logic*, § 113.

syllogism in figure 1); therefore, by contraposition, *If P and Q then R'*, and this proves what is required¹.

It should be noticed further that the argument involved in indirect reduction is based upon the mutual equivalence of the three following syllogisms²:

- (i) premisses, *P and Q*; conclusion, *R'*;
- (ii) premisses, *Q and R*; conclusion, *P'*;
- (iii) premisses, *R and P*; conclusion, *Q'*.

203. *The mnemonic lines Barbara, Celarent, &c.*—The mnemonic hexameter verses (which are spoken of by De Morgan as “the magic words by which the different moods have been denoted for many centuries, words which I take to be more full of meaning than any that ever were made”) are usually given as follows:

¹ The above may be called a process of *partial* contraposition, since *P* retains its position in the antecedent throughout. The validity of the inference may perhaps be made more apparent by writing it as follows:—*P being given, if R then Q'*, therefore, *P being given, if Q then R'*.

² In order that these syllogisms may be valid, *P, Q, R* must be a trio of propositions, which are formally incompatible with one another. Compare section 162; also De Morgan's proof of the second corollary on p. 244, and certain of the proofs contained in section 151. The order of the premisses in the above syllogisms is of course not intended to indicate which is major and which minor.

It will be found that if we take the propositions and syllogisms always in the cyclic order indicated in the text, and if we assume that in the initial syllogism the minor premiss precedes the major in the first figure, while the major precedes the minor in the three remaining figures, then (a) starting with a syllogism in one of the first three figures, the figures 1, 2, 3 will always recur in the cyclic order 1, 2, 3; and in figure 1 the minor premiss will always precede the major, and in figures 2 and 3 the major will always precede the minor, as in the initial syllogism; (b) starting with a syllogism in figure 4, the figure and the order of the premisses will always remain the same.

In interchanging between one syllogism and another in the process of indirect reduction there is always one premiss common to the two syllogisms, and the above results may also be expressed as follows: in interchanging between figures 1 and 2, the common premiss is major in each; in interchanging between figures 1 and 3, the common premiss is minor in each; in interchanging between figures 2 and 3, the common premiss is minor in figure 2 and major in figure 3; if one of the syllogisms is in figure 4, the other will also be in figure 4, the common premiss being first major and then minor, or *vice versa*, as the case may be.

Barbăru, Cēlārent, Dārvi, Fērīoque prioris :
Cēsārē, Cāmēstres, Festinō, Bārōcō, secundae :
 Tertia, *Dāraptī, Disāmīs, Dātīsī, Fēlapton,*
Bōcardō, Fērīsōn, habet: Quarta insuper addit
Brāmāntip, Cāmēnes, Dīmāris, Fēsāpō, Frēsison.

Each valid mood in every figure, unless it be a subaltern mood, is here represented by a separate word; and in the case of a mood in any of the so-called imperfect figures (i.e., figures 2, 3, 4), the mnemonic gives full information for its reduction to figure 1, the so-called perfect figure.

The only meaningless letters are *b* (not initial), *d* (not initial), *l*, *n*, *r*, *t*; the signification of the remainder is as follows:—

The *vowels* give the quality and quantity of the propositions of which the syllogism is composed ; and, therefore, really give the syllogism itself, if the figure is also known. Thus, *Camenes* in figure 4 represents the syllogism—

All P is M,

No M is S.

therefore, *No S is P.*

The *initial letters* in the case of figures 2, 3, 4 shew to which of the moods of figure 1 the given mood is to be reduced, namely, to that which has the same initial letter. The letters *B, C, D, F* were chosen for the moods of figure 1 as being the first four consonants in the alphabet.

Thus, *Camestres* is reduced to *Celarent*,—

All P is M,

No M is S.

No S is M,

All P is M.

therefore,

No S is P.

therefore, *No P is S.*

therefore, *No S is P*¹.

¹ The *order* of inference in this and in other reductions might be made clear by the use of arrows, representing inference, as follows :

$$\begin{array}{ccc}
 \text{All } P \text{ is } M, & \begin{array}{c} \nearrow \\ \searrow \end{array} & \text{No } M \text{ is } S, \\
 \text{No } S \text{ is } M, & & \text{All } P \text{ is } M, \\
 & & \downarrow \\
 \text{No } S \text{ is } P. & \leftarrow & \text{No } P \text{ is } S,
 \end{array}$$

s (in the middle of a word) indicates that in the process of reduction the preceding proposition is to be simply converted. Thus, in reducing *Camestres* to *Celarent*, as shewn above, the minor premiss is simply converted.

s (at the end of a word¹) shews that the conclusion of the *new* syllogism has to be simply converted in order to obtain the given conclusion. This again is illustrated in the reduction of *Camestres*. The final *s* does not affect the conclusion of *Camestres* itself, but the conclusion of *Celarent* to which it is reduced.

p (in the middle of a word) signifies that the preceding proposition is to be converted *per accidens*; as, for example, in the reduction of *Darapti* to *Darii*—

<i>All M is P,</i>	<i>All M is P,</i>
<i>All M is S,</i>	<i>Some S is M,</i>

therefore, *Some S is P.* therefore, *Some S is P.*

p (at the end of a word¹) implies that the conclusion obtained by reduction is to be converted *per accidens*. Thus, in *Bramantip*, the *p* does not relate to the **I** conclusion of the mood itself²; it really relates to the **A** conclusion of the syllogism in *Barbara* which is given by reduction. Thus,—

<i>All P is M,</i>		<i>All M is S,</i>
<i>All M is S,</i>		<i>All P is M,</i>

therefore, *Some S is P.* therefore, *All P is S,*
therefore, *Some S is P.*

m indicates that in reduction the premisses have to be transposed (*metathesis præmissarum*); as just shewn in the case of *Bramantip*, and also in the case of *Camestres*.

c signifies that the mood is to be reduced *indirectly* (i.e., by *reductio per impossibile* in the manner shewn in the preceding section); and the position of the letter indicates that in this process of indirect reduction the first step is to omit the

¹ This peculiarity in the signification of *s* and *p* when they are *final* letters is frequently overlooked.

² Compare however Hamilton, *Logic*, i. p. 264, and Spalding, *Logic*, pp. 230, 1.

premiss preceding it, *i.e.*, the other premiss is to be combined with the contradictory of the conclusion (*conversio syllogismi*, or *ductio per contradictoriam propositionem sive per impossibile*). The letter *c* is by some writers replaced by *k*, *Baroko* and *Bokardo* being given as the mnemonics, instead of *Baroco* and *Bocardo*.

The following lines are sometimes added to the verses given above, in order to meet the case of the subaltern moods :—

Quinque Subalterni, totidem Generalibus orti,
Nomen habent nullum, nec, si bene colligis, usum¹.

¹ The mnemonics have been written in various forms. Those given above are from Aldrich, and they are the ones that are in general use in England. Wallis in his *Institutio Logicæ* (1687) gives for the fourth figure, *Balani*, *Cadere*, *Digami*, *Fegano*, *Fedibo*. P. van Musschenbroek in his *Institutiones Logicæ* (1748) gives *Barbari*, *Calentes*, *Dibatis*, *Fespamo*, *Fresisom*. This variety of forms for the moods of figure 4 is no doubt due to the fact that the recognition of this figure at all was quite exceptional until comparatively recently. Compare sections 209, 210.

According to Ueberweg (*Logic*, § 118) the mnemonics run—

Barbara, *Celarent* primæ, *Darii* *Ferioque*.
Cesare, *Camestres*, *Festino*, *Baroco* secundæ.
Tertia grande sonans recitat *Darapti*, *Felapton*,
Disamis, *Datisti*, *Bocardo*, *Ferison*. *Quartæ*
Sunt *Bamalip*, *Calemes*, *Dimatis*, *Fesapo*, *Fresison*.

Ueberweg gives *Camestros* and *Calemos* for the weakened moods of *Camestres* and *Calemes*. This is not, however, quite accurate. The mnemonics should be *Camestrop* and *Calemop*.

Mr Carveth Read (*Mind*, 1882, p. 440) suggests an ingenious modification of the verses, so as to make each mnemonic immediately suggest the figure to which the corresponding mood belongs, at the same time abolishing all the unmeaning letters. He takes *l* as the sign of the first figure, *n* of the second, *r* of the third, and *t* of the fourth. The lines then run

Ballala, *Celallel*, *Dalii*, *Felioque* prioris.
Cesane, *Camesnes*, *Fesinon*, *Banoco* secundæ.
Tertia *Darapri*, *Drisamis*, *Darisi*, *Ferapro*,
Bocaro, *Ferisor* habet. *Quarta* insuper addit
Bamatip, *Cametes*, *Dimatis*, *Fesapto*, *Fesistot*.

On the same principle, Mrs Ladd Franklin (*Studies in Logic*, Johns Hopkins University, p. 40) suggests that the different figures might be indicated by the letters *r*, *t*, *l*, and *n* respectively. Thus,—

Barbara, *Cegare*, *Darii*, *Ferioque* prioris.
Cesate, *Camestes*, *Festivo*, *Batoko* secundæ.
Tertia *Dalipi*, *Disalmis*, *Dalisi*, *Felapo*,
Bokalo, *Feliso* habet. *Quarta* insuper addit
Bamanip, *Camenes*, *Dimanis*, *Fesampo*, *Fesison*.

204. *The direct reduction of Baroco and Bocardo.*—These moods may be reduced directly to the first figure by the aid of obversion and contraposition as follows¹.

Baroco :—

All P is M,

Some S is not M,

therefore, *Some S is not P,*

is reducible to *Ferio* by the contraposition of the major premiss and the obversion of the minor, thus,—

No not-M is P,

Some S is not-M,

therefore, *Some S is not P.*

Faksoko has been suggested as a mnemonic for this method of reduction, *k* denoting obversion, so that *ks* denotes obversion followed by conversion (i.e., contraposition).

Whately's mnemonic *Fakoro* (*Elements of Logic*, p. 97) does not indicate the obversion of the minor premiss (*r* being with him an unmeaning letter).

Bocardo :—

Some M is not P,

All M is S,

therefore, *Some S is not P,*

¹ *Baroco* and *Bocardo* are also reduced by the process of *ἐκθεσις* to other moods of figures 2 and 3, and thence to figure 1. Ueberweg writes, "*Baroco* may also be referred to *Camestres* when those (some) *S* of which the minor premiss is true are placed under a special notion and denoted by *S'*. Then the conclusion must hold good universally of *S'*, and consequently particularly of *S*. Aristotle calls such a procedure *ἐκθεσις*" (*Logic*, § 113). As regards *Bocardo*, "Aristotle remarks that this mood may be proved without apagogical procedure (*reductio ad impossibile*) by the *ἐκθέσθαι* or *λαμβάνειν* of that part of the middle notion which is true of the major premiss. If we denote this part by *N*, then we get the premisses: *NeP*; *NaS*: from which follows (in *Felapton*) *SoP*; which was to be proved" (§ 115). The procedure is, however, rather more complicated than appears in the above statements. In the case of *Baroco* (*PaM*, *SoM*, ∴ *SoP*), let the *S*'s which are not *M* (of which by hypothesis there are some) be denoted by *X*; then we have *PaM*, *XeM*, ∴ *XeP* (*Camestres*); but *XaS*, and hence we have further *XeP*, *XaS*, ∴ *SoP* (*Felapton*). In the case of *Bocardo* (*MoP*, *MaS*, ∴ *SoP*), let the *M*'s which are not *P* (of which by hypothesis there are some) be denoted by *N*; then we have *MaS*, *NaM*, ∴ *NaS* (*Barbara*); and hence *NeP*, *NaS*, ∴ *SoP* (*Felapton*). The argument in both cases suggests questions connected with the existential import of propositions; but the consideration of such questions must for the present be deferred.

is reducible to *Darii* by the contraposition of the major premiss and the transposition of the premisses, thus,—

All M is S,
Some not-P is M,
 therefore, *Some not-P is S.*

Some not-P is S is not indeed our original conclusion, but the latter can be obtained from it by conversion followed by obversion. This method of reduction may be indicated by *Doksamosk* (which again is obviously preferable to *Dokamo*, suggested by Whately, since the latter would make it appear as if we immediately obtained the original conclusion in *Darii*¹).

205. *Indirect reduction of moods usually reduced ostensibly.*—Just as *Bocardo* and *Baroco* which are usually reduced indirectly may be reduced directly, so other moods which are usually reduced directly may be reduced indirectly. *Bramantip* may be taken as an example:—

All P is M,
All M is S,
 therefore, *Some S is P;*

for, if not, then *No S is P*; and combining this with the given minor premiss we have a syllogism in *Celarent*, namely,

No S is P,
All M is S,
 therefore, *No M is P,*

which yields by conversion *No P is M*. But this is the contrary² of the original major premiss *All P is M*, and it is

¹ Mr Carveth Read (*Mind*, 1882, p. 441) uses the letters *k* and *s* as above; but his mnemonics are required also to indicate the figure to which the moods belong (see the note on p. 279); and he, therefore, arrives at *Faksnoko* and *Doksamrosk*. Spalding (*Logic*, p. 235) suggests *Facoco* and *Docamoc*; but the processes here indicated by the letter *c* are not in all cases the same, and these mnemonics are, therefore, unsatisfactory.

² The reason why we here arrive ultimately at *contrariety* instead of *contradiction* is that *Bramantip* has a strengthened premiss. It will be found that we have a similar case whenever there is in the original syllogism either a strengthened premiss or a weakened conclusion. It will also be found that when the original syllogism has a weakened conclusion, it will suffice to start

impossible that they should be true together. Hence we infer the truth of the original conclusion.

206. *Extension of the doctrine of Reduction.*—The doctrine of reduction may be extended, and it can be shewn not merely that any syllogism may be reduced to figure 1, but also that it may be reduced to any given mood (not being a subaltern mood) of that figure¹. This position will obviously be established if we can shew that *Barbara*, *Celarent*, *Darii*, and *Ferio* are mutually reducible to one another.

Barbara may be reduced to *Celarent* by the obversion of the major premiss and also of the new conclusion thereby obtained. Thus, using arrows, as in the note on p. 277,

$$All\ M\ is\ P, \rightarrow No\ M\ is\ not-P,$$

$$All\ S\ is\ M, \rightarrow All\ S\ is\ M,$$

$$All\ S\ is\ P. \leftarrow No\ S\ is\ not-P,$$

Conversely, *Celarent* is reducible to *Barbara*; and in a similar manner by obversion of major premiss and conclusion *Darii* and *Ferio* are reducible to each other.

It will now suffice if we can shew that *Barbara* and *Darii* are mutually reducible to each other. Clearly the only method possible here is the indirect method.

Take *Barbara*, $\begin{array}{l} MaP, \\ SaM, \\ \hline \therefore SaP; \end{array}$

for, if not, then we have *SoP*; and *MaP*, *SaM*, *SoP* must be true together. From *SoP* by first obverting and then converting (and denoting not-*P* by *P'*) we get *P'iS*, and combining this with *SaM* we have the following syllogism in *Darii*—

with the *subcontrary* (instead of the *contradictory*) of that conclusion. **EA0** in figure 2 may, for example, be indirectly reduced to figure 1 as follows: *PeM*, *SaM*, $\therefore SoP$; for if not, then *SiP* (the subcontrary of *SoP*) must be true, and hence we shall have *PeM*, *SiP*, $\therefore SoM$ (in *Ferio*); but the conclusion of this syllogism contradicts our original minor premiss. We cannot, however, start from the subcontrary of the conclusion if the original syllogism contains a strengthened premiss without a weakened conclusion.

¹ Compare, further, sections 219, 220.

$$\begin{array}{c} SaM, \\ \underline{PiS}, \\ \therefore PiM. \end{array}$$

PiM by conversion and obversion becomes MoP ; and therefore MaP and MoP are true together; but this is impossible, since they are contradictories. Therefore, SoP cannot be true, i.e., the truth of SaP is established.

Similarly, *Darii* may be indirectly reduced to *Barbara*¹.

$$\begin{array}{c} MaP, \quad (i) \\ \underline{SiM}, \quad (ii) \\ \therefore SiP. \quad (iii) \end{array}$$

The contradictory of (iii) is SeP , from which we obtain PaS' . Combining with (i), we have—

$$\begin{array}{c} PaS', \\ \underline{MaP}, \\ \therefore MaS' \text{ in } Barbara. \end{array}$$

But from this conclusion we may obtain SeM , which is the contradictory of (ii).

207. *Dicta for Figures 2, 3, and 4, corresponding to the Dictum for Figure 1.*—The following *dictum*, called the *dictum de diverso*, has been given for Figure 2:—"If one term is contained in, and another excluded from, a third term, they are mutually excluded." This is at least expressed loosely since it would appear to warrant a universal conclusion in *Festino* and *Baroco*. Mansel (*Aldrich*, p. 86) puts this *dictum* in a more satisfactory form:—"If a certain attribute can be predicated, affirmatively or negatively, of every member of a class, any subject of which it cannot be so predicated, does not belong to the class." This proposition may claim to be axiomatic, and it can be applied directly to any syllogism in figure 2².

¹ It has been maintained, that this reduction is unnecessary, and that, to all intents and purposes, *Darii* is *Barbara*, since the "some *S*" in the minor is, and is known to be, the same *some* as in the conclusion.

² Bailey (*Theory of Reasoning*, p. 71) gives the following pair of maxims for figure 2:—"When the whole of a class possess a certain attribute, whatever does not possess the attribute does not belong to the class. When the whole of

The following *dictum*, called the *dictum de exemplo*¹, has been given for Figure 3:—"Two terms which contain a common part partly agree, or if one contains a part which the other does not, they partly differ." This formula also is open to exception. The proposition "If one term contains a part which another does not, they partly differ" applied to *No M is P, All M is S*, would appear to justify *Some P is not S* just as much as *Some S is not P*. Mansel's amendment here is to give two principles for figure 3, the *dictum de exemplo*—"If a certain attribute can be affirmed of any portion of the members of a class, it is not incompatible with the distinctive attributes of that class"; and the *dictum de excepto*—"If a certain attribute can be denied of any portion of the members of a class, it is not inseparable from the distinctive attributes of that class." The force of these axioms, however, is not very clear²; and the following *dicta* which may be regarded as axiomatic, and which will be found to apply respectively to the affirmative and negative moods of figure 3, seem preferable: "If two attributes can both be affirmed of a class, and one at least of them universally so, then these two attributes sometimes accompany each other"; "If one attribute can be affirmed while another is denied of a class, either the affirmation or the denial being

a class is excluded from the possession of an attribute, whatever possesses the attribute does not belong to the class."

¹ The *dictum de diverso* and the *dictum de exemplo* are usually attributed to Lambert. Thomson, however, remarks that Mill and others are in error "in thinking that Lambert invented these *dicta*. More than a century earlier, Keckermann saw that each figure had its own law and its peculiar use, and stated them as accurately, if less concisely, than Lambert" (*Laws of Thought*, p. 173, note). Distinct principles for the second and third figures are laid down also in the *Port Royal Logic*, which was published in 1662.

² In the earlier editions of the present work Mansel's *dictum de exemplo* was interpreted as follows: If a certain attribute (*P*) can be affirmed of any portion of the members of a class (*M*) it is not incompatible with the distinctive attributes (*S*) of that class (*M*); and it was asked in criticism whether it was essential that in the minor premiss we should be predicating the *distinctive attributes* of the class. It is, however, clear that the following is the correct interpretation of the *dictum*: If a certain attribute (*P*) can be affirmed of any portion (*M*) of the members of a class (*S*) it is not incompatible with the distinctive attributes of that class (*S*); and the criticism of course falls to the ground. Similarly in the case of the *dictum de excepto*.

universal, then the former attribute is not always accompanied by the latter¹."

The following *dictum*, called the *dictum de reciproco*, was given by Lambert for Figure 4:—"If no *M* is *B*, no *B* is this or that *M*; if *C* is or is not this or that *B*, there are *B*'s which are or are not *C*." The first part of this *dictum* applies to *Camenes*, and the second part to the remaining moods of the fourth figure; but its application can hardly be regarded as self-evident. Several other axioms have been constructed for figure 4; but they are, as a rule, little more than a bare enumeration of the valid moods of that figure, whilst at the same time they are less self-evident than these moods considered individually. The following axiom, however, suggested by Mr Johnson, is not open to these criticisms: "Three classes cannot be so related, that the first is wholly included in the second, the second wholly excluded from the third, and the third partly or wholly included in the first." This *dictum* declares the mutual incompatibility of each of the following trios of propositions: *XaY*, *YeZ*, *ZiX*; *XaY*, *YeZ*, *ZaX*; and it will be found that these incompatibles yield the six valid moods of the fourth figure².

208. *Is Reduction an essential part of the doctrine of the Syllogism?*—According to the original theory of Reduction, the object of the process is to be sure that the conclusion is a valid inference from the premisses. The validity of a syllogism in figure 1 may be directly tested by reference to the *dictum de omni et nullo*; but this *dictum* has no direct application to syllogisms in the remaining three figures. Thus, Whately says, "As it is on the *Dictum de omni et nullo* that all Reasoning ultimately depends, so, all arguments may be in one way or other brought into some one of the four Moods in the First Figure: and a Syllogism is, in that case, said to be *reduced*" (*Elements of Logic*, p. 93). Professor Fowler puts the same

¹ The following is given by Mr Johnson as a single *dictum* for figure 3, covering both affirmative and negative moods: "A statement may be applied to part of a class, if it applies wholly [or at least partly] to a set of objects that are at least partly [or wholly] included in that class."

² Compare the general line of argument adopted in section 162.

position somewhat more guardedly, "As we have adopted no canon for the 2nd, 3rd, and 4th figures, we have as yet no positive proof that the six moods remaining in each of those figures are valid; we merely know that they do not offend against any of the syllogistic rules. But if we can *reduce* them, *i.e.*, bring them back to the first figure, by shewing that they are only different statements of its moods, or in other words, that precisely the same conclusions can be obtained from equivalent premisses in the first figure, their validity will be proved beyond question" (*Deductive Logic*, p. 97).

Reduction is, on the other hand, regarded by some logicians as both *unnecessary* and *unnatural*. It is, in the first place, said to be *unnecessary*, on the ground that the *dictum de omni et nullo* has no claim to be regarded as the paramount law for all valid inference¹. In the preceding section we have discussed dicta for the other figures, which may be regarded as making them independent of the first, and putting them on a level with it. It may also be maintained that in any mood the validity of a particular syllogism is as self-evident as that of the *dictum de omni et nullo* itself; and that, therefore, although axioms of syllogism are useful as generalisations of the syllogistic process, they are needless in order to establish the validity of any given syllogism. This view is indicated by Ueberweg.

Reduction is, in the second place, said to be *unnatural*, inasmuch as it often involves the substitution of an unnatural and indirect for a natural and direct predication. Figures 2 and 3 at any rate have their special uses, and certain reasonings fall naturally into these figures rather than into the first figure².

¹ Compare Thomson, *Laws of Thought*, p. 172.

² Sir W. Hamilton (*Logic*, vol. 2, p. 438) quotes from Lambert (*Neues Organon*, §§ 230, 231) as follows:—"For, as the syllogisms of every figure admit of being transmuted into those of the first, and partly also into those of any other, if we rightly convert, or interchange, or turn into propositions of equal value, their premisses; consequently, in this point of view, no difference subsists between them. But whether we in every case should perform such commutations in order to bring a syllogism under a favourite figure, or

The following example is given by Thomson (*Laws of Thought*, p. 174): "Thus, when it was desirable to shew by an example that zeal and activity did not always proceed from selfish motives, the natural course would be some such syllogism as the following. The Apostles sought no earthly reward, the Apostles were zealous in their work; therefore, some zealous persons seek not earthly reward." In reducing this syllogism to figure 1, we have to convert our minor into "Some zealous persons were Apostles," which is awkward and unnatural.

Take again this syllogism, "Every reasonable man wishes the Reform Bill to pass, I don't, therefore, I am not a reasonable man." Reduced in the regular way to *Celarent*, the major premiss becomes "No person wishing the Reform Bill to pass is I," yielding the conclusion, "No reasonable man is I."

Further illustrations of this point will be found if we reduce to figure 1, syllogisms with such premisses as the following:—All orchids have opposite leaves, This plant has not opposite leaves; Socrates is poor, Socrates is wise.

The above arguments appear conclusively to establish the position that reduction is not an essential part of the doctrine

to assure ourselves of its correctness,—this is a wholly different question. The latter is manifestly futile. For, in the commutation, we must always undertake a conversion of the premisses, and a converted proposition is assuredly not always of equal evidence with that which we had to convert, while, at the same time, we are not so well accustomed to it. *E.g.*, the proposition, *Some stones attract iron*, every one will admit, because *The magnet is a stone, and attracts iron*. This syllogism is in the third figure. In the first, by conversion of one of its premisses, it would run thus—*All magnets attract iron, Some stones are magnets*, therefore, *Some stones attract iron*. Here we are unaccustomed to the minor proposition, while it appears as if we must pass all stones under review, in order to pick out magnets from among them. On the other hand, *the magnet is a stone* is a proposition which far more naturally suggests itself, and demands no consideration. In like manner:—*A circle is no square*; for *the circle is round*; *the square is not*.....It is thus apparent that we use every syllogistic figure there, where the propositions, as each figure requires them, are more familiar and more current. The difference of the figures rests, therefore, not only on their form, but extends itself, by relation to their employment, also to things themselves, so that we use each figure where its use is more natural."

of the syllogism, at any rate so far as establishing the validity of the different moods is concerned¹.

At the same time, no treatment of the syllogism can be regarded as scientific or complete until the *equivalence* between the moods in the different figures has been shewn; and for this purpose, as well as for its utility as a logical exercise, a full treatment of the problem of reduction should be retained.

209. The Fourth Figure.—Figure 4 was not as such recognised by Aristotle; and its introduction having been attributed by Averroës to Galen, it is frequently spoken of as the *Galenian Figure*. It does not usually appear in works on Logic before the beginning of the last century, and even by modern logicians its use is sometimes condemned. Thus, Bowen (*Logic*, p. 192) holds that “what is called the Fourth Figure is only the First with a converted conclusion; that is, we do not actually reason in the Fourth, but only in the First, and then if occasion requires, convert the conclusion of the First.” This account of figure 4 cannot, however, be accepted, for it will not apply to *Fesapo* or *Fresison*. For example, from

¹ Hamilton (*Logic*, i. p. 433) takes a curious position in regard to the doctrine of reduction. “The last three figures,” he says, “are virtually identical with the first.” This has been recognised by logicians, and hence “the tedious and disgusting rules of their reduction.” But he himself goes further, and extinguishes these figures altogether, as being merely “accidental modifications of the first,” and “the mutilated expressions of a complex mental process.” A somewhat similar position is taken by Kant in his essay *On the Mistaken Subtlety of the Four Figures*. Kant’s argument is virtually based on the two following propositions: (1) Reasonings in figures 2, 3, 4 require to be implicitly, if not explicitly, reduced to figure 1, in order that their validity may be apparent; for example, in *Cesare* we must have covertly performed the conversion of the major premiss in thought, since otherwise our premisses would not be conclusive; (2) No reasonings ever fall naturally into any of the moods of figures 2, 3, 4, which are, therefore, a mere useless invention of logicians. On grounds already indicated, both these propositions must be regarded as erroneous. A further error seems to be involved in the following passage from the same essay of Kant’s: “It cannot be denied that we can draw conclusions legitimately in all these figures. But it is incontestable that all except the first determine the conclusion only by a roundabout way, and by interpolated inferences, and that the very same conclusion would follow from the same middle term in the first figure by pure and unmixed reasoning.” The latter part of this statement certainly cannot be justified in such a case as that of *Baroco*.

the premisses of *Fesapo* (*No P is M* and *All M is S*) no conclusion whatever is obtainable in figure 1¹.

Thomson's ground of rejection is that "in the fourth figure the order of thought is wholly inverted, the subject of the conclusion had only been a predicate, whilst the predicate had been the leading subject in the premiss. Against this the mind rebels; and we can ascertain that the conclusion is only the converse of the real one, by proposing to ourselves similar sets of premisses, to which we shall always find ourselves supplying a conclusion so arranged that the syllogism is in the first figure, with the second premiss first" (*Laws of Thought*, p. 178). As regards the first part of this argument, Thomson himself points out that the same objection applies partially to figures 2 and 3. It is no doubt a reason why as a matter of fact figure 4 is seldom used²; but it affords no sufficient ground for altogether refusing to recognise this figure. The second part of Thomson's argument is, for a reason already stated, unsound. The conclusion, for example, of *Fresison* cannot be "the converse of the real conclusion," since (being an *O* proposition) it is the converse of nothing at all.

It is indeed impossible to treat the syllogism scientifically and completely without admitting in some form or other the

¹ For the most part the critics of the fourth figure seem to identify it altogether with *Bramantip*. The following extract from Father Clarke's *Logic* (p. 337) will serve to illustrate the contumely to which this poor figure is sometimes subjected: "Ought we to retain it? If we do, it should be as a sort of syllogistic Helot, to shew how low the syllogism can fall when it neglects the laws on which all true reasoning is founded, and to exhibit it in the most degraded form which it can assume without being positively vicious. Is it capable of reformation? Not of reformation, but of extinction.....Where the same premisses in the First Figure would prove a universal affirmative, this feeble caricature of it is content with a particular; where the First Figure draws its conclusion naturally and in accordance with the forms into which human thought instinctively shapes itself, this perverted abortion forces the mind to an awkward and clumsy process which rightly deserves to be called 'inordinate and violent'." Father Clarke's own violence appears to be attributable mainly to the fact that figure 4 was not, as such, recognised by Aristotle.

² The reasons why figure 4, "with its premisses looking one way, and its conclusion another," is seldom used, are elaborated by Karslake, *Aids to the Study of Logic*, i. pp. 74, 5.

moods of Figure 4. In an *à priori* separation of figures according to the position of the major and minor terms in the premisses, this figure necessarily appears, and it yields conclusions which are not directly obtainable from the same premisses in any other figure. It is not actually in frequent use, but reasonings may sometimes not unnaturally fall into it; for example, None of the apostles were Greeks, Some Greeks are worthy of all honour, therefore, Some worthy of all honour are not apostles.

210. *Indirect Moods*.—The earliest form in which the mnemonic verses appeared was as follows:—

*Barbara, Celarent, Darii, Ferio, Baralippton,
Celantes, Dabitis, Fapesmo, Frisesomorum,
Cesare, Camestres, Festino, Baroco, Darapti,
Felapton, Disamis, Datisi, Bocardo, Ferison*¹.

Aristotle recognised only three figures: the first figure, which he considered the type of all syllogisms and which he called the perfect figure, the *dictum de omni et nullo* being directly applicable to it alone; and the second and third figures, which he called imperfect figures, since it was necessary to reduce them to the first figure, in order to obtain a test of their validity.

Before the fourth figure, however, was commonly recognised as such, its moods were recognised in another form, namely, as *indirect* moods of the first figure; and the above mnemonics—*Baralippton, Celantes, Dabitis, Fapesmo, Frisesomorum*—represent these moods so regarded².

The conception of indirect moods may be best explained by starting from a definition of figure, which contains no reference to the distinction between major and minor terms, and which accordingly yields only three figures instead of four, namely: Figure 1, in which the middle term is subject in one of the

¹ First given by Petrus Hispanus, afterwards Pope John XXI., who died in 1277.

² From the 14th to the 17th century the mnemonics found in works on Logic usually give the moods of the fourth figure in this form, or else omit them altogether. Wallis (1687) recognises them in both forms, giving two sets of mnemonics.

premises and predicate in the other; Figure 2, in which the middle term is predicate in both premisses; Figure 3, in which the middle term is subject in both premisses. The moods of figure 1 may then be distinguished as direct or indirect according as the position of the terms in the conclusion is the same as their position in the premisses or the reverse¹. Thus, with the premisses *MaP*, *SaM*, we have a direct conclusion *SaP*, and an indirect conclusion *PiS*. These are respectively *Barbara* and *Baralippton*. Similarly, *Celantes* corresponds to *Celarent*, and *Dabitis* to *Darii*. With the premisses *MeP*, *SiM*, we obtain the direct conclusion *SoP*, but nothing can be inferred of *P* in terms of *S*. There is, therefore, no indirect mood corresponding to *Ferio*. On the other hand, *Fapesmo* and *Frisesomorum* (the *Fesapo* and *Fresison* of the fourth figure) have no corresponding direct moods.

Clearly it is no more than a formal difference whether the five moods in question are recognised in the manner just indicated, or as constituting a distinct figure; but, on the whole, the latter alternative seems less likely to give rise to confusion.

The distinction between direct and indirect moods as above expressed is for obvious reasons confined to the first figure. It will be observed, however, that in the traditional names of the indirect moods of the first figure the minor premiss precedes

¹ It follows that if we compare the conclusion of an indirect mood with the conclusion of the corresponding direct mood (where such correspondence exists), we shall find that the terms have changed places. Mansel's definition of an indirect mood as "one in which we do not infer the immediate conclusion, but its converse" (*Aldrich*, p. 78) must, however, be rejected for the reason that it cannot be applied to *Fapesmo* and *Frisesomorum*, which are indirect moods having no corresponding valid direct moods at all. In these we cannot be said to infer "the converse of the immediate conclusion," for there is no immediate conclusion. Mansel deals with these two moods very awkwardly. "*Fapesmo* and *Frisesomorum*," he remarks, "have negative minor premisses, and thus offend against a special rule of the first figure; but this is checked by a counterbalancing transgression. For by simply converting *O*, we alter the distribution of the terms, so as to avoid an illicit process." But the notion that we can counterbalance one violation of law by committing a second cannot be allowed. The truth of course is that, in the first place, the special rules of the first figure as ordinarily given do not apply to the indirect moods; and in the second place, the conclusion *O* is not obtained by conversion at all.

the major, and if we seek to apply a distinction between direct and indirect moods in the case of the second and third figures, it can only be with reference to the conventional order of the premisses. Thus, in the second figure, taking the premisses *PeM*, *SaM*, we may infer either *SeP* or *PeS*, and if we call a syllogism direct or indirect according as the major premiss precedes the minor, or *vice versa*, then *PeM*, *SaM*, *SeP* will be a direct mood, and *PeM*, *SaM*, *PeS* an indirect mood. The former of these syllogisms is *Cesare*, and the latter is *Camestres* with the premisses transposed¹. Hence the latter will immediately become a direct mood by merely changing the order of the premisses; and the artificiality of the distinction is at once apparent. The result will be found to be similar in other cases, and the distinction may, therefore, be rejected so far as figures 2 and 3 are concerned.

EXERCISES.

211. "*Barbara*, *Baroco*, and *Bocardo* cannot be ostensibly reduced to any other figure except by the use of conversion by contraposition." Shew by general reasoning that this follows from the rules relating to the conversion of propositions. Also reduce each of the above moods directly to the fourth figure by the aid of conversion by contraposition. [κ.]

212. Reduce *Ferio* to Figure 2, *Festino* to Figure 3, *Felapton* to Figure 4. [κ.]

213. Reduce *Camestres* to *Datissi*. Why cannot *Camestres* be reduced either directly or indirectly to *Felapton*? [κ.]

214. Assuming that in the first figure the major must be universal and the minor affirmative, shew by *reductio ad absurdum* that the conclusion in the second figure must be negative and in the third particular. [J.]

215. State the following argument in a syllogism of the third figure, and reduce it, both directly and indirectly, to the first:—Some things worthy of being known are not directly useful, for

¹ Take, again, the premisses *MaP*, *MoS*. Here there is no direct conclusion, but only an indirect conclusion *PoS*. This, however, is merely *Bocardo* with the premisses transposed.

every truth is worthy of being known, while not every truth is directly useful. [M.]

216. Express the following argument in as many moods of the third figure as you can, using any process of immediate inference which may be necessary:—Some things which have a practical worth are also of theoretical value; for every science has a theoretical as well as a practical value. [M.]

217. State the figure and mood of the following syllogism; reduce it to the first figure; and examine whether there is anything unnatural in the argument as it stands:—

None who dishonour the king can be true patriots; for a true patriot must respect the law, and none who respect the law would dishonour the king. [J.]

218. "Rejecting the fourth figure and the subaltern moods, we may say with Aristotle; A is proved only in one figure and one mood, E in two figures and three moods, I in two figures and four moods, O in three figures and six moods. For this reason, A is declared by Aristotle to be the most difficult proposition to establish, and the easiest to overthrow; O, the reverse." Discuss the fitness of these data to establish the conclusion. [K.]

219. Shew that any mood may be directly reduced to any other mood, provided (1) that the latter contains neither a strengthened premiss nor a weakened conclusion, and (2) that if the conclusion of the former is universal, the conclusion of the latter is also universal. [K.]

220. Shew that any mood may be directly or indirectly reduced to any other mood, provided that the latter has not either a strengthened premiss or a weakened conclusion, unless the same is true of the former also. [K.]

221. Examine the following statement of De Morgan's:—"There are but six distinct syllogisms. All others are made from them by strengthening one of the premisses, or converting one or both of the premisses, where such conversion is allowable; or else by first making the conversion, and then strengthening one of the premisses." [K.]

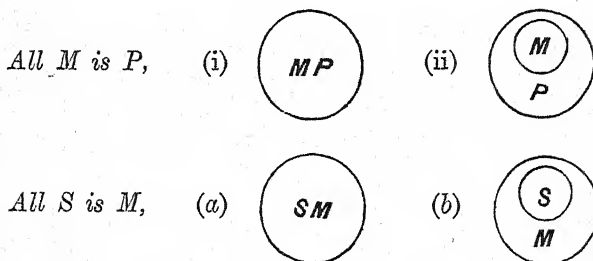
CHAPTER IV.

THE DIAGRAMMATIC REPRESENTATION OF SYLLOGISMS.

222. *The application of the Eulerian diagrams to syllogistic reasonings.*—In shewing the application of the Eulerian diagrams to syllogistic reasonings we may begin with a syllogism in *Barbara* :

All M is P,
All S is M,
 therefore, *All S is P.*

The premisses must first be represented separately by means of the diagrams. They each yield two cases; thus,—



To obtain the conclusion, each of the cases yielded by the major premiss must now be combined with each of those yielded by the minor. This gives four combinations¹, and whatever is true of *S* in terms of *P* in all of them is the conclusion required.

¹ These combinations afford a complete solution of the problem what class-relations between *S*, *M*, and *P* are compatible with the premisses; and similarly in other cases. The syllogistic conclusion is obtained by the elimination of *M*.

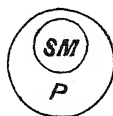
(i) and (a) yield



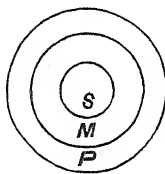
(i) and (b)



(ii) and (a)



(ii) and (b)



In each case *S* either coincides with *P* or is included within *P*; hence *all S is P* may be inferred from the given premisses.

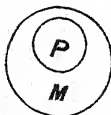
Next, take a syllogism in *Bocardo*. The application of the diagrams is now more complicated. The premisses are

Some M is not P,

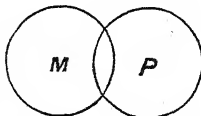
All M is S.

The major premiss yields three possible cases, namely,

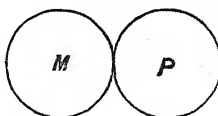
(i)



(ii)

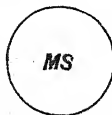


(iii)

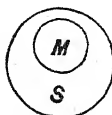


and the minor premiss two possible cases, namely,

(a)

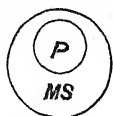


(b)

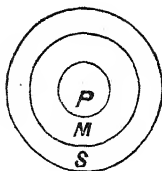


Taking them together we have six combinations, some of which themselves yield more than one case:—

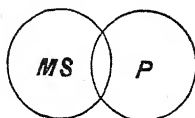
(i) and (a)



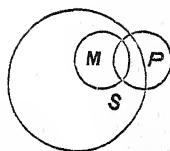
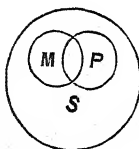
(i) and (b)



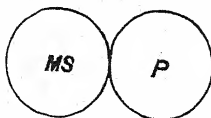
(ii) and (a)



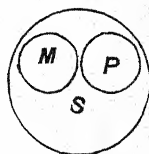
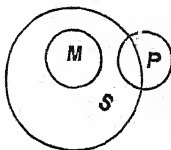
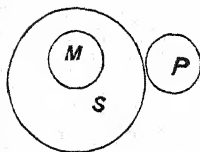
(ii) and (b)



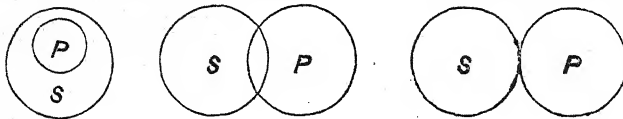
(iii) and (a)



(iii) and (b)



So far as S and P are concerned (M being left out of account) these nine cases are reducible to the following three:



The conclusion, therefore, is *Some S is not P*.

It must be admitted that this is very complex, and that it would be a serious matter if in the first instance we had to work through all the different moods in this manner¹. Still, for purposes of illustration, this very complexity has a certain advantage. It shews how many relations between three terms in respect of extension are left to us, even with two premisses given.

223. *The application of Lambert's diagrammatic scheme to syllogistic reasonings.*—As applied to syllogisms, Lambert's lines are much less cumbrous than Euler's circles. The main point to notice is that it is in general necessary that the line standing for the middle term should not be dotted over any part of its extent². This condition can be satisfied by selecting the appropriate alternative form in the case of **A**, **I**, and **O** propositions, as given in section 90. As examples we may represent *Barbara*, *Baroco*, *Datisi*, and *Fresison* by Lambert's method.

Barbara :—

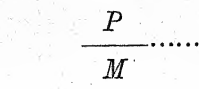
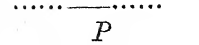
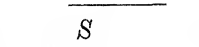
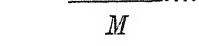
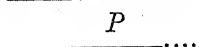
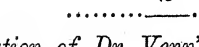
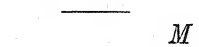
P
—.....
M
—
S
—.....

¹ Ueberweg, however, takes the trouble to establish in this way the validity of the valid moods in the various figures.

² The following representation of *Barbara*

P
..... —.....
M
..... —.....
S
—

illustrates the kind of error that is likely to result if the above precaution is neglected. Supposing this representation were correct we should be justified in inferring *Some P is not S* as well as *All S is P*.

Baroco :—*S**Datisi* :—*Fresison* :—

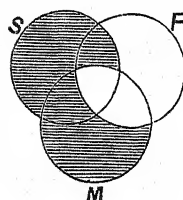
224. *The application of Dr Venn's diagrammatic scheme to syllogistic reasonings.*—Syllogisms in *Barbara* and *Camestres* may be taken in order to shew how Dr Venn's diagrams can be used to illustrate syllogistic reasonings.

The premisses of *Barbara*,

All M is P,

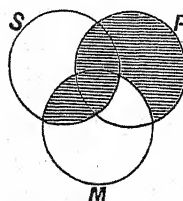
All S is M,

exclude certain compartments as shewn in the following diagram :



This yields at once the conclusion *All S is P*.

Similarly for *Camestres* we have the following :



This scheme is especially adapted to illustrate those syllogistic processes in which all the propositions involved are universal. We have already seen that it is not so suitable when particular propositions have to be represented.

EXERCISES.

225. Represent *Celarent* by the aid of Euler's diagrams. Will the same set of diagrams serve for any other of the syllogistic moods? [K.]

226. Represent by means of the Eulerian diagrams the moods *Festino*, *Datisi*, and *Bramantip*. Shew also by means of these diagrams that *IE* yields no conclusion in any figure. [K.]

227. Represent in Lambert's scheme the moods *Darii*, *Cesare*, *Darapti*, *Bocardo*, *Fesapo*. [K.]

228. Represent in Dr Venn's diagrammatic scheme the moods *Celarent*, *Cesare*, *Camenes*. [K.]

229. Taking the premisses of an ordinary syllogism in *Barbara*, e.g., *all X is Y*, *all Y is Z*, determine precisely and exhaustively what these propositions affirm, what they deny, and what they leave in doubt, concerning the relations of the terms *X*, *Y*, *Z*. [L.]

CHAPTER V.

CONDITIONAL AND HYPOTHETICAL SYLLOGISMS.

230. *The Conditional Syllogism, the Hypothetical Syllogism, and the Hypothetico-Categorical Syllogism.*—The forms of reasoning in which conditional or hypothetical conclusions are inferred from two conditional or two hypothetical premisses are apparently overlooked by some logicians; at any rate, they frequently receive no distinct recognition, the term “hypothetical syllogism” being limited to the case in which one premiss only is hypothetical.

The following definitions may be given:

(1) A *conditional syllogism* is a mediate reasoning consisting of two conditional premisses and a conditional conclusion;

e.g.,—Whenever *C* is *D*, *E* is *F*,
Whenever *A* is *B*, *C* is *D*,
therefore, Whenever *A* is *B*, *E* is *F*.

(2) A *hypothetical syllogism* is a mediate reasoning consisting of two hypothetical premisses and a hypothetical conclusion;

e.g.,—If *Q* is true, *R* is true,
If *P* is true, *Q* is true,
therefore, If *P* is true, *R* is true.

(3) A *hypothetico-categorical syllogism*¹ is a mediate reason-

¹ It seems unnecessary to discuss separately the case in which a conditional and a categorical premiss are combined: *e.g.*, All selfish people are unhappy; If a child is spoilt, he is sure to be selfish; therefore, If a child is spoilt he will be unhappy. Such a syllogism as this is obviously resolvable into an ordinary categorical syllogism by reducing the conditional premiss to the categorical form,

ing consisting of three propositions in which one of the premisses is hypothetical in form, while the other premiss and the conclusion are categorical;

e.g.,—If *P* is true, *Q* is true,
 P is true,
 therefore, *Q* is true.

This nomenclature, so far as the distinction between the hypothetical and the hypothetico-categorical syllogism is concerned, is adopted by Spalding and Ueberweg. Some logicians (*e.g.*, Fowler) give the name "hypothetical syllogism" to all the above forms of reasoning without distinction. Others (*e.g.*, Jevons) define the hypothetical syllogism so as to include the last form only, the others not being recognised as distinct forms of reasoning at all. This view may be to some extent justified by the very close analogy that exists between the syllogism with two conditional or two hypothetical premisses and the categorical syllogism; but the difference in form is worth at least a brief discussion.

231. Distinctions of Mood and Figure in the case of Conditional and Hypothetical Syllogisms.—In the conditional syllogism, the antecedent of the conclusion is equivalent to the minor term of the categorical syllogism, the consequent of the conclusion to the major term, and the element which does not appear in the conclusion at all to the middle term. Distinctions of mood and figure may be recognised in precisely the same way as in the case of the categorical syllogism. For example,

Barbara,—Whenever *C* is *D*, *E* is *F*,
 Whenever *A* is *B*, *C* is *D*,
 therefore, Whenever *A* is *B*, *E* is *F*.

"All spoilt children are selfish"; or it may be resolved into a conditional syllogism by transforming the categorical premiss into the corresponding conditional, "If any one is selfish, he is sure to be unhappy." The following is another example: If water is salt, it will not boil at 212°; Sea water is salt; therefore, Sea water will not boil at 212°. Compare Mr F. B. Tarbell in *Mind*, 1883, p. 578. The hypothetico-categorical syllogism as above defined cannot be so summarily disposed of.

Festino,—Never when *E* is *F*, is it the case that *C* is *D*,
 Sometimes when *A* is *B*, *C* is *D*,
 therefore, Sometimes when *A* is *B*, it is not the case that *E* is *F*.

Darapti,—Whenever *C* is *D*, *E* is *F*,
 Whenever *C* is *D*, *A* is *B*,
 therefore, Sometimes when *A* is *B*, *E* is *F*.

Camenes,—Whenever *E* is *F*, *C* is *D*,
 Never when *C* is *D*, is it the case that *A* is *B*,
 therefore, Never when *A* is *B*, is it the case that *E* is *F*.

It has been shewn that distinctions of quantity do not apply to true hypothetical propositions; for syllogistic purposes, however, all hypothetical propositions may be regarded as equivalent to universals. Hence we cannot have a hypothetical syllogism corresponding to any categorical syllogism that contains a particular premiss or a particular conclusion. Hypothetical syllogisms may, however, be constructed which are analogous respectively to *Barbara*, *Celarent*, *Cesare*, *Camestres*, and *Camenes*. Thus the syllogism given in the preceding section corresponds to *Barbara*; the following corresponds to *Cesare*:

If R is true, Q is not true,
If P is true, Q is true,
 therefore, *If P is true, R is not true.*

232. Fallacies in Hypothetical Syllogisms.—On the mistaken supposition that a pure hypothetical proposition is equivalent to a categorical proposition in which both the subject and the predicate are singular terms, and therefore *ipso facto* distributed, it has been argued that the syllogistic rules relating to the distribution of terms have no application to hypothetical syllogisms; and that the only rules which need be considered in testing such syllogisms are those relating to quality, namely, the rule forbidding two negative premisses, and the rule insisting that a negative premiss and a negative conclusion must always be found together. But it is clearly an error to regard the consequent of a hypothetical proposition as equivalent to a singular term occurring as the predicate of a

categorical proposition. An affirmative hypothetical is not simply convertible, and in respect of distribution, its consequent practically corresponds to the undistributed predicate of an affirmative categorical in which the terms are general. On the other hand, a negative hypothetical is simply convertible; and its consequent corresponds to the distributed predicate of a negative categorical. We may accordingly have fallacies in hypothetical syllogisms corresponding to (1) undistributed middle, (2) illicit major, (3) illicit minor. The following are examples of these fallacies respectively:—

- (1) *If R then Q, If P then Q, therefore, If P then R;*
- (2) *If Q then R, If P then not Q, therefore, If P then not R;*
- (3) *If Q then R, If Q then P, therefore, If P then R.*

233. *The Reduction of Conditional and Hypothetical Syllogisms.*—Conditional syllogisms in figures 2, 3, and 4 may be reduced to figure 1 just as in the case of categorical syllogisms. Thus, the conditional syllogism in *Camenes* given in section 231 may be reduced as follows to *Celarent*:

Never when C is D, is it the case that A is B,

Whenever E is F, C is D,

therefore, Never when E is F, is it the case that A is B,

therefore, Never when A is B, is it the case that E is F.

According to the ordinary rule as indicated in the mnemonic, the premisses have here been transposed, and the conclusion of the new syllogism is converted in order to obtain the original conclusion.

Hypothetical syllogisms in *Cesare*, *Camestres*, and *Camenes* may similarly be reduced to *Celarent*. For example, the hypothetical syllogism in *Cesare* given in section 231 may be reduced by simply converting the major premiss, which then becomes *If Q is true, R is not true.*

234. *The Moods of the Hypothetico-Categorical Syllogism.*—It is usual to distinguish two moods of the hypothetico-categorical syllogism, the *modus ponens* and the *modus tollens*¹.

¹ Ueberweg remarks that the *modus ponens* should more accurately be called the *modus ponendo ponens*, and the *modus tollens* the *modus tollendo tollens* (*Logic*, p. 452).

(1) In the *modus ponens* (also called the *constructive hypothetical syllogism*) the categorical premiss affirms the antecedent of the hypothetical premiss, thereby justifying as a conclusion the affirmation of its consequent. For example,

If P is true then Q is true,
P is true,
 therefore, *Q is true.*

(2) In the *modus tollens* (also called the *destructive hypothetical syllogism*) the categorical premiss denies the consequent of the hypothetical premiss, thereby justifying as a conclusion the denial of its antecedent. For example,

If P is true then Q is true,
Q is not true,
 therefore, *P is not true.*

These moods may be considered to correspond respectively to the first and second figures of the categorical syllogism.

An attempt may be made to reduce the *modus ponens* to pure categorical form as follows:

The case of P being true is a case of Q being true,
The actual case is a case of P being true,
 therefore, *The actual case is a case of Q being true.*

This is not really satisfactory; but it is worth giving in order to bring out the analogy between the above example of the *modus ponens* and the categorical syllogism in *Barbara*. The example of the *modus tollens* given above corresponds to *Camestres*.

The following is a complete enumeration of hypothetico-categorical syllogisms:

- (1) *Modus ponens*, corresponding to *Barbara*,
 - (i) *If P then Q, but P, therefore, Q;*
 - (ii) *If not P then Q, but not P, therefore, Q;*
- (2) *Modus ponens*, corresponding to *Celarent*,
 - (iii) *If P then not Q, but P, therefore, not Q;*
 - (iv) *If not P then not Q, but not P, therefore, not Q;*

(3) *Modus tollens*, corresponding to *Camestres*,(v) *If P then Q, but not Q, therefore, not P;*(vi) *If not P then Q, but not Q, therefore, P;*(4) *Modus tollens*, corresponding to *Cesare*,(vii) *If P then not Q, but Q, therefore, not P;*(viii) *If not P then not Q, but Q, therefore, P¹.***235. Fallacies in Hypothetico-Categorical Syllogisms.**—

There are two principal fallacies to which we are liable in arguing from a hypothetical major premiss:—

(1) It is a fallacy if we regard the affirmation of the consequent as justifying the affirmation of the antecedent. For example,

If P is true then Q is true,

Q is true,

therefore, P is true.

(2) It is a fallacy if we regard the denial of the antecedent as justifying the denial of the consequent. For example,

If P is true then Q is true,

P is not true,

therefore, Q is not true.

These fallacies may be regarded as corresponding respectively to *undistributed middle* and *illicit major* in the case of categorical syllogisms².

¹ Lotze in his classification of hypothetico-categorical syllogisms (*Logic*, § 93) omits this case and one other. He would, however, regard it as what he calls a *modus ponendo ponens*, because both the categorical premiss and the conclusion appear as affirmatives. But it will be observed that the distinction as drawn in the text between the *modus ponens* and the *modus tollens* turns not on the absolute quality of the conclusion, but on whether it affirms the consequent or denies the antecedent of the hypothetical premiss. This is the really important distinction. The above syllogism might be written in the form

If P is not true then it is not the case that Q is true,

but Q is true,

therefore, It is not the case that P is not true.

The analogy with *Cesare* is then more obvious.

² Given "If P and only if P then Q," we may of course argue from Q to P or from not-P to not-Q; and no doubt in the case of ordinary hypotheticals it is often tacitly understood that the consequent is true *only* if the antecedent is true. This must, however, be expressly stated or the argument based upon it cannot be formally valid.

The results reached in this and the preceding section may be summed up in the following canon for the hypothetico-categorical syllogism: Given a hypothetical premiss expressed affirmatively, then the affirmation of the antecedent justifies the affirmation of the consequent; and the denial of the consequent justifies the denial of the antecedent; but not conversely in either case.

236. *The Reduction of Hypothetico-Categorical Syllogisms.*—Any case of the *modus tollens* may be reduced to the *modus ponens*, and *vice versâ*.

Thus, *If P is true then Q is true,*
 Q is not true,
 therefore, *P is not true,*

becomes, by contraposition of the hypothetical premiss,

If Q is not true then P is not true,
 Q is not true,
 therefore, *P is not true;*

and this is the *modus ponens*¹.

237. *Is the reasoning contained in the hypothetico-categorical syllogism² mediate or immediate?*—Kant, Hamilton³, Bain, and others argue that inferences of the kind that we have just been considering are not properly to be regarded as mediate but as immediate inferences.

Now, taking the syllogism—

If A is B, C is D,
 but *A is B,*
 therefore, *C is D,*

the conclusion is at any rate apparently obtained by a combination of two premisses, and the process is moreover one of

¹ A categorical syllogism in *Camestres* may similarly be reduced to *Celarent* without transposing the premisses. Thus, *All P is M, No S is M,* therefore, *No S is P,* becomes, by contraposition of the major and obversion of the minor premiss, *No not-M is P, All S is not-M,* therefore, *No S is P.*

² Similar arguments on both sides may be used in the case where a conditional premiss and a categorical premiss are combined.

³ *Logic*, vol. 2, p. 383. On p. 378, however, Hamilton seems to take the other view.

elimination, namely, of the proposition *A is B*. Hence the burden of proof certainly lies with those who deny the claims of such an inference as this to be called mediate.

Professor Bain (*Logic, Deduction*, p. 117) seems to argue that the so-called hypothetical syllogism is not really mediate inference, because it is "a pure instance of the law of consistency"; in other words, because "the conclusion is implied in what has already been stated." But is not this the case in all formal mediate inference? It cannot be maintained that the categorical syllogism is more than a pure instance of the law of consistency; or that the conclusion in such a syllogism is not implied in what has been already stated. But possibly Dr Bain may mean that the conclusion is implied in the hypothetical premiss alone. Indeed he goes on to say, "'If the weather continues fine, we shall go into the country' is transformable into the equivalent form 'The weather continues fine, and so we shall go into the country.' Any person affirming the one, does not, in affirming the other, declare a new fact, but the same fact." Surely this is not intended to be understood literally. Take the following:—If war is declared, I must return home; If the sun moves round the earth, modern astronomy is a delusion. Are these respectively equivalent to the statements, War has been declared, and so I must return home; The sun moves round the earth, and so modern astronomy is a delusion? Besides, if the proposition *If P is true then Q is true* implies the truth of *P*, what becomes of the possible reasoning, "But *Q is not true*, therefore, *P is not true*"?

Further arguments that have been adduced on the same side are as follows:—

(1) "There is no middle term in the so-called hypothetical syllogism".¹ The answer is that there is something in the premisses which does not appear in the conclusion, and

¹ This is Kant's argument. A more plausible argument would be that there is no *minor* term. But if this is granted, it only shews a want of complete analogy between the hypothetico-categorical syllogism and the categorical syllogism. In attempting to reduce the one to the other, a minor term of the form "the actual case" or something equivalent thereto is made to appear. Compare section 234.

that this corresponds to the middle term of the categorical syllogism.

(2) "In the so-called hypothetical syllogism, the minor and the conclusion indifferently change places".¹ This statement is erroneous. Taking the valid syllogism given at the commencement of this section and transposing the so-called minor and the conclusion, we have a fallacy.

(3) "The major in a so-called hypothetical syllogism consists of two propositions, the categorical major of two terms." This merely tells us that a hypothetical syllogism is not the same in form as a categorical syllogism, but seems to have no bearing on the question whether the so-called hypothetical syllogism is a case of mediate or of immediate inference.

Turning now to the other side of the question no satisfactory answers seem possible to the following arguments in favour of regarding the hypothetico-categorical syllogism as a case of mediate inference. In any such syllogism, the two premisses are quite distinct, neither can be inferred from the other, but both are necessary in order that the conclusion may be obtained. Again, if we compare with it the inferences which are on all sides admitted to be immediate inferences from the hypothetical proposition, the difference between the two cases is apparent. From *If P is true then Q is true*, I can infer immediately *If Q is not true then P is not true*; but I require also to know that *Q is not true* in order to be able to infer that *P is not true*.

¹ This argument is Hamilton's. He remarks that in hypothetical syllogisms, "*the same proposition is reciprocally medium or conclusion*" (*Logic*, vol. 2, p. 379). Dr Ray (*Deductive Logic*, Note C) holds that Hamilton is here wrongly interpreted; and that he meant no more than that with a hypothetical premiss *If A is B, C is D*, a relation between *A* and *B* may be either the other premiss (as in the *modus ponens*) or the conclusion (as in the *modus tollens*). Dr Ray is possibly right. But if so, Hamilton does not express himself clearly. For *A is B* (the premiss of the *modus ponens*) is certainly not *the same proposition as A is not B* (the conclusion of the *modus tollens*). It may be added that the argument in its new form is irrelevant. In the categorical syllogism we have something precisely analogous. For given a major premiss *All M is P*, a relation between *M* and *S* may be the minor premiss (in which case *M* will be the middle term), or it may be the conclusion (in which case *M* will be the major term). Compare the syllogisms: *All M is P, All S is M, therefore, All S is P*; *All M is P, No S is P, therefore, No S is M*.

And whether the hypothetico-categorical syllogism can or cannot be actually reduced to pure categorical form, it can at least be shewn to be analogous to the ordinary categorical syllogism, which is admitted to be a case of mediate reasoning. Moreover there are distinct forms—the *modus ponens* and the *modus tollens*—which are analogous to distinct forms of the categorical syllogism; and fallacies in the hypothetico-categorical syllogism correspond to certain fallacies in the categorical syllogism.

Professor Bowen indeed remarks (*Logic*, p. 265):—"The reduction of a Hypothetical Judgment to a Categorical shews very clearly the Immediacy of the reasoning in what is called a Hypothetical Syllogism. Thus, *If A is B, C is D*, is equivalent to *All cases of A is B are cases of C is D*, therefore,

Some cases of A is B are cases of } *C is D.*
This case of A is B is a case of }

But this overlooks the fact that a new judgment is required before we can tell that this is a case of *A* being *B*. The mere statement that *some cases of A is B are cases of C is D* is clearly not equivalent to the conclusion of the hypothetical syllogism¹. By analogy we should have to argue that the following categorical syllogism in *Barbara* is an immediate inference: *All M is P, This is M*, therefore, *This is P*. Thus the argument again proves too much.

The argument in favour of regarding the *modus tollens*—*If P is true then Q is true, but Q is not true*, therefore, *P is not true*—as mediate inference is still more forcible; but of course the *modus ponens* and the *modus tollens* stand or fall together².

Professor Croom Robertson (*Mind*, 1877, p. 264) has suggested an explanation as to the manner in which this controversy may have arisen. He distinguishes the *hypothetical* "if" from the *inferential* "if," the latter being equivalent to *since*, *seeing that*, *because*. No doubt by the aid of a certain accentua-

¹ Professor Bowen obviously has in view a conditional as distinguished from a pure hypothetical major premiss. But this distinction does not materially affect the present argument.

² In section 244 it will be shewn further that the hypothetical syllogism and the disjunctive syllogism also stand or fall together.

tion the word "if" may be made to carry with it this force. Professor Robertson quotes a passage from *Clarissa Harlowe* in which the remark "If you have the value for my cousin that you say you have, you must needs think her worthy to be your wife," is explained by the speaker to mean, "*Since* you have &c." Using the word in this sense, the conclusion *C is D* certainly follows immediately from the bare statement *If A is B, C is D*; or rather this statement itself affirms the conclusion. When, however, the word "if" carries with it this inferential implication, we cannot regard the proposition in which it occurs as strictly hypothetical. We have rather a condensed mode of expression including two statements in one; it may indeed be argued that in the single statement thus interpreted we have a hypothetical syllogism expressed elliptically¹.

EXERCISES.

238. Shew how the *modus ponens* may be reduced to the *modus tollens*. [K.]

239. Test the following: "If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it." [V.]

240. Examine technically the following argument:—

If you needed food, I would give you money; but as you do not care to work, you cannot need food; therefore, I will give you no money. [J.]

241. Construct conditional syllogisms in *Cesare*, *Bocardo*, *Fesapo*, and reduce them to the first figure. [K.]

242. Name the mood and figure of the following, and shew that either one may be reduced to the other form:

(1) *If R is true, Q is true,*
If P is true, Q is not true,
 therefore, *If P is true, R is not true;*

¹ Compare Mansel's *Aldrich*, p. 103.

(2) *If Z is true, Y is true,*
If Y is true, X is not true,
therefore, *If X is true, Z is not true.*

[K.]

243. Let X, Y, Z, P, Q, R be six propositions.

- Given (1) *If X is true, P is true;*
(2) *If Y is true, Q is true;*
(3) *If Z is true, R is true;*
(4) *Of X, Y, Z one at least is true;*
(5) *Of P, Q, R not more than one is true;*

prove syllogistically

- (i) *If P is true, X is true;*
(ii) *If Q is true, Y is true;*
(iii) *If R is true, Z is true;*
(iv) *Of P, Q, R, one at least is true;*
(v) *Of X, Y, Z, not more than one is true.*

[K.]

CHAPTER VI.

DISJUNCTIVE SYLLOGISMS.

244. *The Disjunctive Syllogism.*—A disjunctive (or alternative) syllogism may be defined as a formal reasoning in which a categorical premiss is combined with an alternative premiss so as to yield a conclusion which is either categorical or else alternative with fewer alternants than are contained in the alternative premiss¹.

For example, *A is either B or C,*
 A is not B,
 therefore, *A is C ;*
 Either P or Q or R is true,
 P is not true,
 therefore, *Either Q or R is true.*

The categorical premiss in each of the above syllogisms denies one of the alternants of the alternative premiss, and the conclusion affirms the remaining alternant or alternants. Reasonings of this type are accordingly described as examples of the *modus tollendo ponens*.

It follows from the resolution of alternative propositions given in section 141 that the force of an alternative as a premiss in an argument is equivalent either to that of a conditional or to that of a hypothetical proposition.

¹ Archbishop Thomsop's definition of the disjunctive syllogism—"An argument in which there is a disjunctive judgment" (*Laws of Thought*, p. 197)—must be regarded as too wide if, as is usually the case, an affirmative judgment with a disjunctive predicate is considered disjunctive. It would include such a syllogism as the following—*B is either C or D, A is B, therefore A is either C or D.* The argument here in no way turns upon the alternation contained in the major premiss, and the reasoning may be regarded as an ordinary categorical syllogism in *Barbara*, the major term being complex. A more general treatment of reasonings involving disjunctive judgments is given in Part iv.

Thus, *Either A is B or C is D,*
A is not B,
 therefore, *C is D;*

may be resolved into the form

If A is not B, C is D,
A is not B,
 therefore, *C is D;*

or into the form

If C is not D, A is B,
A is not B,
 therefore, *C is D*¹.

A corollary from the above is that those who deny the character of mediate reasoning to the hypothetico-categorical syllogism must also deny it to the disjunctive syllogism, or else must refuse to recognise the resolution of the disjunctive proposition into one or more hypotheticals.

In the above example it is not quite clear from the form of the major premiss whether we have a true hypothetical or a conditional. But in the following examples, which are added to illustrate the distinction, it is evident that the alternative propositions are equivalent to a true hypothetical and to a conditional respectively:

Either all A's are B's or all A's are C's,
This A is not B,
 therefore, *All A's are C's;*
All A's are either B or C,
This A is not B,
 therefore, *This A is C*².

¹ Logicians have not, as a rule, given any distinctive recognition to arguments consisting of two disjunctive premisses and a disjunctive conclusion; and Mr Welton goes so far as to remark that "both premisses of a syllogism cannot be disjunctive since from two assertions as indefinite as disjunctive propositions necessarily are, nothing can be inferred" (*Logic*, p. 327). It is, however, clear that this is erroneous, if an argument consisting of two hypothetical premisses and a hypothetical conclusion is possible, and if a hypothetical can be reduced to the disjunctive form. As an example we may express in disjunctives the hypothetical syllogism given on p. 300: *Either Q is not true or R is true, Either P is not true or Q is true, therefore, Either P is not true or R is true.*

² When the alternative major premiss is complex, not compound (that is,

245. *The modus ponendo tollens*.—In addition to the *modus tollendo ponens*, some logicians recognise as valid a *modus ponendo tollens*, in which the categorical premiss affirms one of the alternants of the alternative premiss, and the conclusion denies the other alternant or alternants. Thus,

A is either B or C,
A is B,
 therefore, *A is not C.*

The argument here proceeds on the assumption that the alternants are mutually exclusive; but this, on the interpretation of alternative propositions adopted in section 140, is not necessarily the case. Hence the recognition or denial of the validity of the *modus ponendo tollens* in its ordinary form depends upon our interpretation of the alternative proposition itself¹.

No doubt exclusiveness is often intended to be implied and is understood to be implied. For example, "He was either first or second in the race, He was second, therefore, He was not first." This reasoning would ordinarily be accepted as valid. But its validity really depends not on the expressed major premiss, but on the understood premiss, "No one can be both first and second in a race." The following reasoning is in fact equally valid with the one stated above, "He was second in the race, therefore, He was not first." The alternative premiss is, therefore, quite immaterial to the reasoning; we could do just as well without it, for the really vital premiss, "No one can be both first and second in a race," is true, and would be accepted as such, quite irrespective of the truth of the alternative proposition, "He was either first or second." In other cases the mutual exclusiveness of the alternants may be tacitly

equivalent to a conditional, not to a true hypothetical)—as in the second of the above examples—the syllogism may of course be reduced to pure categorical form. Thus,

Every A which is not B is C,
This A is an A which is not B,
 therefore, *This A is C.*

¹ It will be observed that, interpreting the alternants as not necessarily exclusive of one another, the *modus ponendo tollens* in the above form is equivalent to one of the fallacies in the hypothetico-categorical syllogism mentioned in section 235.

understood, although not obvious *a priori* as in the above example. But in no case can a special implication of this kind be recognised when we are dealing with purely symbolic forms. If we hold that the *modus ponendo tollens* as above stated is *formally* valid, we must be prepared to interpret the alternants as *in every case* mutually exclusive.

If, however, we take a major premiss which is not alternative at all, but *disjunctive*, in the stricter sense explained in section 138, then we may have a formally valid reasoning which has every right to be described as a *modus ponendo tollens*. Thus,

P and Q are not both true;
but P is true;
 therefore, *Q is not true*¹.

The following table of the *ponendo ponens*, &c., in their valid and invalid forms may be useful:

	<i>Valid</i>	<i>Invalid</i>
<i>Ponendo Ponens</i>	If <i>P</i> then <i>Q</i> , but <i>P</i> , ∴ <i>Q</i> .	If <i>P</i> then <i>Q</i> , but <i>Q</i> , ∴ <i>P</i> .
<i>Tollendo Tollens</i>	If <i>Q</i> then <i>P</i> , but not <i>P</i> , ∴ not <i>Q</i> .	If <i>Q</i> then <i>P</i> , but not <i>Q</i> , ∴ not <i>P</i> .
<i>Tollendo Ponens</i>	Either <i>P</i> or <i>Q</i> , but not <i>P</i> , ∴ <i>Q</i> .	Not both <i>P</i> and <i>Q</i> , but not <i>Q</i> , ∴ <i>P</i> .
<i>Ponendo Tollens</i>	Not both <i>P</i> and <i>Q</i> , but <i>P</i> , ∴ not <i>Q</i> .	Either <i>P</i> or <i>Q</i> , but <i>Q</i> , ∴ not <i>P</i> .

The above valid forms are mutually reducible to one another, and the same is true of the invalid forms.

¹ This is in the stricter sense a *disjunctive* syllogism, the *modus tollendo*

246. *The Dilemma.*—The proper place of the dilemma amongst hypothetical and disjunctive arguments is difficult to determine, inasmuch as conflicting definitions are given by different logicians. The following definition may be taken as perhaps on the whole the most satisfactory:—A *dilemma* is a formal argument containing a premiss in which two or more hypotheticals are conjunctively affirmed, and a second premiss in which the antecedents of these hypotheticals are alternatively affirmed or their consequents alternatively denied¹. These premisses are usually called the major and the minor respectively².

Dilemmas are called *constructive* or *destructive* according as the minor premiss alternatively affirms the antecedents, or denies the consequents, of the major³.

Since it is a distinguishing characteristic of the dilemma that the minor should be alternative, it follows that the hypotheticals into which the major premiss of a *constructive dilemma* may be resolved must contain at least two distinct antecedents. They may, however, have a common consequent. The conclusion of the dilemma will then categorically affirm

ponens being an *alternative* syllogism. The reader must, however, be careful to remember that the latter is what is ordinarily meant by the disjunctive syllogism in logical text-books.

¹ In the strict use of the term, a *dilemma* implies only two alternants in the alternative premiss; if there are more than two alternants we have a *trilemma*, or a *tetralemma*, or a *polylemma*, as the case may be.

² This application of the terms *major* and *minor* is somewhat arbitrary. The *dilemmatic* force of the argument is indeed made more apparent by stating the alternative premiss (*i.e.*, the so-called *minor* premiss) first. It will, however, be remembered that, in the view of some logicians, the force of the ordinary categorical syllogism also is made more apparent by stating the minor premiss first. Compare section 147.

³ A further form of argument may be distinguished in which the alternation contained in the so-called minor premiss is affirmed only hypothetically, and in which, therefore, the conclusion also is hypothetical. For example,

If *A* is *B*, *E* is *F*; and if *C* is *D*, *E* is *F*;

If *X* is *Y*, either *A* is *B* or *C* is *D*;

therefore, If *X* is *Y*, *E* is *F*.

This might be called the *hypothetical dilemma*. It admits of varieties corresponding to the varieties of the ordinary dilemma; but no detailed treatment of it seems called for.

this consequent, and will correspond with it in form¹. The dilemma itself is in this case called *simple*. If, on the other hand, the major premiss contains more than one consequent, the conclusion will necessarily be alternative, and the dilemma is called *complex*.

Similarly, in a *destructive dilemma* the hypotheticals into which the major can be resolved must have more than one consequent, but they may or may not have a common antecedent; and the dilemma will be *simple* or *complex* accordingly.

We have then four forms of dilemma as follows :

(i) The *simple constructive dilemma*.

If *A* is *B*, *E* is *F*; and if *C* is *D*, *E* is *F*;
but Either *A* is *B* or *C* is *D*;
therefore, *E* is *F*.

(ii) The *complex constructive dilemma*.

If *A* is *B*, *E* is *F*; and if *C* is *D*, *G* is *H*;
but Either *A* is *B* or *C* is *D*;
therefore, Either *E* is *F* or *G* is *H*².

(iii) The *simple destructive dilemma*.

If *A* is *B*, *C* is *D*; and if *A* is *B*, *E* is *F*;
but Either *C* is not *D* or *E* is not *F*;
therefore, *A* is not *B*.

(iv) The *complex destructive dilemma*.

If *A* is *B*, *E* is *F*; and if *C* is *D*, *G* is *H*;
but Either *E* is not *F* or *G* is not *H*;
therefore, Either *A* is not *B* or *C* is not *D*³.

¹ It will usually be a simple categorical; but see the following note.

² The following is a simple, not a complex, constructive dilemma :

If *A* is *B*, *E* is *F* or *G* is *H*; and if *C* is *D*, *E* is *F* or *G* is *H*;
but Either *A* is *B* or *C* is *D*;
therefore, Either *E* is *F* or *G* is *H*.

The hypotheticals which here constitute the major premiss have a common consequent; but since this is itself alternative, the conclusion appears in the alternative form. This case is analogous to the following—All *M* is *P* or *Q*, All *S* is *M*, therefore, All *S* is *P* or *Q*—where the conclusion of an intrinsically categorical syllogism also appears in the alternative form. Compare the note on p. 312.

³ The following is a simple, not a complex, destructive dilemma :

In the case of dilemmas, as in the case of hypothetico-categorical syllogisms, the constructive form may be reduced to the destructive form, and vice versâ. All that has to be done is to contraposit the hypotheticals which constitute the major premiss. One example will suffice. Taking the simple constructive dilemma above given, and contraposing the major, we have—

If E is not F, A is not B; and if E is not F, C is not D;
 but Either A is B or C is D;
 therefore, E is F;

and this is a dilemma in the simple destructive form.

The definition of the dilemma above given is practically identical with that given by Dr Fowler (*Deductive Logic*, p. 116). Mansel (*Aldrich*, p. 108) defines the dilemma as “a syllogism having a conditional (hypothetical) major premiss *with more than one antecedent*, and a disjunctive minor.” Equivalent definitions are given by Whately and Jevons. According to this view, while the constructive dilemma may be either simple or complex, the destructive dilemma must always be complex, since in the corresponding simple form (as in the example given on p. 317) there is *only one antecedent* in the major. This exclusion seems arbitrary and is a ground for rejecting the definition in question. Whately, indeed, regards the name dilemma as necessarily implying *two antecedents*; but it should rather be regarded as implying *two alternatives*, each of which is equally distasteful. Whately goes on to assert that the excluded form is merely a destructive hypothetical syllogism, similar to the following,

If A is B, C is D;
 C is not D;
 therefore, A is not B.

But the two really differ precisely as the simple constructive

If both P and Q are true then X is true, and under the same hypothesis Y is true;
 but Either X or Y is not true;
 therefore, Either P or Q is not true.

Here we have but a single antecedent. For *If both P and Q are true then X is true* cannot be resolved into two distinct hypotheticals.

dilemma given on p. 317, differs from the constructive hypothetical syllogism,

If A is B, E is F;

A is B;

therefore, *E is F.*

Besides, it is clear that the form under discussion is not merely a destructive hypothetical syllogism such as has been already discussed, since the premiss which is combined with the hypothetical premiss is not categorical but alternative¹.

The following definition is sometimes given:—"The dilemma (or trilemma or polylemma) is an argument in which a choice is allowed between two (or three or more) alternatives, but it is shewn that whichever alternative is taken the same conclusion follows." This definition, which no doubt gives point to the expression "the horns of a dilemma," includes the simple constructive dilemma and the simple destructive dilemma; but it does not allow that either of the complex dilemmas is properly

¹ The argument—

If A is B, C is D and E is F;

but *Either C is not D or E is not F;*

therefore, *A is not B;*

must be distinguished from the following—

If A is B, C is D and E is F;

but *C is not D and E is not F;*

therefore, *A is not B.*

The former is a simple destructive dilemma, but in the latter no alternative is given at all, and the reasoning is equivalent to two simple hypothetical syllogisms, yielding the same conclusion, namely,

(1) *If A is B, C is D;*

but *C is not D;*

therefore, *A is not B.*

(2) *If A is B, E is F;*

but *E is not F;*

therefore, *A is not B.*

Similarly, the simple constructive dilemma given on p. 317 must be distinguished from the following argument:

If A is B, E is F; and if C is D, E is F;

but *A is B and C is D;*

therefore, *E is F.*

Here again we practically have two simple hypothetical syllogisms both yielding the same conclusion.

so-called, since in each case we are left with the same number of alternants in the conclusion as are contained in the alternative premiss. On the other hand, it embraces forms that are excluded by both the preceding definitions; for example, the following reasoning—which should rather be classed simply as a destructive hypothetico-categorical syllogism—

If A is, either B or C is;
but Neither B nor C is;
 therefore, *A is not*¹.

Jevons (*Elements of Logic*, p. 168) remarks that “dilemmatic arguments are more often fallacious than not, because it is seldom possible to find instances where two alternatives exhaust all the possible cases, unless indeed one of them be the simple negative of the other.” In other words, many dilemmatic arguments will be found to contain a premiss involving a fallacy of incomplete alternation. It should, however, be observed that in strictness a syllogistic argument is not itself to be called fallacious because it contains a false premiss. The fallacy that Jevons has in view is a material rather than a formal fallacy.

¹ Compare Ueberweg, *Logic*, § 123.

Hamilton (*Logic*, i. p. 350) defines the dilemma as “a syllogism in which the sumption (major) is at once hypothetical and disjunctive, and the subsumption (minor) sublates the whole disjunction, as a consequent, so that the antecedent is sublated in the conclusion.” This involved definition appears to have chiefly in view the form just given; but it excludes the following—which is one of the typical forms of dilemma according to all the preceding definitions—*If A is then C is, and if B is then C is; but either A or B is; therefore, C is.*

Thomson (*Laws of Thought*, p. 203) gives the following,—“A dilemma is a syllogism with a conditional (hypothetical) premiss, in which either the antecedent or the consequent is disjunctive.” This definition, however, is probably wider than Thomson himself intended. It would include the following argument—*If A is B then C is D or E is F, but A is B, therefore, C is D or E is F.*

EXERCISES.

247. Express the following argument symbolically, and determine to what type it belongs:—

The cause must either precede the effect, or be simultaneous with it, or succeed it. The last supposition is absurd; and the second would render it impossible to distinguish the cause from the effect. On the first supposition the cause must cease before the effect comes into being; but, surely, that which is not cannot be a cause. Either, then, there is no cause for any effect, or we are unable to discover it. [C.]

248. What can be inferred from the premisses, *Either A is B or C is D, Either C is not D or E is F*? Exhibit the reasoning (a) in the form of a hypothetical syllogism, (b) in the form of a dilemma. [K.]

249. Discuss the logical conclusiveness of fatalistic reasoning like this:—If I am fated to be drowned now, there is no use in my struggling; if not, there is no need of it. But either I am fated to be drowned now or I am not; so that it is either useless or needless for me to struggle against it. [B.]

~~All A is e~~
 A All men are either wise or ^{not} wise
 E No man are either wise or

CHAPTER VII.

IRREGULAR AND COMPOUND SYLLOGISMS.

250. *The Enthymeme.*—By the enthymeme, Aristotle meant what has been called the “rhetorical syllogism” as opposed to the apodeictic, demonstrative, theoretical syllogism. The following is from Mansel’s notes to *Aldrich* (pp. 209 to 211): “The enthymeme is defined by Aristotle, συλλογισμὸς ἐξ εἰκότων ἢ σημείων. The εἰκὸς and σημείον themselves are propositions; the former stating a *general probability*, the latter a *fact*, which is known to be an indication, more or less certain, of the truth of some further statement, whether of a single fact or of a general belief. The former is a proposition nearly, though not quite, *universal*; as ‘Most men who envy hate’: the latter is a *singular* proposition, which however is not regarded as a sign, except relatively to some other proposition, which it is supposed may be inferred from it. The εἰκὸς, when employed in an enthymeme, will form the *major premiss* of a syllogism such as the following:

Most men who envy hate,
This man envies,
therefore, This man (probably) hates.

“The reasoning is logically faulty; for, the major premiss not being absolutely universal, the middle term is not distributed.

“The σημείον will form one premiss of a syllogism which may be in any of the three figures, as in the following examples:

Figure 1. All ambitious men are liberal,
Pittacus is ambitious,
therefore, Pittacus is liberal.

Figure 2. All ambitious men are liberal,
Pittacus is liberal,
therefore, Pittacus is ambitious.

Figure 3. Pittacus is liberal,
Pittacus is ambitious,
therefore, All ambitious men are liberal.

"The syllogism in the first figure alone is logically valid. In the second, there is an undistributed middle term; in the third, an illicit process of the minor."

On this subject the student may be referred to the remainder of the note from which the above extract is taken, and to Hamilton, *Discussions*, pp. 152 to 156¹.

An *enthymeme* is now usually defined as a syllogism incompletely stated, one of the premisses or the conclusion being understood but not expressed². The arguments of everyday

¹ Karslake (*Aids to the Study of Logic*, Book II.) after mentioning that with Aristotle the conditions of the existence of *Demonstration* are "that its *matter* be certain, its *method* Deduction, and its *end* scientific certainty" (p. 32), gives the following account of the Aristotelian enthymeme: "Whereas the mathematician, e.g., wishing to *demonstrate* his point, will give all the grounds on which his conclusion rests, and the grounds on which these grounds rest, and so on, till he comes to some primary principles which all will admit; the rhetorician, on the contrary, will not pursue his proofs in all their ramifications up to their primary source, but will *assume* a great deal in order to avoid unnecessarily complicating his speech..... 'What the Example is to Induction, that the Enthymeme is to Deduction' is Aristotle's language always: and since the Example is Induction upon practical matters, the Enthymeme must be Deduction upon practical matters correspondingly. Its *matter* is practical: its *method* Deduction: its *end* persuasion. It is therefore one branch, but the most important branch, of what is called by Aristotle the Dialectical or Topical Syllogism, that is, of reasoning in those cases where the grounds are not *certain*, but *probable* only, and where the end sought after is *opinion*; and therefore it is pretty much the correlative of *Demonstration*, extending nearly over the whole sphere of probable, as *Demonstration* extended over the whole of certain, truth" (pp. 52, 3).

² This account of the enthymeme appears to have been originally based on the erroneous idea that the name signified the retention of one premiss *in the mind*, ἐν θυμῷ. Thus, in the *Port Royal Logic*, an enthymeme is described as "a syllogism perfect in the mind, but imperfect in the expression, since some one of the propositions is suppressed as too clear and too well known, and as being easily supplied by the mind of those to whom we speak" (p. 229). As regards the real origin of the name enthymeme, see Mansel's *Aldrich*, p. 218.

life are for the most part enthymematic; and the same may be said of fallacious arguments, which are seldom completely stated, or their want of cogency would be more quickly recognised.

An enthymeme is said to be of the *first order* when the major premiss is suppressed; of the *second order* when the minor premiss is suppressed; and of the *third order* when the conclusion is suppressed.

Thus, "Balbus is avaricious, and therefore, he is unhappy," is an enthymeme of the first order; "All avaricious persons are unhappy, and therefore, Balbus is unhappy," is an enthymeme of the second order; "All avaricious persons are unhappy, and Balbus is avaricious," is an enthymeme of the third order.

251. *The Polysyllogism.*—A chain of syllogisms, that is, a series of syllogisms so linked together that the conclusion of one becomes a premiss of another, is called a *polysyllogism*. In a polysyllogism, any individual syllogism the conclusion of which becomes the premiss of a succeeding one is called a *prosyllogism*; any individual syllogism one of the premisses of which is the conclusion of a preceding syllogism is called an *episyllogism*. Thus,—

All C is D,	}	prosyllogism,
All B is C,		
therefore, All B is D,	}	episyllogism.
but All A is B,		
therefore, All A is D.		

The same syllogism may of course be both an episyllogism and a prosyllogism, as would be the case with the above episyllogism if the chain were continued further.

A chain of reasoning¹ is said to be *progressive* (or *synthetic* or *episyllogistic*) when the progress is from prosyllogism to episyllogism. Here the premisses are first given, and we pass on by successive steps of inference to the conclusions which

¹ The distinction which follows is ordinarily applied to chains of reasoning only; but the reader will observe that it admits of application to the case of the simple syllogism also.

they yield. A chain of reasoning is, on the other hand, said to be *regressive* (or *analytic* or *prosyllogistic*) when the progress is from episyllogism to prosyllogism. Here the ultimate conclusion is first given and we pass back by successive steps of proof to the premisses on which it may be based.

In the analysis of an argument, it is no doubt important to enquire whether the object had in view is to establish a given thesis or to determine what follows from given premisses. But from the purely formal standpoint, the above distinction resolves itself merely into a difference in order of statement. In the systematic treatment of forms of inference it is convenient and usual to adopt the *progressive* order¹.

252. *The Epicheirema.*—An *epicheirema* is a polysyllogism with one or more prosyllogisms briefly indicated only. That is, one or more of the syllogisms of which the polysyllogism is composed are enthymematic. The following is an example,

All B is D, because it is C,
All A is B,
 therefore, *All A is D*².

253. *The Sorites.*—A *sorites* is a polysyllogism in which all the conclusions are omitted except the final one, the premisses being given in such an order that any two successive propositions contain a common term. Two forms of sorites are usually recognised, namely, the so-called *Aristotelian sorites* and the *Goclenian sorites*. In the former, the premiss stated first contains the subject of the conclusion, while the term

¹ On the distinction between progressive and regressive arguments, see Ueberweg, *Logic*, § 124.

² A distinction has been drawn between *single* and *double* epicheiremas according as reasons are enthymematically given in support of *one* or *both* of the premisses of the ultimate syllogism. The example given in the text is a single epicheirema; the following is an example of a double epicheirema:

All P is Y, because it is X;
All S is P, because all M is P;
 therefore, *All S is Y.*

The epicheirema is sometimes defined as if it were essentially a *regressive* chain of reasoning. But this is hardly correct, if, as is usually the case, examples such as the above are given; for it is clear that in these examples the argument is only partly regressive.

common to any two successive premisses occurs first as predicate and then as subject; in the latter, the premiss stated first contains the predicate of the conclusion, while the term common to any two successive premisses occurs first as subject and then as predicate. The following are examples:

Aristotelian Sorites—All *A* is *B*,

All *B* is *C*,

All *C* is *D*,

All *D* is *E*,

therefore, All *A* is *E*.

Goclenian Sorites—All *D* is *E*,

All *C* is *D*,

All *B* is *C*,

All *A* is *B*,

therefore, All *A* is *E*.

It will be found that, in the case of the *Aristotelian* sorites, if the argument is drawn out in full, the first premiss and the suppressed conclusions all appear as *minor* premisses in successive syllogisms. Thus, the *Aristotelian* sorites given above may be analysed into the three following syllogisms—

(1) All *B* is *C*,
All *A* is *B*,
therefore, All *A* is *C*;

(2) All *C* is *D*,
All *A* is *C*,
therefore, All *A* is *D*;

(3) All *D* is *E*,
All *A* is *D*,
therefore, All *A* is *E*.

Here the premiss originally stated first is the minor premiss of (1), the conclusion of (1) is the minor premiss of (2), that of (2) the minor premiss of (3); and so it would go on if the number of propositions constituting the sorites were increased.

In the *Goclenian* sorites, the premisses are the same, but their order is reversed, and the result of this is that the premiss originally stated first and the suppressed conclusions

become *major* premisses in successive syllogisms. Thus, the Goclenian sorites given above may be analysed into the three following syllogisms—

- (1) *All D is E,*
All C is D,
 therefore, *All C is E;*
- (2) *All C is E,*
All B is C,
 therefore, *All B is E;*
- (3) *All B is E,*
All A is B,
 therefore, *All A is E.*

Here the premiss originally stated first is the major premiss of (1), the conclusion of (1) is the major premiss of (2); and so on.

The so-called Aristotelian sorites¹ is that to which the greater prominence is usually given; but it will be observed that the order of premisses in the Goclenian form is that which really corresponds to the customary order of premisses in a simple syllogism².

¹ This form of sorites ought not properly to be called *Aristotelian*; but it is generally so described in logical text-books. The name *sorites* is not to be found in any logical treatise of Aristotle, though in one place he refers vaguely to the form of reasoning which the name is now employed to express. The distinct exposition of this form of reasoning is attributed to the Stoics, and it is called by the name *sorites* by Cicero; but it was not till much later that the name came into general use amongst logicians in this sense. The form of sorites called the Goclenian was first given by Professor Rudolf Goclenius of Marburg (1547 to 1628) in his *Isagoge in Organum Aristotelis*, 1598. Compare Hamilton, *Logic*, i. p. 375; and Ueberweg, *Logic*, § 125. It may be added that the term *sorites* (which is derived from *σωρός*, a heap) was used by ancient writers in a different sense, namely, to designate a particular sophism, based on the difficulty which is sometimes found in assigning an exact limit to a notion. "It was asked,—was a man bald who had so many thousand hairs; you answer, No: the antagonist goes on diminishing and diminishing the number, till either you admit that he who was not bald with a certain number of hairs, becomes bald when that complement is diminished by a single hair; or you go on denying him to be bald, until his head be hypothetically denuded." A similar puzzle is involved in the question,—On what day does a lamb become a sheep? Sorites in this sense is also called *sophisma polyzeteseos* or *fallacy of continuous questioning*. See Hamilton, *Logic*, i. p. 464.

² The mistake is sometimes made of speaking of the Goclenian sorites as a

A sorites may of course consist of conditional or hypothetical propositions; and it is not at all unusual to find propositions of these kinds combined in this manner. Theoretically a sorites might also consist of alternative propositions; but it is not likely that this combination would ever occur naturally.

254. *The special rules of the Sorites.*—The following special rules may be given for the ordinary Aristotelian sorites, as defined in the preceding section:—

(1) Only one premiss can be negative; and if one is negative, it must be the last.

(2) Only one premiss can be particular; and if one is particular, it must be the first.

Any Aristotelian sorites may be represented in skeleton form, the quantity and quality of the premisses being left undetermined, as follows:—

S	M_1
M_1	M_2
M_2	M_3
.....	
.....	
M_{n-2}	M_{n-1}
M_{n-1}	M_n
M_n	P
<hr/>	
S	P

(1) There cannot be more than one negative premiss, for if there were—since a negative premiss in any syllogism necessitates a negative conclusion—we should in analysing the sorites somewhere come upon a syllogism containing two negative premisses.

Again, if one premiss is negative, the final conclusion must be negative. Hence, P must be distributed in this conclusion. Therefore, it must be distributed in its premiss, *i.e.*, the last premiss, which must accordingly be negative. If any premiss then is negative, this is the one.

regressive form of argument. It is clear, however, that in both forms of sorites we pass continuously from premisses to conclusions, not from conclusions to premisses.

(2) Since it has been shewn that all the premisses, except the last, must be affirmative, it is clear that if any, except the first, were particular, we should somewhere commit the fallacy of undistributed middle.

The special rules of the Goclenian sorites, as defined in the preceding section, may be obtained by transposing "first" and "last" in the above.

255. *The possibility of a Sorites in a Figure other than the First.*—It will have been noticed that in analysing both the Aristotelian and the Goclenian sorites all the resulting syllogisms are in figure 1. Such sorites may accordingly be said to be themselves in figure 1. The question arises whether a sorites is possible in any other figure.

The usual answer to this question is that the first or last syllogism of a sorites may be in figure 2 or 3 (*e.g.*, in figure 2 we may have *A is B, B is C, C is D, D is E, F is not E, therefore, A is not F*) but that it is impossible that all the steps should be in either of these figures¹. "Every one," says Mill, "who

¹ Sir William Hamilton indeed professes to give sorites in the second and third figures, which have, he says, been overlooked by other logicians (*Logic*, II. p. 403). It appears, however, that by a sorites in the second figure he means such a reasoning as the following:—*No B is A, No C is A, No D is A, No E is A, All F is A, therefore, No B, or C, or D or E, is F*; and by a sorites in the third figure such as the following:—*A is B, A is C, A is D, A is E, A is F, therefore, Some B, and C, and D, and E, are F*. He does not himself give these examples; but that they are of the kind which he intends may be deduced from his not very lucid statement, "In second and third figures, there being no subordination of terms, the only sorites competent is that by repetition of the same middle. In first figure, there is a new middle term for every new progress of the sorites; in second and third, only one middle term for any number of extremes. In first figure, a syllogism only between every second term of the sorites, the intermediate term constituting the middle term. In the others, every two propositions of the common middle term form a syllogism." But it is clear that in the accepted sense of the term these are not sorites at all. In each case the conclusion is a mere summation of the conclusions of a number of syllogisms having a common premiss; in neither case is there any chain argument. Hamilton's own definition of the sorites, involved as it is, might have saved him from this error. He gives for his definition, "When, on the common principle of all reasoning,—that the part of a part is a part of the whole,—we do not stop at the second gradation, or at the part of the highest part, and conclude that part of the whole, but proceed to some indefinitely remoter part, as *D, E, F, G, H, &c.*, which, on the general principle, we connect in the

understands the laws of the second and third figures (or even the general laws of the syllogism) can see that no more than one step in either of them is admissible in a sorites, and that it must either be the first or the last" (*Examination of Hamilton*, pp. 514, 5).

This treatment of the question seems, however, open to refutation by the simple method of constructing examples. Take, for instance, the following sorites:—

- (i) *Some S is not M₁,*
All M₂ is M₁,
All M₃ is M₂,
All M₄ is M₃,
All P is M₄,

therefore, *Some S is not P.*

- (ii) *Some M₄ is not P,*
All M₄ is M₃,
All M₃ is M₂,
All M₂ is M₁,
All M₁ is S,

therefore, *Some S is not P.*

Analysing the first of the above, and inserting the suppressed conclusions in square brackets, we have—

- Some S is not M₁,*
All M₂ is M₁,
 [therefore, *Some S is not M₂.]*
All M₃ is M₂,
 [therefore, *Some S is not M₃.]*
All M₄ is M₃,
 [therefore, *Some S is not M₄.]*
All P is M₄,
 therefore, *Some S is not P.*

conclusion with its remotest whole,—this complex reasoning is called a *Chain-Syllogism* or *Sorites*" (*Logic*, i. p. 366). In connexion with Hamilton's treatment of this question, Mill very justly remarks, "If Sir W. Hamilton had found in any other writer such a misuse of logical language as he is here guilty of, he would have roundly accused him of total ignorance of logical writers" (*Examination of Hamilton*, p. 515).

This is the only resolution of the sorites possible unless the order of the premisses is transposed, and it will be seen that all the resulting syllogisms are in figure 2 and in the mood *Baroco*. The sorites may accordingly be said to be in the same mood and figure. It is analogous to the Aristotelian sorites, the subject of the conclusion appearing in the premiss stated first, and the suppressed premisses being all *minors* in their respective syllogisms.

The corresponding analysis of (ii) yields the following :—

Some M₄ is not P,
All M₄ is M₃,
 [therefore, *Some M₃ is not P,*]
All M₃ is M₂,
 [therefore, *Some M₂ is not P,*]
All M₂ is M₁,
 [therefore, *Some M₁ is not P,*]
All M₁ is S,
 therefore, *Some S is not P.*

These syllogisms are all in figure 3 and in the mood *Bocardo*; and the sorites itself may be said to be in the same mood and figure. It is analogous to the Goclenian sorites, the predicate of the conclusion appearing in the premiss stated first, and the suppressed premisses being *majors* in their respective syllogisms.

It will be observed that the rules given in the preceding section have not been satisfied in either of the above sorites, the reason being that the rules in question correspond to the special rules of figure 1, and do not apply unless the sorites is in that figure. For such sorites as are possible in figures 2, 3, and 4, other rules might be framed corresponding to the special rules of these figures in the case of the simple syllogism.

It is not maintained that sorites in other figures than the first are likely to be met with in common use, but their construction is of some theoretical interest¹.

¹ The examples given in the text have been purposely chosen so as to admit of only one analysis, which was not the case with the examples given in previous editions. The old examples were, however, perfectly valid, and

256. *Ultra-total Distribution of the Middle Term.*—The ordinary syllogistic rule relating to the distribution of the middle term does not contemplate the recognition of any signs of quantity other than *all* and *some*; and if other signs are recognised, the rule must be modified. For example, the admission of the sign *most* yields the following valid reasoning, although the middle term is not distributed in either of the premisses:—

Most M is P,
Most M is S,
 therefore, *Some S is P.*

Interpreting *most* in the sense of *more than half*, it clearly further light may be thrown on the general question by a brief reply to certain criticisms passed upon those examples. The following was given for figure 2 (the suppressed conclusions being inserted in square brackets), and it was said to be analogous to the Aristotelian sorites:—

All A is B,
No C is B,
 [therefore, *No A is C*],
All D is C,
 [therefore, *No A is D*],
All E is D,
 therefore, *No A is E.*

It has, to begin with, been objected that the above is Goelenian, and not Aristotelian, in form, “the subject of each premiss after the first being the predicate of the succeeding one.” This overlooks the more fundamental characteristic of the Aristotelian sorites, that the first premiss and the suppressed conclusions are all *minors* in their respective syllogisms. It has further been objected that the following analysis might serve in lieu of the one above given:—*AaB, CeB, [∴ CeA,] DaC, [∴ DeA,] EaD, ∴ AeE.* No doubt this analysis is a possible one, but the objection to it is its heterogeneous character. The first premiss and the first suppressed conclusion are majors, while the last suppressed conclusion is a minor. Again, the first syllogism is in figure 2, the second in figure 1, and the third in figure 4. It must be granted that what has been above called a heterogeneous analysis is in some cases the only one available, but it is better to adopt something more homogeneous where possible. If the first premiss of a sorites contains the subject, and the last the predicate, of the conclusion, then the last premiss is necessarily the major of the final syllogism; and hence the rule may be laid down that we can work out such a sorites homogeneously only by treating the first premiss and all the suppressed conclusions as minors, and all the remaining premisses as majors, in their respective syllogisms. A corresponding rule may be laid down if the first premiss contains the predicate, and the last the subject, of the conclusion.

follows from the above premisses that there must be some *M* which is both *S* and *P*. But we cannot say that in either premiss the term *M* is distributed.

In order to meet cases of this kind, Sir W. Hamilton (*Logic*, vol. II., p. 362) gives the following modification of the rule relating to the distribution of the middle term: "The quantifications of the middle term, whether as subject or predicate, taken together, must exceed the quantity of that term taken in its whole extent"; in other words, we must have an ultra-total distribution of the middle term in the two premisses taken together. Hamilton then continues somewhat too dogmatically, "The rule of the logicians, that the middle term should be once at least distributed, is untrue. For it is sufficient if, in both the premisses together, its quantification be more than its quantity as a whole (ultra-total). Therefore, a *major part* (a *more* or *most*) in one premiss, and a *half* in the other, are sufficient to make it effective."

De Morgan (*Formal Logic*, p. 127) writes as follows: "It is said that in every syllogism the middle term must be universal in one of the premisses, in order that we may be sure that the affirmation or denial in the other premiss may be made of some or all of the things about which affirmation or denial has been made in the first. This law, as we shall see, is only a particular case of the truth: it is enough that the two premisses together affirm or deny of more than all the instances of the middle term. If there be a hundred boxes, into which a hundred *and one* articles of two different kinds are to be put, not more than one of each kind into any one box, some one box, if not more, will have two articles, one of each kind, put into it. The common doctrine has it, that an article of one particular kind must be put into every box, and then some one or more of another kind into one or more of the boxes, before it may be affirmed that one or more of different kinds are found together." De Morgan himself works the question out in detail in his treatment of *the numerically definite syllogism* (*Formal Logic*, pp. 141 to 170). The following may be taken as an example of numerically definite reasoning:—If 70 per cent. of *M* are *P*, and 60 per cent are *S*, then at least

30 per cent. are both S and P ¹. The argument may be put as follows: On the average, of 100 M 's 70 are P and 60 are S ; suppose that the 30 M 's which are not P are S , still 30 S 's are to be found in the remaining 70 M 's which are P 's; and this is the desired conclusion. Problems of this kind constitute a borderland between formal logic and algebra. Some further examples will be given in chapter 9 (section 293).

257. The Quantification of the Predicate and the Syllogism.—It will be convenient to consider briefly in this chapter the application of the doctrine of the quantification of the predicate to the syllogism; the result is the reverse of simplification². The most important points that arise may be brought out by considering the validity of the following syllogisms: in figure 1, UUU , $IU\eta$, AYI ; in figure 2, ηUO , AUA ; in figure 3, YAI . In the next section we will proceed more systematically, U and ω being left out of account.

(1) UUU in figure 1 is valid:—

¹ Using other letters, this is the example given by Mill, *Logic*, book 2, chapter 2, § 1, *note*, and quoted by Herbert Spencer, *Principles of Psychology*, vol. 2, p. 88. The more general problem of which the above is a special instance is as follows: Given that there are n M 's in existence, and that a M 's are S while b M 's are P , to determine what is the least number of S 's that are also P 's. It is clear that we have no conclusion at all unless $a + b > n$, *i.e.*, unless there is ultra-total distribution of the middle term. If this condition is satisfied, then supposing the $(n - b)$ M 's which are *not*- P are all of them found amongst the MS 's, there will still be some MS 's left which are P 's, namely, $a - (n - b)$. Hence the least number of S 's that are also P 's must be $a + b - n$.

² In connexion with his doctrine of the quantification of the predicate, Hamilton distinguishes between the *figured syllogism* and the *unfigured syllogism*. In the *figured syllogism*, the distinction between subject and predicate is retained, as in the text. By a rigid quantification of the predicate, however, the distinction between subject and predicate may be dispensed with; and such being the case there is no ground left for distinction of figure (which depends upon the position of the middle term as subject or predicate in the premisses). This gives what Hamilton calls the *unfigured syllogism*. For example:—Any bashfulness and any praiseworthy are not equivalent, All modesty and some praiseworthy are equivalent, therefore, Any bashfulness and any modesty are not equivalent; All whales and some mammals are equal, All whales and some water animals are equal, therefore, Some mammals and some water animals are equal. A distinct canon for the unfigured syllogism is given by Hamilton as follows:—“In as far as two notions either both agree, or one agreeing the other does not, with a common third notion; in so far these notions do or do not agree with each other.”

All M is all P,
All S is all M,
 therefore, *All S is all P.*

It will be observed that whenever one of the premisses is **U**, the conclusion may be obtained by substituting *S* or *P* (as the case may be) for *M* in the other premiss.

Without the use of quantified predicates, the above reasoning may be expressed by means of the two following syllogisms:

<i>All M is P,</i>	<i>All M is S,</i>
<i>All S is M,</i>	<i>All P is M,</i>
therefore, <i>All S is P;</i>	therefore, <i>All P is S.</i>

(2) **IU_η** in figure 1 is invalid, if *some* is used in its ordinary logical sense. The premisses are *Some M is some P* and *All S is all M*. We may, therefore, obtain the legitimate conclusion by substituting *S* for *M* in the major premiss. This yields *Some S is some P*.

If, however, *some* is here used in the sense of *some only*, *No S is some P* follows from *Some S is some P*, and the original syllogism is valid, although a negative conclusion is obtained from two affirmative premisses.

This syllogism is given as valid by Thomson (*Laws of Thought*, § 103); but apparently only through a misprint for **IE_η**. In his scheme of valid syllogisms (thirty-six in each figure) Thomson seems consistently to interpret *some* in its ordinary logical sense. Using the word in the sense of *some only*, several other syllogisms would be valid that he does not give as such¹.

(3) **AYI** in figure 1, *some* being used in its ordinary logical sense, is equivalent to **AAI** in figure 3 in the ordinary syllogistic scheme, and it is therefore valid. But it is invalid if *some* is used in the sense of *some only*, for the conclusion now implies that *S* and *P* are partially excluded from each other as well as partially coincident, whereas this is not implied by the premisses. With this use of *some*, the correct conclusion can be expressed only by stating an alternative between *SuP*, *SaP*, *SyP*, and *SiP*. This case may serve to illustrate the

¹ Compare section 106.

complexities in which we should be involved if we were to attempt to use *some* consistently in the sense of *some only*¹.

(4) $\neg\mathbf{UO}$ in figure 2 is valid :—

No P is some M,
All S is all M,
therefore, *Some S is not any P.*

Without the use of quantified predicates, we can obtain an equivalent argument in *Bocardo*, thus—

Some M is not P,
All M is S,
therefore, *Some S is not P.*

(5) \mathbf{AUA} in figure 2 runs as follows :—

All P is some M,
All S is all M,
therefore, *All S is some P.*

Here we have neither undistributed middle nor illicit process of major or minor, nor is any rule of quality broken, and yet the syllogism is invalid². Applying the rule given above that “whenever one of the premisses is \mathbf{U} , the conclusion may be obtained by substituting *S* or *P* (as the case may be) for *M* in the other premiss,” we find that the valid conclusion is *Some S is all P*. More generally, it follows from this rule of substitution that *if one premiss is \mathbf{U} while in the other premiss the middle term is undistributed, then the term combined with the middle term in the \mathbf{U} premiss must be undistributed in the conclusion*. This appears to be the one additional syllogistic rule required if we recognise \mathbf{U} propositions in syllogistic reasonings.

All danger of fallacy is avoided by breaking up the \mathbf{U} proposition into two \mathbf{A} propositions. In the case before us we have—*All P is M, All M is S; All P is M, All S is M*. From the first of these pairs of premisses we get the con-

¹ Compare Monck, *Logic*, p. 154.

² We should have a corresponding case if we were to infer *No S is P* from the premisses given in the preceding example.

clusion *All P is S*; in the second pair the middle term is undistributed, and therefore no conclusion is yielded at all.

(6) **YAI** in figure 3 is valid:—

Some M is all P,
All M is some S,
 therefore, *Some S is some P.*

The conclusion is however weakened, since from the given premisses we might infer *Some S is all P*¹. It will be observed that when we quantify the predicate, the conclusion of a syllogism may be weakened in respect of its predicate as well as in respect of its subject. In the ordinary doctrine of the syllogism this is for obvious reasons not possible.

Without quantification of the predicate the above reasoning may be expressed in *Bramantip*, thus,

All P is M,
All M is S,
 therefore, *Some S is P.*

We could get the full conclusion, *All P is S*, in *Barbara*.

258. *Table of valid moods resulting from the recognition of Y and η in addition to A, E, I, O.*—If we adopt the sixfold schedule of propositions obtained by adding *Only S is P* (**Y**) and *Not only S is P* (η) to the ordinary fourfold schedule, as in section 112, every proposition is simply convertible, and, therefore, a valid mood in any figure is reducible to any other figure by the simple conversion of one or both of the premisses. Hence if the valid moods of any one figure are determined, those of the remaining figures may be immediately deduced therefrom.

It will be found that in each figure there are twelve valid moods, which are neither strengthened nor weakened. This result may be established by either of the two alternative methods which follow.

I. We may enquire what combinations of premisses will suffice to establish conclusions of the forms **A, Y, E, I, O, η** , respectively.

¹ Or, retaining the original conclusion, we might replace the major premiss by *Some M is some P*; hence, from another point of view, the syllogism may be regarded as strengthened.

It will suffice, as we have already seen, to consider some one figure. We may, therefore, take figure 1, so that the position of the terms will be—

$$\begin{array}{cc} M & P \\ S & M \\ \hline S & P \end{array}$$

(i) To prove *SaP*, both premisses must be affirmative; and, in order to avoid illicit minor, the minor premiss must be *SaM*. It follows that the major must be *MaP* or there would be undistributed middle. Hence **AAA** is the only valid mood yielding an **A** conclusion.

(ii) To prove *SyP*, both premisses must be affirmative; and, in order to avoid illicit major, the major premiss must be *MyP*. It follows that the minor must be *SyM*, in order to avoid undistributed middle. Hence **YYY** is the only valid mood yielding a **Y** conclusion.

(iii) To prove *SeP*, the major must be (1) *MeP* or (2) *MyP* or (3) *MoP* in order to avoid illicit major. If (1), the minor must be *SaM* or there would be either two negative premisses or illicit minor; if (2), it must be *SeM* or there would be undistributed middle or illicit minor; if (3), it must be affirmative and distribute both *S* and *M*, which is impossible. Hence **EAE** and **YEE** are the only valid moods yielding an **E** conclusion.

(iv) To prove *SiP*, both premisses must be affirmative, and since *SaM* would necessarily be a strengthened premiss, the minor must be (1) *SiM* or (2) *SyM*. If (1), the major must be *MaP* or there would be undistributed middle; and if (2), it must be *MiP* or there would be a strengthened premiss. Hence **AII** and **IYI** are the only valid moods yielding an **I** conclusion.

(v) To prove *SoP*, the major must be (1) *MeP* or (2) *MyP* or (3) *MoP* or there would be illicit major. If (1), the minor must be *SiM* or there would be a strengthened premiss; if (2), it must be *SoM* or there would be either two affirmative premisses with a negative conclusion or undistributed middle or a strengthened premiss; and if (3), it must be *SyM* or there would be two negative premisses or undistributed middle.

Hence **EIO**, **YOO**, **OYO** are the only valid moods yielding an **O** conclusion.

(vi) To prove $S\eta P$, the minor must be (1) SeM or (2) SaM or (3) $S\eta M$ or there would be illicit minor. If (1), the major must be MeP or there would be a strengthened premiss; if (2), the major must be $M\eta P$ or there would be undistributed middle or two affirmative premisses with a negative conclusion or a strengthened premiss; and if (3), the major must be MaP or there would be undistributed middle or two negative premisses. Hence **IE η** , **$\eta A\eta$** , **A $\eta\eta$** are the only valid moods yielding an η conclusion.

By converting one or both of the premisses we may at once deduce from the above a table of valid (unstrengthened and unweakened) moods for all four figures as follows:—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
AAA	YAA	AYA	YYA
YYY	AYY	YAY	AAηY
EAE	EAE	EYE	EYE
YEE	AEE	YEE	AEE
AII	YII	AII	YII
IYI	IYI	IAI	IAI
EIO	EIO	EIO	EIO
YOO	AOO	YηO	AηO
OYO	ηYO	OAO	ηAO
IEη	IEη	IEη	IEη
$\eta A\eta$	O$A\eta$	$\eta Y\eta$	O$Y\eta$
A$\eta\eta$	Y$\eta\eta$	AOη	YOη

II. The above table may also be obtained by (1) taking all the combinations of premisses that are *a priori* possible, (2) establishing special rules for the particular figure selected, which (taken together with the rules of quality) will enable us to exclude the combinations of premisses which are either invalid or strengthened whatever the conclusion may be, (3) assigning the valid unweakened conclusion in the remaining cases.

The following are all possible combinations of premisses, valid and invalid:

AA (b)	YA	IA	EA (b)	OA	η A (b) (c)
AY	YY (a)	IY (a)	EY	OY (a)	η Y
AI	YI (a)	II (a)	EI	OI (a)	η I (c)
AE (b)	YE	IE	[EE] (b)	[OE]	[η E] (b)
AO	YO (a)	IO (a)	[EO]	[OO] (a)	[η O]
A η (b) (c)	Y η	I η (c)	[E η] (b)	[O η]	[$\eta\eta$] (b) (c)

The combinations in square brackets are excluded by the rule that from two negative premisses nothing follows.

Taking the third figure, in which the middle term is subject in each premiss, and remembering that the subject is distributed in **A**, **E**, η and in these only, while the predicate is distributed in **Y**, **E**, **O** and in these only, the following special rules are obtainable:

(a) One premiss must be **A**, **E**, or η , or the middle term would not be distributed in either premiss;

(b) One premiss must be **Y**, **I**, or **O**, or the middle term would be distributed in both premisses, and there would hence be a strengthened premiss;

(c) If either premiss is negative, one of the premisses must be **Y**, **E**, or **O**, for otherwise (since the conclusion must be negative, distributing one of its terms) there would be illicit process either of major or minor.

These rules exclude the combinations of premisses marked respectively (a), (b), (c) above.

Assigning the valid unweakened conclusion in the case of each of the twelve combinations which remain, we have the following: **AYA**, **AI η** , **AO η** , **YAY**, **YEE**, **Y η O**, **IAI**, **IE η** , **EYE**, **EIO**, **OA η** , **η Y η** . From this, the table of valid (unstrengthened and unweakened) moods for all four figures may be expanded as before¹.

¹ By adopting a special set of symbols, suggested by Mr Johnson, the solution of the problem discussed in the above section may be generalised, so as to be quite independent of figure. Writing capitals for distributed terms, small letters for undistributed terms; + for affirmation, - for negation; and attaching to each proposition a number, determined by counting 4 for affirmation, 3 for distribution of the middle term, 2 for distribution of an extreme term; the six possible major premisses and the six possible minor premisses may be written as follows in descending order of probative strength:

259. *Formal Inferences not reducible to ordinary Syllogisms*¹.—The following is an example of what is usually called

7	6	5	4	3	2
MP^+	mP^+	MP^-	mp^+	Mp^-	mP^-
Ms^+	mS^+	MS^-	ms^+	Ms^-	mS^-

The conditions are that in the premisses there shall be one and only one extra distribution (which must include the distribution of the middle term), and one and only one extra affirmation, as compared with the conclusion. Hence the following rule for combining the premisses may be deduced: The sum of the premisses must be an *odd* number, *not less than nine*. For the sum of the premisses cannot be even, since the middle term would then be either twice distributed or twice undistributed; the combinations $5+2$, $3+2$, $2+5$, $2+3$ have to be rejected, because of double negatives; and the combinations $4+3$, $3+4$, because s and p being both undistributed, a negative conclusion is inadmissible.

The following combinations then remain valid: $7+6$; $7+4$; $7+2$; $6+7$; $6+5$; $6+3$; $5+6$; $5+4$; $4+7$; $4+5$; $3+6$; $2+7$; and the conclusion may in each case be obtained by multiplying the signs, and dropping M and m .

These results are shewn in the following diagram :

	7 MP^+	6 mP^+	5 MP^-	4 mp^+	3 Mp^-	2 mP^-
7 Ms^+		sP^+		sp^+		sP^-
6 mS^+	Sp^+		SP^-		Sp^-	
5 MS^-		SP^-		Sp^-		
4 ms^+	sp^+		sP^-			
3 Ms^-		sP^-				
2 mS^-	Sp^-					

There are thus twelve moods satisfying the required conditions in each figure. For example, $7+6$ yields in the four figures respectively, AAA , YAA , AYA , YYA .

¹ Attempts to reduce *immediate* inferences to syllogistic form have been already considered in section 76. In the present section, non-syllogistic *mediate* inferences will be considered.

the argument *à fortiori*:

B is greater than C,
A is greater than B,
 therefore, *A is greater than C.*

As this stands, it is clearly not in the ordinary syllogistic form since it contains four terms; some logicians, however, profess to reduce it to the ordinary syllogistic form as follows:

B is greater than C,
 therefore, *Whatever is greater than B is greater than C,*
 but *A is greater than B,*
 therefore, *A is greater than C.*

With De Morgan, we may treat this as a mere evasion, or as a *petitio principii*. The principle of the argument *à fortiori* is really assumed in passing from *B is greater than C* to *Whatever is greater than B is greater than C*.

The following attempted resolution¹ must be disposed of similarly:

Whatever is greater than a greater than C is greater than C,
A is greater than a greater than C,
 therefore, *A is greater than C.*

At any rate, it is clear that this cannot be the whole of the reasoning, since *B* no longer appears in the premisses at all.

The point at issue may perhaps be most clearly indicated by saying that whilst the ordinary syllogism may be based upon the *dictum de omni et nullo*, the argument *à fortiori* cannot be made to rest entirely upon this axiom. A new principle is required and one which must be placed on a par with the *dictum de omni et nullo*, not in subordination to it. This new principle may be expressed in the form, *Whatever is greater than a second thing which is greater than a third thing is itself greater than that third thing.*

Mansel (*Aldrich*, pp. 199, 200) treats the argument *à fortiori* as an example of a *material consequence* on the ground that it depends upon "some understood proposition or propositions, connecting the terms, by the addition of which the mind is enabled to reduce the consequence to logical form." He would

¹ Compare Mansel's *Aldrich*, p. 200.

effect the reduction in one of the ways already referred to. This, however, begs the question that the syllogistic is the only *logical* form. As a matter of fact the cogency of the argument *à fortiori* is just as intuitively evident as that of a syllogism in *Barbara* itself. Why should no relation be regarded as *formal* unless it can be expressed by the word *is*? Touching on this case, De Morgan remarks that the formal logician has a right to confine himself to any part of his subject that he pleases; "but he has no right except the right of fallacy to call that part the whole" (*Syllabus*, p. 42).

There are an indefinite number of other arguments which for similar reasons cannot be reduced to syllogistic form. For example,—*A* equals *B*, *B* equals *C*, therefore, *A* equals *C*¹; *X* is a contemporary of *Y*, and *Y* of *Z*, therefore, *X* is a contemporary of *Z*; *A* is the brother of *B*, *B* is the brother of *C*, therefore, *A* is the brother of *C*; *A* is to the right of *B*, *B* is to the right of *C*, therefore, *A* is to the right of *C*; *A* is in tune with *B*, and *B* with *C*, therefore, *A* is in tune with *C*. All these arguments depend upon principles which may be placed on a par with the *dictum de omni et nullo*, and which are equally axiomatic in the particular systems to which they belong.

The claims that have been put forward on behalf of the syllogism as the exclusive form of all deductive reasoning must accordingly be rejected.

¹ In regard to this argument De Morgan writes, "This is not an instance of common syllogism: the premisses are '*A is an equal of B*'; '*B is an equal of C*.' So far as common syllogism is concerned, that '*an equal of B*' is as good for the argument as '*B*' is a *material* accident of the meaning of '*equal*.' The logicians accordingly, to reduce this to a common syllogism, state the effect of composition of relation in a major premiss, and declare that the case before them is an example of that composition in a minor premiss. As in, *A is an equal of an equal* (of *C*); *Every equal of an equal is an equal*; therefore, *A is an equal of C*. This I treat as a mere evasion. Among various sufficient answers this one is enough: *men do not think as above*. When *A=B*, *B=C*, is made to give *A=C*, the word *equals* is a *copula* in thought, and not a *notion* attached to a *predicate*. There are processes which are not those of common syllogism in the logician's major premiss above: but waiving this, logic is an analysis of the form of thought, possible and actual, and the logician has no right to declare that other than the actual is actual" (*Syllabus*, pp. 31, 2).

Such claims have been made, for example, by Whately. Syllogism, he says, is "the form to which *all* correct reasoning may be ultimately reduced" (*Logic*, p. 12). Again, he remarks, "An argument thus stated regularly and at full length, is called a Syllogism; which, therefore, is evidently not a peculiar *kind of argument*, but only a peculiar *form of expression*, in which every argument may be stated" (*Logic*, p. 26)¹.

Spalding seems to have the same thing in view when he says,— "An inference, whose antecedent is constituted by more propositions than one, is a mediate inference. The simplest case, that in which the antecedent propositions are two, is the syllogism. The syllogism is the norm of all inferences whose antecedent is more complex; and all such inferences may, by those who think it worth while, be resolved into a series of syllogisms" (*Logic*, p. 158).

J. S. Mill endorses these claims. "All valid ratiocination," he observes, "all reasoning by which from general propositions previously admitted, other propositions equally or less general are inferred, may be exhibited in some of the above forms," *i.e.*, the syllogistic moods (*Logic*, I. p. 191).

What is required to fill the logical gap which is created by the admission that the syllogism is *not* the norm of all valid formal inference has been called the *logic of relatives*². The function of the logic of relatives is to "take account of relations generally, instead of those merely which are indicated by the ordinary logical copula *is*" (Venn, *Symbolic Logic*, p. 400). The line which this branch of logic may take, if it is ever fully worked out, is indicated by the following passage from De Morgan (*Syllabus*, pp. 30, 31):—"A *convertible* copula is one in which the copular relation exists between two names *both*

¹ Compare also Whately, *Logic*, pp. 24, 5, and p. 34. Professor Ray expresses himself equally strongly. "The syllogism," he remarks, "is the type of all valid reasoning; for no reasoning will be valid, unless it can be thrown into the form of a syllogism. As a matter of fact, in daily life, men draw inferences in many different ways, but only those among them will be valid, and properly deserving of the name, which are capable of being ultimately reduced to the syllogistic form, the rest being nothing but suggestions of association, fancy, imagination, &c., wrongly called inferences" (*Deductive Logic*, p. 255).

² Compare pp. 118, 119.

ways: thus 'is fastened to,' 'is joined by a road with,' 'is equal to,' &c. are *convertible* copulæ. If 'X is equal to Y' then 'Y is equal to X,' &c. A *transitive* copula is one in which the copular relation joins X with Z whenever it joins X with Y and Y with Z. Thus 'is fastened to' is usually understood as a transitive copula: 'X is fastened to Y' and 'Y is fastened to Z' give 'X is fastened to Z.'" The student may further be referred to Venn, *Symbolic Logic*, pp. 399 to 404; and also to Mr Johnson's articles on the *Logical Calculus* in *Mind*, 1892, especially pp. 26 to 28 and 244 to 250.

EXERCISES.

260. Take any enthymeme (in the modern sense) and supply premisses so as to expand it into (a) a syllogism, (b) an epicheirema, (c) a sorites; and name the mood, order, or variety of each product.

[c.]

261. Shew that if either of two given propositions will suffice to expand a given enthymeme of the first or second order into a valid syllogism, then the two propositions will be equivalent to each other, provided that neither of them constitutes a strengthened premiss.

[j.]

262. Discuss the character of the following sorites, in each case indicating how far more than one analysis is possible: (i) *Some D is E, All D is C, All C is B, All B is A, therefore, Some A is E*; (ii) *Some A is B, No C is B, All D is C, All E is D, therefore, Some A is not E*; (iii) *All E is D, All D is C, All C is B, All B is A, therefore, Some A is E*; (iv) *No D is E, Some D is C, All C is B, All B is A, therefore, Some A is not E*.

[k.]

263. Examine the validity of the following moods:—

In figure 1, UAU, YOO, EYO;

In figure 2, AAA, AYY, UO ω ;

In figure 3, YEE, OYO, A ω O.

[c.]

264. Enquire in what figures, if any, the following moods are valid, noting cases in which the conclusion is weakened:—AUI; YAY; UO η ; IU η ; UEO.

[l.]

265. Is it possible that there should be three propositions such that each in turn is deducible from the other two?

[v.]

266. Determine special rules for figures 1, 2, and 4, corresponding to the special rules for figure 3 given in section 258.

[k.]

CHAPTER VIII.

EXAMPLES OF ARGUMENTS AND FALLACIES.

267. In how many different moods may the argument implied in the following proposition be stated?

"No one can maintain that all persecution is justifiable who admits that persecution is sometimes ineffective."

How would the formal correctness of the reasoning be affected by reading "deny" for "maintain"? [v.]

268. What conclusions (if any) can be drawn from each pair of the following sentences taken two and two together?

- (1) None but gentlemen are members of the club;
- (2) Some members of the club are not officers;
- (3) All members of the club are invited to compete;
- (4) All officers are invited to compete.

Point out the mood and figure in each case in which you make a valid syllogism; and state your reasons when you consider that no valid syllogism is possible. [v.]

269. No one can maintain that all republics secure good government who bears in mind that good government is inconsistent with a licentious press.

What premisses must be supplied to express the above reasoning in *Ferio*, *Festino*, and *Ferison* respectively? [v.]

270. Write the following arguments in syllogistic form, and reduce them to the first figure:—

(α) Falkland was a royalist and a patriot; therefore, some royalists were patriots.

(β) All who are punished should be responsible for their actions; therefore, if some lunatics are not responsible for their actions, they should not be punished.

(γ) All who have passed the Little-Go have a knowledge of Greek; hence A.B. cannot have passed the Little-Go, for he has no knowledge of Greek. [K.]

271. "It is impossible to maintain that the virtuous alone are happy, and at the same time that selfishness is compatible with happiness but incompatible with virtue."

State the above argument syllogistically in as many different moods as possible. [J.]

272. Give the technical name of the following argument:—
Payment by results sounds extremely promising; but payment by results necessarily means payment for a minimum of knowledge; payment for a minimum of knowledge means teaching in view of a minimum of knowledge; teaching in view of a minimum of knowledge means bad teaching. [K.]

273. From *P* follows *Q*; and from *R* follows *S*; but *Q* and *S* cannot both be true; shew that *P* and *R* cannot both be true.

[De Morgan.]

274. Every English peer is entitled to sit in the House of Lords, and every member of the House of Commons must be elected to Parliament by a constituency; but no one entitled to a seat in the House of Lords is thus elected to Parliament. What can we conclude from these premisses about an English peer? [M.]

275. If (1) it is false that whenever *X* is found *Y* is found with it, and (2) not less untrue that *X* is sometimes found without the accompaniment of *Z*, are you justified in denying that (3) whenever *Z* is found there also you may be sure of finding *Y*? And however this may be, can you in the same circumstances judge anything about *Y* in terms of *Z*? [R.]

276. Can the following arguments be reduced to syllogistic form?

(1) The sun is a thing insensible;
The Persians worship the sun;
Therefore, the Persians worship a thing insensible.

(2) The Divine law commands us to honour kings;
Louis xiv. is a king;
Therefore, the Divine law commands us to honour Louis xiv.

[Port Royal Logic.]

277. Examine the following arguments; where they are valid, reduce them if you can to syllogistic form; and where they are invalid, explain the nature of the fallacy:—

(1) We ought to believe the Scripture;
Tradition is not Scripture;
Therefore, we ought not to believe tradition.

(2) Every good pastor is ready to give his life for his sheep;
Now, there are few pastors in the present day who are ready to give their lives for their sheep;
Therefore, there are in the present day few good pastors.

(3) Those only who are friends of God are happy;
Now, there are rich men who are not friends of God;
Therefore, there are rich men who are not happy.

(4) The duty of a Christian is not to praise those who commit criminal actions;
Now, those who engage in a duel commit a criminal action;
Therefore, it is the duty of a Christian not to praise those who engage in duels.

(5) The gospel promises salvation to Christians;
Some wicked men are Christians;
Therefore, the gospel promises salvation to wicked men.

(6) He who says that you are an animal speaks truly;
He who says that you are a goose says that you are an animal;
Therefore, he who says that you are a goose speaks truly.

(7) You are not what I am;
I am a man;
Therefore, you are not a man.

(8) We can only be happy in this world by abandoning ourselves to our passions, or by combating them;

If we abandon ourselves to them, this is an unhappy state, since it is disgraceful, and we could never be content with it;

If we combat them, this is also an unhappy state, since there is nothing more painful than that inward war which we are continually obliged to carry on with ourselves;

Therefore, we cannot have in this life true happiness.

(9) Either our soul perishes with the body, and thus, having no feelings, we shall be incapable of any evil; or if the soul survives the body, it will be more happy than it was in the body;

Therefore, death is not to be feared.

[*Port Royal Logic.*]

278. Examine the following arguments :—

(1) "He that is of God heareth my words: ye therefore hear them not, because ye are not of God."

(2) All the fish that the net inclosed were an indiscriminate mixture of various kinds: those that were set aside and saved as valuable, were fish that the net inclosed: therefore, those that were set aside and saved as valuable, were an indiscriminate mixture of various kinds.

(3) Testimony is a kind of evidence which is very likely to be false: the evidence on which most men believe that there are pyramids in Egypt is testimony: therefore, the evidence on which most men believe that there are pyramids in Egypt is very likely to be false.

(4) If Paley's system is to be received, one who has no knowledge of a future state has no means of distinguishing virtue and vice: now one who has no means of distinguishing virtue and vice can commit no sin: therefore, if Paley's system is to be received, one who has no knowledge of a future state can commit no sin.

(5) If Abraham were justified, it must have been either by faith or by works: now he was not justified by faith (according to James), nor by works (according to Paul): therefore, Abraham was not justified.

(6) For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary; and it is ineffectual, for the idle, and such as are indifferent to mental improvement: therefore, the incitement of academical honours is either unnecessary or ineffectual.

(7) He who is most hungry eats most; he who eats least is most hungry: therefore, he who eats least eats most.

(8) A monopoly of the sugar-refining business is beneficial to sugar-refiners: and of the corn-trade to corn-growers: and of the silk-manufacture to silk-weavers, &c., &c.; and thus each class of men are benefited by some restrictions. Now all these classes of men make up the whole community: therefore, a system of restrictions is beneficial to the community. [Whately, *Logic*.]

279. The following are a few examples in which the reader can try his skill in detecting fallacies, determining the peculiar form of syllogisms, and supplying the suppressed premisses of enthymemes.

(1) None but those who are contented with their lot in life can justly be considered happy. But the truly wise man will always make himself contented with his lot in life, and, therefore, he may justly be considered happy.

(2) All intelligible propositions must be either true or false. The two propositions "Cæsar is living still," and "Cæsar is dead," are both intelligible propositions; therefore, they are both true, or both false.

(3) Many things are more difficult than to do nothing. Nothing is more difficult to do than to walk on one's head. Therefore, many things are more difficult than to walk on one's head.

(4) None but Whigs vote for Mr B. All who vote for Mr B. are ten-pound householders. Therefore, none but Whigs are ten-pound householders.

(5) If the Mosaic account of the cosmogony is strictly correct, the sun was not created till the fourth day. And if the sun was not created till the fourth day, it could not have been the cause of the alternation of day and night for the first three days. But either the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now, or else the sun must have been the cause of the alternation of day and night for the first three days. Hence it follows that either the Mosaic account of the cosmogony is not strictly correct, or else the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now.

(6) Suffering is a title to an excellent inheritance; for God chastens every son whom he receives.

(7) It will certainly rain, for the sky looks very black.

[Solly, *Syllabus of Logic*.]

280. Dr Johnson remarked that "a man who sold a penknife was not necessarily an ironmonger." Against what logical fallacy was this remark directed? [c.]

281. Examine the following arguments, pointing out any fallacies that they contain:

(a) The more correct the logic, the more certainly will the conclusion be wrong if the premisses are false. Therefore, where the premisses are wholly uncertain the best logician is the least safe guide.

(b) The spread of education among the lower orders will make them unfit for their work: for it has always had that effect on those among them who happen to have acquired it in previous times.

(c) This pamphlet contains seditious doctrines. The spread of seditious doctrines may be dangerous to the State. Therefore, this pamphlet must be suppressed. [c.]

282. Discuss the nature of the reasoning contained, or apparently intended, in the following sentences:—

It is impossible to prove that persecution is justifiable if you cannot prove that some non-effective measures are justifiable; for no persecution has ever been effective.

This deed may be genuine though it is not stamped, for some unstamped deeds are genuine. [c.]

283. State the following arguments in logical form, and examine their validity:—

(1) Poetry must be either true or false: if the latter, it is misleading; if the former, it is disguised history, and savours of imposture as trying to pass itself off for more than it is. Some philosophers have therefore wisely excluded poetry from the ideal commonwealth.

(2) If we never find skins except as the teguments of animals, we may safely conclude that animals cannot exist without skins. If colour cannot exist by itself, it follows that neither can anything that is coloured exist without colour. So if language without thought is unreal, thought without language must also be so.

(3) Had an armistice been beneficial to France and Germany, it would have been agreed upon by those powers; but such has not been the case; it is plain therefore that an armistice would not have been advantageous to either of the belligerents.

(4) If we are marked to die, we are enow
To do our country loss: and, if to live,
The fewer men, the greater share of honour. [o.]

284. Examine logically the following arguments:—

(a) If truthfulness is never found save with scrupulousness, and if truthfulness is incompatible with stupidity, it follows that stupidity and scrupulousness can never be associated.

(b) You say that there is no rule without an exception. I answer that, in that case, what you have just said must have an exception, and so prove that you have contradicted yourself.

(c) Knowledge gives power; consequently, since power is desirable, knowledge is desirable. [L.]

285. Examine the following arguments, stating them in syllogistic form, and pointing out fallacies, if any :—

(a) Some who are truly wise are not learned; but the virtuous alone are truly wise; the learned, therefore, are not always virtuous.

(b) If all the accused were innocent, some at least would have been acquitted; we may infer, then, that none were innocent, since none have been acquitted.

(c) Every statement of fact deserves belief; many statements, not unworthy of belief, are asserted in a manner which is anything but strong; we may infer, therefore, that some statements not strongly asserted are statements of fact.

(d) That many persons who commit errors are blameworthy is proved by numerous instances in which the commission of errors arises from gross carelessness. [M.]

286. Examine technically the following arguments :—

(1) Those who hold that the insane should not be punished ought in consistency to admit also that they should not be threatened; for it is clearly unjust to punish any one without previously threatening him.

(2) If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do?

(3) Knavery and folly always go together; so, knowing him to be a fool, I distrusted him.

(4) If I deny that poverty and virtue are inconsistent, and you deny that they are inseparable, we can at least agree that some poor are virtuous.

(5) How can you deny that the infliction of pain is justifiable if punishment is sometimes justifiable and yet always involves pain?

[V.]

287. Detect the fallacy in the following argument :—

“A vacuum is impossible, for if there is nothing between two bodies they must touch.” [N.]

288. Examine technically the following arguments :—

(a) "Tis only the present that pains,
And the present will pass."

(b) All legislative restraint is either unjust or unnecessary; since, for the sake of a single man's interests, to restrain all the rest of the community is unjust, and to restrain the man himself is unnecessary.

(c) Only Conservatives—and not all of them—are Protectionists; only Liberals—and not all of them—are Home Rulers; but both parties contain supporters of women's franchise. Hence only Unionists—and not all of them—are Protectionists, while the supporters of women's franchise contain both Unionists and Free-traders.

(d) No school-boy can be expected to understand Constitutional History, and none but school-boys can be expected to remember dates: so that no one can be expected both to remember dates and to understand Constitutional History.

(e) To be wealthy is not to be healthy; not to be healthy is to be miserable; therefore, to be wealthy is to be miserable.

(f) Whatever any man desires is desirable; every man desires his own happiness; therefore, the happiness of every man is desirable. [J.]

289. Examine the validity of the following arguments :—

(1) I knew he was a Bohemian, for he was a good musician, and Bohemians are always good musicians.

(2) Bullies are always cowards, but not always liars; liars, therefore, are not always cowards.

(3) If all the soldiers had been English, they would not all have run away; but some did run away; and we may, therefore, infer that some of them at least were not English.

(4) None but the good are really to be envied; all truly wise men are good; therefore, all truly wise men are to be envied.

(5) You cannot affirm that all his acts were virtuous, for you deny that they were all praiseworthy, and you allow that nothing that is not praiseworthy is virtuous.

(6) Since the end of poetry is pleasure, that cannot be un-poetical with which all are pleased.

(7) *Most M is P, Most S is M, therefore, Some S is P.*

(8) Old Parr, healthy as the wild animals, attained to the age of 152 years; all men might be as healthy as the wild animals; therefore, all men might attain to the age of 152 years.

(9) It is quite absurd to say "I would rather not exist than be unhappy," for he who says "I will this, rather than that," chooses something. Non-existence, however, is no something, but nothing, and it is impossible to choose rationally when the object to be chosen is nothing.

(10) Because the quality of having warm red blood belongs to all known birds, it must be part of their specific nature; but unknown birds have the same specific nature as known birds; therefore, the quality of having warm red blood must belong to the unknown as well as the known birds, *i.e.*, be a universal and essential property of the species.

[κ.]

CHAPTER IX.

PROBLEMS ON THE SYLLOGISM.

290. *Bearing of the existential import of propositions upon the validity of syllogistic reasonings.*—We may as before take different suppositions with regard to the existential import of propositions, and proceed to consider how far the validity of the various syllogistic moods is affected by each in turn.

(1) *Let every proposition imply the existence both of its subject and of its predicate*¹. In this case, the existence of the major, middle, and minor terms is in every case guaranteed by the premisses, and therefore no further assumption with regard to existence is required in order that the conclusion may be legitimately obtained². We may regard the above supposition as that which is tacitly made in the ordinary doctrine of the syllogism.

(2) *Let every proposition imply the existence of its subject.* Under this supposition, as we have already seen, an affirmative proposition ensures the existence of its predicate also; but not so a negative proposition. It follows that any mood will be valid unless the minor term is in its premiss the predicate of a negative proposition. This cannot happen either in figure 1 or in figure 2, since in these figures the minor is always subject in its premiss; nor in figure 3, since in this figure the minor premiss is always affirmative. In figure 4, the only moods with

¹ It will be observed that this is not quite the same as supposition (1) in sections 117 to 119.

² If, however, we are to be allowed to proceed as in section 154 (where from *all P is M, all S is M*, we inferred *some not-S is not-P*) we must posit the existence not merely of the terms directly involved, but also of their contradictories.

a negative minor are *Camenes* and its weakened form **AEO**. Our conclusion then is that on the given supposition every ordinarily recognised mood is valid except these two¹.

(3) *Let no proposition imply the existence either of its subject or of its predicate.* Taking *S, M, P*, as the minor, middle, and major terms respectively, the conclusion will imply that if there is any *S* there is some *P* or *not-P* (according as it is affirmative or negative). Will the premisses also imply this? If so, then the syllogism is valid; but not otherwise.

It has been shewn in section 160 that a universal affirmative conclusion, *All S is P*, can be proved only by means of the premisses, *All M is P, All S is M*; and it is clear that these premisses themselves imply that if there is any *S* there is some *P*. On our present supposition, then, a syllogism is valid if its conclusion is universal affirmative.

Again, as shewn in section 160, a universal negative conclusion, *No S is P*, can be proved only in the following ways:—

- (i) *No M is P* (or *No P is M*),
All S is M,

therefore, *No S is P*;

- (ii) *All P is M*,
No S is M (or *No M is S*),

therefore, *No S is P*.

In (i) the minor premiss implies that if *S* exists then *M*

¹ Reduction to figure 1 appears to be affected by this supposition, since it makes the contraposition of **A** and the conversion of **E** in general invalid. The contraposition of **A** is involved in the direct reduction of *Baroco* (*Faksoko*). The process is, however, in this particular case valid, as the existence of *not-M* is given by the minor premiss. The conversion of **E** is involved in the reduction of *Cesare*, *Camestres*, and *Festino* from figure 2; and of *Camenes*, *Fesapo*, and *Fresison* from figure 4. Since, however, one premiss must be affirmative the existence of the middle term is thereby guaranteed, and hence the simple conversion of **E** in the second figure, and in the major of the fourth becomes valid. Also the conversion of the conclusion resulting from the reduction of *Camestres* is legitimate, since the original minor term is subject in its premiss. Hence *Camenes* (and its weakened form) are the only moods whose reduction is rendered illegitimate by the supposition under consideration. This result agrees with that reached in the text.

exists, and the major premiss that if M exists then *not-P* exists. In (ii) the minor premiss implies that if S exists then *not-M* exists, and the major premiss that if *not-M* exists then *not-P* exists (as shewn in section 118). Hence a syllogism is valid if its conclusion is universal negative.

Next, let the conclusion be particular. In figure 1, the implication of the conclusion with regard to existence is contained in the premisses themselves, since the minor term is the subject of an affirmative minor premiss, and the middle term the subject of the major premiss. In figure 2, we may consider the weakened moods disposed of in what has been already said with regard to universal conclusions; for under our present supposition subalternation is a valid process. The remaining moods with particular conclusions in this figure are *Festino* and *Baroco*. In the former, the minor premiss implies that if S exists then M exists, and the major that if M exists then *not-P* exists; in the latter, the minor premiss implies that if S exists then *not-M* exists, and the major that if *not-M* exists then *not-P* exists.

All the ordinarily recognised moods, then, of figures 1 and 2 are valid. But it is otherwise with moods yielding a particular conclusion in figures 3 and 4, with the single exception of the weakened form of *Camenes* (which is itself the only mood with a universal conclusion in these figures). Subalternation being a valid process, the legitimacy of the latter follows from the legitimacy of *Camenes* itself. But in all other cases in figures 3 and 4, the minor term is the predicate of an affirmative minor premiss. Its existence, therefore, carries no further implication of existence with it in the premisses. It does so in the conclusion. Hence all the moods of figures 3 and 4, with the exception of **AEE** and **AEO** in the latter figure, are invalid. Take, as an example, a syllogism in *Darapti*,—

All M is P,

All M is S,

therefore, *Some S is P.*

The conclusion implies that if S exists P exists; but consistently with the premisses, S may be existent while M and P

are both non-existent. An implication is, therefore, contained in the conclusion which is not justified by the premisses.

Hence on the supposition that no proposition implies the existence either of its subject or of its predicate all the ordinarily recognised moods of figures 1 and 2 are valid, but none of those of figures 3 and 4 excepting *Camenes* and the weakened form of *Camenes*¹.

(4) *Let particulars imply, while universals do not imply, the existence of their subjects.* The legitimacy of moods with universal conclusions may be established as in the preceding case. Taking moods with particular conclusions, it is obvious that they will be valid if the minor premiss is particular, having the minor term as its subject; or if the minor premiss is particular affirmative, whether the minor term is its subject or predicate. *Disamis*, *Bocardo*, and *Dimaris* are also valid, since the major premiss in each case guarantees the existence of *M*, and the minor implies that if *M* exists then *S* exists. The above will be found to cover all the valid moods in which one premiss is particular. There remain only the moods in which from two universals we infer a particular. It is clear that all these moods must be invalid, for their conclusions will imply the existence of the minor term, and this cannot be guaranteed by the premisses².

On the supposition then that particulars imply, while universals do not imply, the existence of their subjects, the moods rendered invalid are all the weakened moods, together with *Darapti*, *Felapton*, *Bramantip*, and *Fesapo*³, each of which contains a strengthened premiss. More briefly, any ordinarily recognised mood is on this supposition valid, unless it contains either a strengthened premiss or a weakened conclusion.

¹ An express statement concerning existence may, however, render the rejected moods legitimate. If, for instance, the existence of the middle term is expressly given, then *Darapti* becomes valid. Compare note 1 on p. 252.

² Hypothetical conclusions (of the form *If S exists then &c.*) will of course still be legitimate.

³ It will be observed that the letter *p* occurs in the mnemonic for each of these moods, indicating that their reduction to figure 1 involves *conversion per accidens*. On the supposition under discussion this process is invalid, and we may find here a confirmation of the above result.

291. *Connexion between the truth and falsity of premisses and conclusion in a valid syllogism.*—By saying that a syllogism is valid we mean that the truth of its conclusion follows from the truth of its premisses; and it is an immediate inference from this that if the conclusion is false one or both of the premisses must be false. The converse does not, however, hold good in either case. The truth of the premisses does not follow from the truth of the conclusion; nor does the falsity of the conclusion follow from the falsity of either or both of the premisses.

The above statements would probably be accepted as self-evident; still it is more satisfactory to give a formal proof of them, and such a proof is afforded by means of the three following theorems¹.

(1) *Given a valid syllogism, then in no case will the combination of either premiss with the conclusion establish the other premiss.*

We have to shew that if one premiss and the conclusion of a valid syllogism be taken as a new pair of premisses they do not in any case suffice to establish the other premiss.

Were it possible for them to do so, then the premiss given true would have to be affirmative, for if it were negative, the original conclusion would be negative, and combining these we should have two negative premisses which could yield no conclusion.

Also, the middle term would have to be distributed in the premiss given true. This is clear if it is not distributed in the other premiss; and since the other premiss is the conclusion of the new syllogism, if it is distributed there, it must also be distributed in the premiss given true or we should have an illicit process in the new syllogism.

Therefore, the premiss given true, being affirmative, and distributing the middle term, cannot distribute the other term which it contains². Neither therefore can this term be dis-

¹ It is assumed throughout this section that our schedule of propositions does not include U. The theorems hold good, however, for the sixfold schedule, including Y and η , as well as for the ordinary fourfold schedule.

² This statement, though not holding good for U, holds good for Y as well as A.

tributed in the original conclusion. But this is the term which will be the middle term of the new syllogism, and we shall therefore have undistributed middle.

Hence the truth of one premiss and the conclusion of a valid syllogism does not establish the truth of the other premiss; and *à fortiori* the truth of the conclusion cannot by itself establish the truth of both the premisses¹.

(2) *The contradictories of the premisses of a valid syllogism will not in any case suffice to establish the contradictory of the original conclusion.*

The premisses of the original syllogism must be either (a) both affirmative, or (β) one affirmative and one negative.

In case (a), the contradictories of the original premisses will both be negative; and from two negatives nothing follows.

In case (β), the contradictories of the original premisses will be one negative and one affirmative; and if this combination yields any conclusion, it will be negative. But the original conclusion must also be negative, and therefore its contradictory will be affirmative.

In neither case then can we establish the contradictory of the original conclusion.

(3) *One premiss and the contradictory of the other premiss of a valid syllogism will not in any case suffice to establish the contradictory of the original conclusion².*

This follows at once from the first of the theorems established in this section. Let the premisses of a valid syllogism be *P* and *Q*, and the conclusion *R*. *P* and the contradictory of *Q* will not prove the contradictory of *R*; for if they did, it would follow that *P* and *R* would prove *Q*; but this has been shewn not to be the case.

¹ Other methods of solution more or less distinct from the above might be given. A somewhat similar problem is discussed by Solly, *Syllabus of Logic*, pp. 123 to 126, 132 to 136. We have shewn that one premiss and the conclusion of a valid syllogism will never suffice to prove the other premiss, but it of course does not follow that they will never yield any conclusion at all; for a consideration of this question, see the following section.

² It does not follow that one premiss and the contradictory of the other premiss of a valid syllogism will never yield any conclusion at all. See the following section.

We have now established by strictly formal reasoning Aristotle's dictum that although it is not possible syllogistically to get a false conclusion from true premisses, it is quite possible to get a true conclusion from false premisses¹. In other words, the falsity of one or both of the premisses does not establish the falsity of the conclusion of a syllogism². The second of the above theorems deals with the case in which both the premisses are false; the third with that in which one only of the premisses is false.

292.—*Arguments from the truth of one premiss and the falsity of the other premiss in a valid syllogism, or from the falsity of one premiss to the truth of the conclusion, or from the truth of one premiss to the falsity of the conclusion.*—In this section we shall consider three problems, mutually involved in one another, which are in a manner related to the theorems contained in the preceding section. It has, for example, been shewn that one premiss and the contradictory of the other premiss will not in any case suffice to establish the contradictory of the original conclusion; the object of the first of the following problems is to enquire in what cases they can establish any conclusion at all.

(i) *To find a pair of valid syllogisms having a common premiss, such that the remaining premiss of the one contradicts the remaining premiss of the other*³.

¹ Hamilton (*Logic*, i. p. 450) considers the doctrine "that if the conclusion of a syllogism be true, the premisses may be either true or false, but that if the conclusion be false, one or both of the premisses must be false" to be extralogical, if it is not absolutely erroneous. He is clearly wrong, since the doctrine in question admits of a purely formal proof.

² "In all cases where *T* is not given in direct perception, but deduced from premisses, what really depends on the correctness of those premisses is not the truth of *T*, but only our insight into that truth. Without correct premisses *T* cannot indeed be *proved*, but nevertheless it can be true and its truth is independent of any errors we may commit, when reflecting about it, and subsists even when conclusively deduced from premisses materially false. This point deserves notice, for it is a common mistake in reasoning to take the invalidity of the proof which is offered for *T* as a proof of the falsehood of *T* itself, and to confuse the refutation of an argument with the disproof of a fact" (Lotze, *Logic*, § 240).

³ This problem was suggested by the following question of Mr O'Sullivan's, which puts the same problem in another form: Given that one premiss of a

We have to find cases in which P and Q , P and Q' (the contradictory of Q), are the premisses of two valid syllogisms. In working out this problem and the problems that follow, it must be remembered that if two propositions are contradictories, they will differ in quality, and also in the distribution of their terms, so that any term distributed in either of them is undistributed in the other and *vice versa*. We may, therefore, assume that Q is affirmative and Q' negative. Let P contain the terms X and Y , while Q and Q' contain the terms Y and Z , so that Y is the middle term, and X and Z the extreme terms, of each syllogism.

Since Q' is negative, P must be affirmative; and since Y must be undistributed either in Q or in Q' , it must be distributed in P .

Hence $P = YaX$.

Q' must distribute Z ; for the conclusion (being negative) must distribute one term, and X is undistributed in P . It follows that Z is undistributed in Q .

Hence $Q = YaZ$ or YiZ or ZiY ;

$Q' = YoZ$ or YeZ or ZeY .

The following syllogisms, therefore, are such that if one premiss (that in black type) is retained, while the other is replaced by its contradictory, a conclusion is still obtainable:—

In figure 1: **AII**;

In figure 3: **AAI**, **AAI**, **IAI**, **AII**, **EAO**, **OAQ**;

In figure 4: **IAI**, **EAQ**.

(ii) *To find a pair of valid syllogisms, having a common conclusion, such that a premiss in the one contradicts a premiss in the other.*

Let Q and Q' (which we may assume to be respectively affirmative and negative) be the premisses in question, and P' the conclusion; also let Q and Q' contain the terms Y and Z , while P' contains the terms X and Y , so that Z is the middle term, and X and Y the extreme terms, of each syllogism.

It follows immediately that P' is negative; also that Y

valid syllogism is false and the other true, determine generally in what cases a conclusion can be drawn from these data.

must be undistributed in P' , since it is necessarily undistributed either in Q or in Q' .

Hence $P' = YoX$.

Since X is distributed in P' it must also be distributed in the premiss which is combined with Q' ; and as this premiss must be affirmative, it cannot also distribute Z , which must therefore be distributed in Q' (and undistributed in Q).

Hence $Q = YaZ$ or YiZ or ZiY ;

$Q' = YoZ$ or YeZ or ZeY .

The following syllogisms, therefore, are such that the same conclusion is obtainable from another pair of premisses, of which one contradicts one of the original premisses (namely, that in black type):—

In figure 1: EAO, EIO ;

In figure 2: EAO, AEO, EIO, AOO ;

In figure 3: EIO ;

In figure 4: AEO, EIO .

(iii) *To find a pair of valid syllogisms having a common premiss, such that the conclusion of one contradicts the conclusion of the other*¹.

Let P be the common premiss, Q and Q' (respectively affirmative and negative) the contradictory conclusions; also let P contain the terms X and Y , while Q and Q' contain the terms Y and Z , so that X is the middle term, and Y and Z the extreme terms, of each syllogism.

Since Q is affirmative, P must be affirmative; and since either Q or Q' will distribute Y , P must distribute Y .

Hence $P = YaX$.

The premiss which, combined with P , proves Q must be affirmative and must distribute X ; it cannot therefore distribute Z , and Z must accordingly be undistributed in Q (and distributed in Q').

¹ This problem was suggested by the following question of Mr Pantón's, which puts the same problem in another form: If the conclusion be substituted for a premiss in a valid mood, investigate the conditions which must be fulfilled in order that the new premisses should be legitimate.

Hence $Q = YaZ$ or YiZ or ZiY ;
 $Q' = YoZ$ or YeZ or ZeY .

The following syllogisms, therefore, are such that the contradictory of the conclusion is obtainable, although one of the premisses (that in black type) is retained :—

In figure 1: AAA, AAI, EAE, EAO;

In figure 2: EAE, EAO, AEE;

In figure 4: AAI, AEE¹.

The three sets of moods above worked out are mutually derivable from one another. Thus,

(i)	(ii)	(iii)
$P \text{ and } Q \therefore R = Q \text{ and } R' \therefore P' = R' \text{ and } P \therefore Q'$		
$P \text{ and } Q' \therefore T' = Q' \text{ and } T \therefore P' = T' \text{ and } P \therefore Q$		

In this table (i) represents the possible cases in which, one premiss being retained, the other premiss may be replaced by its contradictory. We can then deduce (ii) the cases in which, the conclusion being retained, one premiss may be replaced by its contradictory; and (iii) the cases in which, one premiss being retained, the conclusion may be replaced by its contradictory. We might of course equally well start from (ii) or from (iii), and thence deduce the two others.

Comparing the first syllogism of (i) with the second syllogism of (iii) and *vice versa*, we see further that (i) gives the cases in which, one premiss being retained, the conclusion may be replaced by the other premiss; and that (iii) gives the cases in which, one premiss being retained, the other premiss may be replaced by the conclusion.

¹ It will be observed that each of the above problems yields *nine* cases. Between them they cover all the 24 valid moods; but there are three moods (namely, EAO in figures 1 and 2 and AAI in figure 3) which occur twice over. The 15 unstrengthened and unweakened moods are equally distributed, namely, the four yielding *I* conclusions (together with OAO) falling under (i); the six yielding *O* conclusions (except OAO) under (ii); the five yielding *A* or *E* conclusions under (iii). All the moods of figure 1 (except those with an *I* premiss) fall under (iii); all the moods of figure 2 (except those with an *E* conclusion) under (ii); all the moods of figure 3 (except the one not having an *A* premiss) under (i).

The following is another method of stating and solving all three problems: *To determine in what cases it is possible to obtain two incompatible trios of propositions, each trio containing three and only three terms and each including a proposition which is identical with a proposition in the other and also a proposition which is the contradictory of a proposition in the other.*

Let the propositions be P, Q, R' , and P, Q', T ; and let P contain the terms X and Y ; Q and Q' , the terms Y and Z ; R and T , the terms Z and X . Suppose Q to be affirmative, and Q' negative.

Then since one of each trio of propositions must be negative, and not more than one can be so (as shewn in section 162), P and T must be affirmative, and R' negative.

Again, since each of the terms X, Y, Z must be distributed once at least in each trio of propositions (as shewn in section 162), and since Y must be undistributed either in Q or in Q' , Y must be distributed in P .

Hence $P = YaX$.

X , being undistributed in P , must be distributed in R' and T .

Hence $T = XaZ$.

Z , being undistributed in T , must be distributed in Q' , and therefore undistributed in Q , and distributed in R' .

Hence $Q = YaZ$ or YiZ or ZiY ;

$Q' = YoZ$ or YeZ or ZeY ;

$R' = XeZ$ or ZeX .

We have then the following solution of our problem:—

YaX, YaZ or YiZ or ZiY, XeZ or ZeX ;

YaX, YoZ or YeZ or ZeY, XaZ .

293. *Numerical Moods of the Syllogism*¹.—The following

¹ This section was suggested by the following question of Mr Johnson's:—
 “Shew the validity of the following syllogisms: (i) All M 's are P 's, At least n S 's are M 's, therefore, At least n S 's are P 's; (ii) All P 's are M 's, Less than n S 's are M 's, therefore, Less than n S 's are P 's; (iii) Less than n M 's are P 's, At least n M 's are S 's, therefore, Some S 's are not P 's. Deduce from the above the ordinary non-numerical moods of the first three figures.”

are examples of numerical moods in the different figures of the syllogism :—

- Figure 1.* (i) *All M's are P's,*
At least n S's are M's,
 therefore, *At least n S's are P's;*
- (ii) *Less than n M's are P's,*
All S's are M's,
 therefore, *Less than n S's are P's;*
- (iii) *Less than n M's are P's,*
At least n S's are M's,
 therefore, *Some S's are not P's;*

- Figure 2.* (iv) *All P's are M's,*
Less than n S's are M's,
 therefore, *Less than n S's are P's;*
- (v) *Less than n P's are M's,*
All S's are M's,
 therefore, *Less than n S's are P's;*
- (vi) *Less than n P's are M's,*
At least n S's are M's,
 therefore, *Some S's are not P's;*

- Figure 3.* (vii) *Less than n M's are P's,*
At least n M's are S's,
 therefore, *Some S's are not P's;*
- (viii) *All M's are P's,*
At least n M's are S's,
 therefore, *At least n S's are P's;*
- (ix) *At least n M's are P's,*
All M's are S's,
 therefore, *At least n S's are P's;*

- Figure 4.* (x) *At least n P's are M's,*
All M's are S's,
 therefore, *At least n S's are P's;*
- (xi) *All P's are M's,*
Less than n M's are S's,
 therefore, *Less than n S's are P's;*

- (xii) *Less than n P's are M's,*
At least n M's are S's,
Some S's are not P's.

therefore,

The above moods may be established as follows:—

(i) From *All M's are P's*; it follows that *Every S which is M is also P*, and since *At least n S's are M's*, it follows further that *At least n S's are P's*.

Denoting the major premiss of (i) by *A*, the minor by *B*, and the conclusion by *C*, we obtain immediately the following syllogisms:—

$$\begin{array}{rcl} A, & & C', \\ C', & & B, \\ \hline B'; & & A'; \end{array}$$

and these are respectively equivalent to (iv) and (vii)¹.

(v) is obtainable from (iv) by transposing the premisses and converting the conclusion;

(ii) from (v) by converting the major premiss;

(iii) from (vii) by converting the minor premiss;

(vi) from (iii) by converting the major premiss;

(viii) from (i) by converting the minor premiss;

(ix) from (viii) by transposing the premisses and converting the conclusion;

(x) from (i) by transposing the premisses and converting the conclusion;

(xi) from (iv) by converting the minor premiss;

(xii) from (vii) by converting the major premiss.

¹ The argument here involved may be set out more at length as follows:—

(iv) *All P's are M's, (a)*

Less than n S's are M's, (b)

therefore, *Less than n S's are P's; (c)*

for, if not, then *At least n S's are P's; (d)*

and by (i), (a) and (d) yield the conclusion *At least n S's are M's*; but this contradicts (b), and hence we have proved indirectly the desired conclusion.

(vii) *Less than n M's are P's, (e)*

At least n M's are S's, (f)

therefore, *Some S's are not P's; (g)*

for, if not, then *All S's are P's; (h)*

and by (i), (h) and (f) yield the conclusion *At least n M's are P's*; but this contradicts (e), and hence we have proved indirectly the desired conclusion.

The ordinary non-numerical moods of the different figures may be deduced from the above results as follows:—

Figure 1. (i) Putting n = total number of S 's, we have MaP , SaM , $\therefore SaP$, that is, *Barbara*; and putting $n = 1$, we have MaP , SiM , $\therefore SiP$, that is, *Darii*.

(ii) Putting $n = 1$, MeP , SaM , $\therefore SeP$ (*Celarent*).

(iii) Putting $n = 1$, MeP , SiM , $\therefore SoP$ (*Ferio*).

AAI and *EAO* follow *à fortiori*.

Figure 2. (iv) Putting n = total number of S 's, PaM , SoM , $\therefore SoP$ (*Baroco*); putting $n = 1$, PaM , SeM , $\therefore SeP$ (*Camestres*).

(v) Putting $n = 1$, PeM , SaM , $\therefore SeP$ (*Cesare*).

(vi) Putting $n = 1$, PeM , SiM , $\therefore SoP$ (*Festino*).

AEO and *EAO* follow *à fortiori*.

Figure 3. (vii) Putting n = total number of M 's, MoP , MaS , $\therefore SoP$ (*Bocardo*); putting $n = 1$, MeP , MiS , $\therefore SoP$ (*Ferison*).

(viii) Putting $n = 1$, MaP , MiS , SiP (*Datisi*).

(ix) Putting $n = 1$, MiP , MaS , $\therefore SiP$ (*Disamis*).

Darapti and *Felapton* follow *à fortiori*.

Figure 4. (x) Putting $n = 1$, PiM , MaS , $\therefore SiP$ (*Dimaris*).

(xi) Putting $n = 1$, PaM , MeS , $\therefore SeP$ (*Camenes*).

(xii) Putting $n = 1$, PeM , MiS , $\therefore SoP$ (*Fresison*).

Bramantip, *AEO*, and *Fesapo* follow *à fortiori*.

EXERCISES.

294. "Whatever P and Q may stand for, we may shew *à priori* that some P is Q . For All PQ is Q by the law of identity, and similarly All PQ is P ; therefore, by a syllogism in *Darapti*, Some P is Q ." How would you deal with this paradox? [K.]

A solution is afforded by the discussion contained in section 290; and this example seems to shew that the enquiry—how far assumptions with regard to existence are involved in syllogistic processes—is not irrelevant or unnecessary.

295. What conclusion can be drawn from the following propositions? The members of the board were all either bondholders or shareholders, but not both; and the bondholders, as it happened, were all on the board. [v.]

We may take as our premisses :

No member of the board is both a bondholder and a shareholder,

All bondholders are members of the board ;

and these premisses yield a conclusion (in *Celarent*),

No bondholder is both a bondholder and a shareholder,
that is, No bondholder is a shareholder.

296. The following rules were drawn up for a club:—
(i) The financial committee shall be chosen from amongst the general committee; (ii) No one shall be a member both of the general and library committees, unless he be also on the financial committee; (iii) No member of the library committee shall be on the financial committee.

Is there anything self-contradictory or superfluous in these rules? [VENN, *Symbolic Logic*, p. 261.]

Let F = member of the financial committee,

G = member of the general committee,

L = member of the library committee.

The above rules may then be expressed symbolically as follows:—

- (i) All F is G ;
- (ii) If any L is G , that L is F ;
- (iii) No L is F .

From (ii) and (iii) we obtain (iv) No L is G .

The rules may therefore be written in the form,

- (1) All F is G ,
- (2) No L is G ,
- (3) No L is F .

But in this form (3) is deducible from (1) and (2).

Hence all that is contained in the rules as originally stated may be expressed by (1) and (2); that is, the rules as originally stated were partly superfluous, and they may be reduced to

- (1) The financial committee shall be chosen from amongst the general committee;
- (2) No one shall be a member both of the general and library committees.

If (ii) is interpreted as implying that there are some individuals who are on both the general and library committees, then it follows that (ii) and (iii) are inconsistent with each other.

297. Given that the middle term is distributed twice in the premisses of a syllogism, determine *directly* (i.e., without any reference to the mnemonic verses or the special rules of the figures) in what different moods it might possibly be. [K.]

The premisses must be either both affirmative, or one affirmative and one negative.

In the first case, both premisses being affirmative can distribute their subjects only. The middle term must, therefore, be the subject in each, and both must be universal. This limits us to the one syllogism—

All M is P,
All M is S,
therefore, *Some S is P.*

In the second case, one premiss being negative, the conclusion must be negative and will, therefore, distribute the major term. Hence, the major premiss must distribute the major term, and also (by hypothesis) the middle term. This condition can be fulfilled only by its being one or other of the following—*No M is P* or *No P is M*. The major being negative, the minor must be affirmative, and in order to distribute the middle term must be *All M is S*.

In this case we get two syllogisms, namely,—

No M is P,
All M is S,
therefore, *Some S is not P;*
No P is M,
All M is S,
therefore, *Some S is not P.*

The given condition limits us, therefore, to three syllogisms (one affirmative and two negative); and by reference to the mnemonic verses we may now identify these with *Darapti* and *Felapton* in figure 3, and *Fesapo* in figure 4.

298. If the major premiss and the conclusion of a valid syllogism agree in quantity, but differ in quality, find the mood and figure. [T.]

Since we cannot have a negative premiss with an affirmative

conclusion, the major premiss must be affirmative and the conclusion negative. It follows immediately that, in order to avoid illicit major, the major premiss must be *All P is M* (where *M* is the middle term and *P* the major term). The conclusion, therefore, must be *No S is P* (*S* being the minor term); and this requires that, in order to avoid undistributed middle and illicit minor, the minor premiss should be *No S is M* or *No M is S*. Hence the syllogism is in *Camestres* or in *Camenes*.

299. Given a valid syllogism with two universal premisses and a particular conclusion, such that the same conclusion cannot be inferred, if for either of the premisses is substituted its subaltern, determine the mood and figure of the syllogism. [K.]

Let *S*, *M*, *P* be respectively the minor, middle, and major terms of the given syllogism. Then, since the conclusion is particular, it must be either *Some S is P* or *Some S is not P*.

First, if possible, let it be *Some S is P*.

The only term which need be distributed in the premisses is *M*. But since we have two universal premisses, *two* terms must be distributed in them as subjects¹. One of these must be superfluous; and, therefore, for one of the premisses we may substitute its subaltern, and still get the same conclusion.

The conclusion cannot then be *Some S is P*.

Secondly, if possible, let the conclusion be *Some S is not P*.

If the subject of the minor premiss is *S*, we may clearly substitute its subaltern without affecting the conclusion. The subject of the minor premiss must therefore be *M*, which will thus be distributed in this premiss. *M* cannot also be distributed in the major, or else it is clear that its subaltern might be substituted for the minor and nevertheless the same conclusion inferred. The major premiss must, therefore, be affirmative with *M* for its predicate. This limits us to the syllogism—

All P is M,

No M is S,

therefore, *Some S is not P*;

and this syllogism, which is AEO in figure 4, does fulfil the given conditions, for it becomes invalid if either of the premisses is made particular.

¹ We here include the case in which the middle term is itself twice distributed.

The above amounts to a general proof of the proposition laid down in section 192:—*Every syllogism in which there are two universal premisses with a particular conclusion is a strengthened syllogism, with the single exception of AEO in figure 4.*

300. Given two valid syllogisms in the same figure in which the major, middle, and minor terms are respectively the same, shew, without reference to the mnemonic verses, that if the minor premisses are subcontraries, the conclusions will be identical. [K.]

The minor premiss of one of the syllogisms must be O, and the major premiss of this syllogism must, therefore, be A and the conclusion O. The middle and the major terms having then to be distributed in the premisses, this syllogism is determined, namely,—

All P is M,
Some S is not M,
therefore, *Some S is not P.*

Since the other syllogism is to be in the same figure, its minor premiss must be *Some S is M*; the major must therefore be universal, and in order to distribute the middle term it must be negative. This syllogism therefore is also determined, namely,—

No P is M,
Some S is M,
therefore, *Some S is not P.*

The conclusions of the two syllogisms are thus shewn to be identical.

301. Find out in which of the valid syllogisms the combination of one premiss with the subcontrary of the conclusion would establish the subcontrary of the other premiss. [J.]

In the original syllogism (α) let X (universal) and Y (particular) prove Z (particular), the minor, middle, and major terms being S , M , and P , respectively. Then we are to have another syllogism (β) in which X and Z , (the sub-contrary of Z) prove Y_1 (the sub-contrary of Y). In β , S or P will be the middle term.

It is clear that only one term can be distributed in α if the conclusion is affirmative, and only two if the conclusion is negative. Hence S cannot be distributed in α , and it follows that it cannot be distributed in the premisses of β . The middle term of β must

therefore be P , and as X must consequently contain P it must be the major premiss of a and Y the minor premiss.

Z must be either SiP or SoP . *First*, let $Z = SiP$. Then it is clear that $X = MaP$, $Z_1 = SoP$, $Y_1 = SoM$, $Y = SiM$. *Secondly*, let $Z = SoP$. Then $Z_1 = SiP$, $X = PaM$ or MeP or PeM (since it must distribute P), $Y_1 = SiM$ (if X is affirmative) or SoM (if X is negative), $Y = SoM$ or SiM accordingly.

Hence we have four syllogisms satisfying the required conditions as follows:—

MaP	MeP	PeM	PaM
SiM	SiM	SiM	SoM
SiP	SoP	SoP	SoP

It will be observed that these are all the moods of the first and second figures, in which one premiss is particular.

302. Is it possible that there should be a valid syllogism such that, each of the premisses being converted, a new syllogism is obtainable giving a conclusion in which the old major and minor terms have changed places? Prove the correctness of your answer by general reasoning, and if it is in the affirmative, determine the syllogism or syllogisms fulfilling the given conditions. [κ.]

If such a syllogism be possible, it cannot have two affirmative premisses, or (since A can only be converted *per accidens*) we should have two particular premisses in the new syllogism.

Therefore, *the original syllogism must have one negative premiss.* This cannot be O , since O is inconvertible.

Therefore, *one premiss of the original syllogism must be E.*

First, let this be the major premiss. Then the minor premiss must be affirmative, and its converse (being a particular affirmative), will not distribute either of its terms. But this converse will be the *major* premiss of the new syllogism, which also must have a negative conclusion. We should then have illicit major in the new syllogism; and hence the above supposition will not give us the desired result.

Secondly, let the minor premiss of the original syllogism be E . The major premiss in order to distribute the old major term must be A , with the major term as subject. We get then the following, satisfying the given conditions:—

All P is M,

No M is S, or No S is M,

therefore, *No S is P, or Some S is not P;*

that is, we really have four syllogisms, such that both premisses being converted, thus,

No S is M, or No M is S,

Some M is P,

we have a new syllogism giving a conclusion in which the old major and minor terms have changed places, namely,

Some P is not S.

Symbolically,—

$$\begin{array}{rcl}
 PaM, & & SeM, \\
 MeS, & & \text{or } MeS, \\
 \text{or } SeM, & & MiP, \\
 \hline
 \therefore SeP & & \therefore PoS. \\
 \text{or } SoP & &
 \end{array}$$

If it be required to retain the *quantity* of the original conclusion, this must be *SoP*; in this case then we have only two syllogisms fulfilling the given conditions.

303. Shew that if the proportion of *B*'s out of the class *A* is greater than that out of the class *not-A*, then the proportion of *A*'s out of the class *B* will be greater than that out of the class *not-B*¹. [J.]

Let the number of *A*'s be denoted by $N(A)$, the number of *AB*'s by $N(AB)$, &c.

Then, since *Every A is AB or Ab* (by the law of excluded middle) and *No A is both AB and Ab* (by the law of contradiction), it follows that

$$N(A) = N(AB) + N(Ab).$$

We have to shew that

$$\frac{N(AB)}{N(B)} > \frac{N(Ab)}{N(b)}$$

follows from
$$\frac{N(AB)}{N(A)} > \frac{N(Ab)}{N(a)}.$$

This can be done by substituting

$$N(AB) + N(Ab) \text{ for } N(A), \text{ \&c.}$$

¹ This and the following problem cannot properly be called problems on the syllogism. They are given as examples in numerical logic.

$$\begin{aligned}
\text{Thus,} \quad & \frac{N(AB)}{N(A)} > \frac{N(aB)}{N(a)}, \\
\therefore \quad & \frac{N(a)}{N(aB)} > \frac{N(A)}{N(AB)}, \\
\therefore \quad & \frac{N(aB) + N(ab)}{N(aB)} > \frac{N(AB) + N(Ab)}{N(AB)}, \\
\therefore \quad & \frac{N(ab)}{N(aB)} > \frac{N(Ab)}{N(AB)}, \\
\therefore \quad & \frac{N(ab)}{N(Ab)} > \frac{N(Ab)}{N(AB)}, \\
\therefore \quad & \frac{N(Ab) + N(ab)}{N(Ab)} > \frac{N(AB) + N(aB)}{N(AB)}, \\
\therefore \quad & \frac{N(b)}{N(Ab)} > \frac{N(B)}{N(AB)}, \\
\therefore \quad & \frac{N(AB)}{N(B)} > \frac{N(Ab)}{N(b)}.
\end{aligned}$$

304. Given the number (U) of objects in the Universe, and the number of objects in each of the classes $x_1, x_2, x_3, \dots, x_n$, shew that the least number of objects in the class $(x_1 x_2 x_3 \dots x_n)$

$$= U - N(\bar{x}_1) - N(\bar{x}_2) - N(\bar{x}_3) \dots - N(\bar{x}_n),$$

where $N(\bar{x}_1)$ means the number of things which are *not* x_1 ; $N(\bar{x}_2)$, the number that are *not* x_2 ; &c. [J.]

Given $N(\bar{x}_1)$, $N(\bar{x}_2)$, &c., the number of objects in the class $(\bar{x}_1$ or $\bar{x}_2 \dots$ or $\bar{x}_n)$ is greatest when no object belongs to any pair of the classes $\bar{x}_1, \bar{x}_2, \dots$; and in this case it = $N(\bar{x}_1) + N(\bar{x}_2) \dots + N(\bar{x}_n)$.

Hence the least number in the contradictory class, $x_1 x_2 \dots x_n$,

$$= U - N(\bar{x}_1) - N(\bar{x}_2) \dots - N(\bar{x}_n).$$

305. Prove that with three given propositions (of the forms A, E, I, O) it is never possible to construct more than one valid syllogism. [K.]

306. On the supposition that no proposition implies the existence either of its subject or of its predicate, find in what cases the reduction of syllogisms to figure 1 is invalid. [K.]

307. *Some M is not P; All S is all M.* What conclusion follows from the combination of these premisses?

Can you infer anything about either *S* or *P* from the knowledge that both the above propositions are false? [K.]

308. (i) *Either all M is all P or Some M is not P*; (ii) *Some S is not M.* What is all that can be inferred (a) about *S* in terms of *P*, (b) about *P* in terms of *S*, from the knowledge that both the above statements are false? [K.]

309. (a) "A good temper is proof of a good conscience, and the combination of these is proof of a good digestion, which again always produces one or the other." Shew that this is precisely equivalent to the following: "A good temper is proof of a good digestion, and a good digestion of a good conscience."

(b) Examine (by diagrams or otherwise) the following argument:—"Patriotism and humanitarianism must be either incompatible or inseparable; and though family-affection and humanitarianism are compatible, yet either may exist without the other; hence, family affection may exist without patriotism." Reduce the argument, if you can, to ordinary syllogistic form; and determine whether the premisses state anything more than is necessary to prove the conclusion. [J.]

310. "All scientific persons are willing to learn; all unscientific persons are credulous; therefore, some who are credulous are not willing to learn, and some who are willing to learn are not credulous."

Shew that the ordinary rules of immediate and mediate inference justify this reasoning; but that a certain assumption is involved in thus avoiding the apparent illicit process. Shew also that, accepting the validity of obversion and simple conversion, we have an analogous case in any inference of a particular from a universal. [J.]

311. An invalid syllogism of the second figure with a particular premiss is found to break the general rules of the syllogism in this respect only, that the middle term is undistributed. If the particular premiss is false and the other true, what do we know about the truth or falsity of the conclusion? [K.]

312. A syllogism is found to offend against none of the syllogistic rules except that with two affirmative premisses it has a negative conclusion. Determine the mood and figure of the syllogism. [K.]

313. Given two valid syllogisms in the same figure in which the major, middle, and minor terms are respectively the same, shew, without reference to the mnemonic verses, that if the minor premisses are contradictories, the conclusions will not be contradictories. [K.]

314. Is it possible that there should be two syllogisms having a common premiss such that their conclusions, being combined as premisses in a new syllogism, may give a universal conclusion? If so, determine what the two syllogisms must be. [N.]

315. Three given propositions form the premisses and conclusion of a valid syllogism which is neither strengthened nor weakened. Shew that if two of the propositions are replaced by their contradictories, the argument will still be valid, provided that the proposition remaining unaltered is either a universal premiss or a particular conclusion. [J.]

316. Find out the valid syllogisms that may be constructed without using a universal premiss of the same quality as the conclusion.

Shew how these syllogisms may be directly reduced to one another; and represent diagrammatically the combined information that they yield, on the supposition that they have the same minor, middle, and major terms respectively. [J.]

PART IV.

A GENERALIZATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX PROPOSITIONS¹.

CHAPTER I.

THE COMBINATION OF TERMS.

317. *Complex Terms.*—A simple term may be defined as a term which does not consist of a combination of other terms; for example, *A*, *P*, *X*. The combination of simple terms yields a complex term; and the combination may be either *conjunctive* or *alternative*.

A complex term resulting from the conjunctive combination of other terms may be called a conjunctive term², and it will be found convenient to denote such a term by the simple juxtaposition of the other terms involved³. This kind of combination is sometimes called *determination*⁴; and we may speak of the elements combined in a conjunctive term as the *determinants*

¹ The following pages deal with problems that have ordinarily been relegated to symbolic logic. They do not, however, directly treat of symbolic logic itself, if that term is understood in its ordinary sense, namely, as designating the branch of the science in which symbols of *operation* are used. Of course in one sense all formal logic is symbolic.

² What is here called a conjunctive term is called by Jevons a *combined term* (*Pure Logic*, § 40).

³ The conjunctive combination of terms is in symbolic logic usually represented by the sign of *multiplication*.

⁴ Compare Schröder, *Der Operationskreis des Logikkalküls*, p. 6.

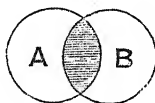
of that term. Thus, A and B are the determinants of the conjunctive term AB ¹.

A complex term resulting from the alternative combination of other terms may be called an *alternative term*²; and we may speak of the elements combined in such a term as the *alternants* of that term. Thus, A and B are the alternants of the alternative term A or B .

In the following pages, in accordance with the view indicated in section 140, the alternants in an alternative term are not regarded as necessarily exclusive of one another (except of course where they are formal contradictories). Thus, if we speak of anything as being A or B we do not intend to exclude the possibility of its being both A and B . In other words, A or B does not exclude AB ³.

It is necessary here to discuss briefly the logical signification of the words *and*, *or*. In the predicate of a proposition their signification is clear; they indicate conjunctive and

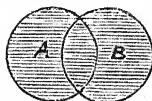
¹ Since AB stands for the class made up of all the individuals that belong both to the class A and to the class B , it is represented by the shaded portion of the following diagram :



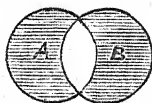
If the classes A and B lie entirely outside one another, then AB is a non-existent class.

² It is called by Jevons a *plural term* (*Pure Logic*, § 63). The alternative combination of terms is in symbolic logic usually represented by the sign of addition.

³ On this view, the shaded portion of the following diagram represents A or B :



On the other interpretation of alternatives, we should have to shade our diagram as follows in order to represent the same term :



alternative combination respectively, for example, *P is Q and R*, *P is Q or R*. But when they occur in the subject of a proposition there is in each case an ambiguity to which attention must be called.

Thus, there would be a gain in brevity if we could write a proposition with an alternative term as subject in the form *P or Q is R*. The latter expression would, however, more naturally be interpreted to mean *P is R or Q is R*, the force of the *or* being understood, not as yielding a single categorical proposition with an alternative subject-term, but as a brief mode of connecting alternatively two propositions with a common predicate. Hence, when we intend the former, the more definite mode of statement, *Whatever is either P or Q is R*, or *Anything that is either P or Q is R*, should be adopted.

There is also ambiguity in the form *P and Q is R*. This would naturally be interpreted, not as a single proposition with a conjunctive subject-term (*PQ is R*), but as a brief mode of connecting conjunctively two propositions with a common predicate, namely, *P is R and Q is R*. In order, therefore, to express unambiguously a proposition with a conjunctive subject-term, it will be well either to adopt the method of simple juxtaposition without any connecting word as, for example, *PQ is R*, or else to employ one of the more cumbrous forms, *Whatever is both P and Q is R*, or *Anything that is both P and Q is R*¹.

318. Order of Combination in Complex Terms.—The order of combination in a complex term is indifferent whether the combination be conjunctive or alternative².

¹ It will be observed that both in this case and in the case of *or*, we get rid of the ambiguity by making the words occur in the predicate of a subordinate sentence. Mr Johnson expresses the substance of the last three paragraphs in the text by pointing out that "common speech adopts the convention: *Subjects are externally synthesised and predicates are internally synthesised*" (*Mind*, 1892, p. 239). In other words, *and* and *or* occurring in a predicate are understood as expressing a conjunctive or an alternative term; but occurring in a subject they are understood as expressing a conjunctive or an alternative proposition.

² This is sometimes spoken of as the law or property of *commutativeness*. Compare Boole, *Laws of Thought*, p. 31, and Jevons, *Principles of Science*, chapter 2, § 8.

Thus, AB and BA have the same signification. It comes to the same thing whether out of the class A we select the B 's or out of the class B we select the A 's.

Again, A or B and B or A have the same signification. It is a matter of indifference whether we form a class by adding the B 's to the A 's or by adding the A 's to the B 's.

319. *The Opposition of Complex Terms.*—However complex a term may be, the criterion of contradictory opposition given in section 28 must still apply: "A pair of contradictory terms are so related that between them they exhaust the entire universe to which reference is made, whilst in that universe there is no individual of which both can be at the same time affirmed." In what follows it will be found convenient to denote the contradictory of any simple term by the corresponding small letter. Thus for *not-A* we may write a , and for *not-B* we may write b .

Now whatever is not AB must be either a or b , whilst nothing that is AB can be either a or b . Hence

$$\begin{cases} AB, \\ a \text{ or } b, \end{cases}$$

constitute a pair of contradictories. Similarly,

$$\begin{cases} A \text{ or } B, \\ ab, \end{cases}$$

are a pair of contradictories. And the same will hold good if A and B stand for terms which are already themselves complex (although relatively simple as compared with AB or A or B).

If, then, two terms are conjunctively combined into a complex term (of which they will constitute the determinants), the contradictory of this complex term is found by alternatively combining the contradictories of the two determinants. And, conversely, if two terms are alternatively combined into a complex term (of which they will constitute the alternants), the contradictory of this complex term is found by conjunctively combining the contradictories of the two alternants.

In each case, we substitute for the relatively simple terms involved their contradictories, and (as the case may be) change

conjunctive combination into alternative combination, or alternative combination into conjunctive combination.

But whatever degree of complexity a term may reach, it will consist of a series of conjunctive and alternative combinations; and it may be successively resolved into the combination of pairs of relatively simple terms till it is at last shewn to result from the combination of absolutely simple terms. For example,—

$$ABC \text{ or } DE \text{ or } FG$$

results from the alternative combination of the pair—

$$\left\{ \begin{array}{l} ABC \text{ or } DE, \\ FG; \end{array} \right.$$

$ABC \text{ or } DE$ results from the alternative combination of the pair—

$$\left\{ \begin{array}{l} ABC, \\ DE; \end{array} \right.$$

FG results from the conjunctive combination of the pair—

$$\left\{ \begin{array}{l} F, \\ G; \end{array} \right.$$

and ABC, DE , may be resolved similarly.

Hence the successive application of the above rule for finding the contradictory of a complex term where we are dealing with a single pair of determinants or alternants will result in our ultimately substituting for each simple term involved its contradictory, and reversing the nature of their combination throughout¹. We may, therefore, lay down the following rule for obtaining the contradictory of any complex term: Replace each constituent simple term by its contradictory, and throughout substitute conjunctive combination for alternative

¹ Thus, taking the term $ABC \text{ or } DE \text{ or } FG$, and in the first instance denoting the contradictory of a complex term by a bar drawn across it, we have successively

$$\begin{aligned} & \overline{ABC \text{ or } DE \text{ or } FG} \\ &= \overline{ABC} (\overline{DE \text{ or } FG}) \\ &= (\overline{AB \text{ or } c}) \overline{DE} . \overline{FG} \\ &= (a \text{ or } b \text{ or } c) (d \text{ or } e) (f \text{ or } g). \end{aligned}$$

*combination and vice versa*¹. This rule is of simple application, and it is of fundamental importance in the treatment of complex propositions adopted in the following pages.

Thus, the contradictory of

$A \text{ or } BC$

is $a \text{ and } (b \text{ or } c)$,

i.e., $ab \text{ or } ac$;

and the contradictory of

$ABC \text{ or } ABD$

is $(a \text{ or } b \text{ or } c) \text{ and } (a \text{ or } b \text{ or } d)$,

which, by the aid of rules presently to be given, is reducible to the form

$a \text{ or } b \text{ or } cd$.

It is possible for two complex terms to be formally *inconsistent* or *repugnant* without being true contradictories. This will be the case if they contain contradictory determinants without between them exhausting the universe of discourse. The terms AB and bC afford an example: nothing can be both AB and bC (for, if this were so, something would be both B and *not-B*), but we cannot say *a priori* that everything is either AB or bC (since something may be Abc , which is neither AB nor bC).

If two conjunctive terms are such that every determinant in the one has corresponding to it in the other its contradictory, these two terms may be regarded as in the strictest sense logical contraries². Thus, AbC , aBc may be spoken of as contraries. An alternative term, such as $AB \text{ or } ab$, does not seem to admit of a contrary in this distinctive sense.

320. Duality of Formal Equivalences in the case of Complex Terms.—It will be shewn in the following sections that certain complex terms are formally equivalent to other complex terms or to simple terms (for example, $A \text{ or } aB = A \text{ or } B$, $A \text{ or } AB = A$); and it is important to notice at the outset that such formal equivalences always go in pairs. For if two

¹ Compare Schröder, *Der Operationskreis des Logikkalküls*, p. 18.

² Compare section 29.

terms are equivalent, their contradictories must also be equivalent; and hence, applying the rule for obtaining contradictories given in the preceding section, we are enabled to formulate the simple law that *to every formal equivalence there corresponds another formal equivalence in which conjunctive combination is throughout substituted for alternative combination and vice versa*¹. This law may be more precisely established as follows:—A formal equivalence that holds good for any given set of simple terms must equally hold good for any other set of simple terms; and, therefore, whatever holds good for the terms A , B , &c. must equally hold good for their contradictories a , b , &c. Hence, given any equivalence, we may first replace each simple term by its contradictory, and then take the contradictory of each side of the equivalence. The result of this double transformation will be that we shall obtain another equivalence in which every conjunctive combination has been replaced by an alternative combination, and conversely, while the term-symbols involved have remained unchanged. This proves what was required.

The application of the above law will be fully illustrated in the sections that immediately follow.

321. Laws of Distribution.—In order to combine a simple term conjunctively with an alternative term, we must conjunctively combine it with every alternant of the alternative². A and $(B \text{ or } C)$ ³ denotes whatever is A and at the same time either B or C , and it is, therefore, equivalent to $AB \text{ or } AC$. It follows that in order to combine two alternative terms conjunctively, we must conjunctively combine every alternant of the one with every alternant of the other. Thus, $(A \text{ or } B)(C \text{ or } D)$ denotes whatever is either A or B and at the same time either C or D , and it is equivalent to $AC \text{ or } AD \text{ or } BC \text{ or } BD$ ⁴.

¹ This is pointed out by Schröder, *Der Operationskreis des Logikkalküls*, p. 3. The two equivalences which are thus mutually deducible the one from the other may be said to be *reciprocal*.

² Compare Jevons, *Principles of Science*, chapter 5, § 7.

³ In such a case as this the use of brackets is necessary in order to avoid ambiguity. Thus, A and B or C might mean $AB \text{ or } C$, or as above $AB \text{ or } AC$.

⁴ Whether or not we introduce algebraic symbols into logic, there is here a very close analogy with algebraic multiplication which cannot be disguised.

We have then

$$A (B \text{ or } C) = AB \text{ or } AC,$$

and applying the law of duality of formal equivalences given in the preceding section, we have at once another equivalence, namely,

$$A \text{ or } BC = (A \text{ or } B) (A \text{ or } C)^1.$$

These two equivalences are called by Schröder the *Laws of Distribution*². They are of the greatest importance in the manipulation and simplification of complex terms.

322. Laws of Tautology.—The following rules may be laid down for the omission of superfluous terms from a complex term :

(a) *The repetition of any given determinant is superfluous.*

Out of the class A to select the A 's is a process that leaves us just where we began. In other words, what is both A and A is identical with what is A . Thus, such terms as AA , ABB , are tautologous; the former merely denotes the class A , and the latter the class AB . Hence the above rule, which is called by Jevons the *Law of Simplicity*³.

(b) *The repetition of any given alternant is superfluous.*

To say that anything is $A \text{ or } A$ is equivalent to saying simply that it is A . Hence such terms as $A \text{ or } A$, $A \text{ or } BC \text{ or } BC$, are tautologous; and we have the above rule, which is called by Jevons the *Law of Unity*⁴.

It will be seen by reference to the rule given in section 320 that the Law of Simplicity ($AA = A$) and the Law of Unity ($A \text{ or } A = A$) are reciprocal, that is, the former is deducible from the latter and *vice versa*. For the only difference between them is that conjunctive combination in the one is replaced by alternative combination in the other⁵.

¹ This equivalence might also be established independently by the aid of certain of the equivalences given in the following sections.

² *Der Operationskreis des Logikkalküls*, pp. 9, 10.

³ See *Pure Logic*, § 42; and *Principles of Science*, chapter 2, § 8. The corresponding equation $x^2 = x$ is in Boole's system fundamental; see *Laws of Thought*, p. 31.

⁴ See *Pure Logic*, § 69; and *Principles of Science*, chapter 5, § 4.

⁵ It may assist the reader in following the reasoning in section 320 if we work through this particular case independently. If $AA = A$, then $aa = a$, for

323. Laws of Development and Reduction.—Important formal equivalences are yielded by the laws of contradiction and excluded middle.

By the law of contradiction a term containing contradictory determinants (for example, Bb) cannot represent any existing class. Hence the term A or Bb is equivalent to A simply; in other words, the *conjunctive combination of contradictories* may be indifferently introduced or omitted as an *alternant*.

Again, by the law of excluded middle a term containing contradictory alternants (for example, B or b) represents the entire universe of discourse. Hence the term A (B or b) is equivalent to A simply; in other words, the *alternative combination of contradictories* may be indifferently introduced or omitted as a *determinant*.

The above equivalences, namely,

$$A \text{ or } Bb = A,$$

$$A (B \text{ or } b) = A,$$

are reciprocal; that is to say, either is deducible from the other by the rule given in section 320.

Applying further the Laws of Distribution given in section 321 we have the following:

$$A = A \text{ or } Bb = (A \text{ or } B) (A \text{ or } b),$$

$$A = A (B \text{ or } b) = AB \text{ or } Ab.$$

These may be taken as formulæ for the *development* and the *reduction* of terms. Thus, the substitution of $(A \text{ or } B) (A \text{ or } b)$ for A may be called the *development of a term by means of the law of contradiction*; and the substitution of $AB \text{ or } Ab$ for A the *development of a term by means of the law of excluded middle*. In both the above cases the term A is developed with reference to the term B . Similarly by developing A with reference to B and C , we should have $(A \text{ or } B \text{ or } C) (A \text{ or } B \text{ or } c) (A \text{ or } b \text{ or } C) (A \text{ or } b \text{ or } c)$ if we make use of the law of contradiction, or $ABC \text{ or } ABc \text{ or } AbC \text{ or } Abc$ if we make use of the law of excluded middle. Development by means of the law of

whatever is formally valid in the case of A must also be formally valid in the case of any other term. But if two terms are equivalent, their contradictories must be equivalent. Hence from $aa=a$, it follows that $A \text{ or } A=A$. And it is clear that we might pass similarly from $A \text{ or } A=A$ to $AA=A$.

excluded middle is by far the more useful of the two processes in the manipulation of complex terms, and it may be understood that this is meant when the development of a term is spoken of without further qualification.

Conversely, the process of passing from $(A \text{ or } B)$ ($A \text{ or } b$) to A , or from $AB \text{ or } Ab$ to A , may be called the *reduction of a term* by means of the law of contradiction or the law of excluded middle, as the case may be.

Following Jevons, we may speak of an alternative term of the type $AB \text{ or } Ab$ as a *dual term*¹, and of the substitution of A for $AB \text{ or } Ab$ as the *reduction of a dual term*².

324. Laws of Absorption.—It may be shewn that any alternant which is merely a subdivision of another alternant may be indifferently introduced or omitted from a complex term. Thus, AB being a subdivision of A , the terms $A \text{ or } AB$ and A are equivalent. This rule (which is called by Schröder the *Law of Absorption*³) may be established as follows: By the development of A with reference to B , $A \text{ or } AB$ becomes $AB \text{ or } Ab \text{ or } AB$; but, by the law of unity, this is equivalent to $AB \text{ or } Ab$; and this is a dual term reducible to A .

Applying the rule given in section 320 we obtain a second law of absorption, namely, $A (A \text{ or } B) = A$, which is the reciprocal of the first law of absorption, $A \text{ or } AB = A$.

325. Laws of Exclusion and Inclusion.—The contradictory of any alternant in a complex term may be indifferently introduced or omitted as a determinant of any other alternant; that is to say, the terms $A \text{ or } aB$ and $A \text{ or } B$ are equivalent. This may be established as follows: By the law of absorption $A \text{ or } aB$ is equivalent to $A \text{ or } AB \text{ or } aB$, and by the reduction of the dual term here contained, this yields $A \text{ or } B$. The above equivalence may be called the *Law of Exclusion* on the

¹ See *Pure Logic*, § 103. Corresponding to this, the law of excluded middle is usually called by Jevons the *Law of Duality*; compare *Pure Logic*, § 99, and *Principles of Science*, chapter 5, § 5.

² Jevons (*Principles of Science*, chapter 6, § 9) also speaks of it as the "abstraction of indifferent circumstances."

³ *Der Operationskreis des Logikkalküls*, p. 12. This Law of Absorption is equivalent to one of Boole's "Methods of Abbreviation" (*Laws of Thought*, p. 130). Compare, also, Jevons, *Pure Logic*, § 70.

ground that by passing from A or B to A or aB we make the alternants mutually exclusive.

The reciprocal equivalence $A (a \text{ or } B) = AB$ may be expressed as follows: The contradictory of any determinant in a complex term may be indifferently introduced or omitted as an alternant of any other determinant. This equivalence may be called the *Law of Inclusion* on the ground that by passing from AB to $A (a \text{ or } B)$ we make the determinants collectively inclusive of the entire universe of discourse.

326. *Summary of Formal Equivalences of Complex Terms.*—The following is a summary of the formal equivalences contained in the five preceding sections (those that are bracketed together being in each case related to one another reciprocally in the manner indicated in section 320):—

- | | | |
|-------------------------------------------------------------------|---|---------------------------------------------|
| (1) $A (B \text{ or } C) = AB \text{ or } AC,$ | } | <i>Laws of Distribution;</i> |
| (2) $A \text{ or } BC = (A \text{ or } B) (A \text{ or } C),$ | | |
| (3) $AA = A,$ | } | <i>Laws of Tautology (Law of Simplicity</i> |
| (4) $A \text{ or } A = A,$ | | |
| | | <i>and Law of Unity);</i> |
| (5) $A = A \text{ or } Bb = (A \text{ or } B) (A \text{ or } b),$ | } | <i>Laws of Develop-</i> |
| (6) $A = A (B \text{ or } b) = AB \text{ or } Ab,$ | | |
| | | <i>ment and Reduction;</i> |
| (7) $A \text{ or } AB = A,$ | } | <i>Laws of Absorption;</i> |
| (8) $A (A \text{ or } B) = A,$ | | |
| (9) $A \text{ or } B = A \text{ or } aB,$ | } | <i>Law of Exclusion and Law of</i> |
| (10) $AB = A (a \text{ or } B),$ | | |
| | | <i>Inclusion.</i> |

By bearing the above equivalences in mind, the labour of manipulating complex propositions may be very much diminished.

327. *The Conjunctive Combination of Alternative Terms.*—The first law of distribution gives the general rule for the conjunctive combination of alternatives. But with a view to such combination special attention may be called (i) to the second law of distribution, namely, $(A \text{ or } B) (A \text{ or } C) = A \text{ or } BC$; and (ii) to the equivalence $(A \text{ or } B) (AC \text{ or } D) = AC \text{ or } AD \text{ or } BD$, which may be established as follows: By the first law of distribution $(A \text{ or } B) (AC \text{ or } D)$ is equivalent to $AAC \text{ or } ABC \text{ or } AD \text{ or } BD$; but by the law of simplicity $AAC = AC$, and by the law of absorption $AC \text{ or } ABC = AC$; hence our original term is equivalent to $AC \text{ or } AD \text{ or } BD$, which was to be proved.

From the above equivalences we obtain the two following practical rules which are of the greatest assistance in simplifying the process of conjunctively combining alternatives:

(1) If two alternatives which are to be conjunctively combined have an alternant in common, this alternant may be at once written down as one alternant of the result, and we need not go through the form of combining it with any of the remaining alternants of either alternative;

(2) If two alternatives are to be conjunctively combined and an alternant of one is a subdivision of an alternant of the other, then the former alternant may be at once written down as one alternant of the result, and we need not go through the form of combining it with the remaining alternants of the other alternative¹.

EXERCISES.

328. Simplify the following terms: (i) AD or acD ; (ii) Ad or Ae or aB or aC or bC or aE or bE or bd or be or cd or ce . [K.]

(i) By rule (1) in section 326, AD or acD is equivalent to $(A$ or $ac) D$; and this by rule (9) is equivalent to $(A$ or $c) D$; which again by rule (1) is equivalent to AD or cD ².

(ii) Since bE or be is a dual term, it may be reduced to b , and hence Ad or Ae or aB or aC or bC or aE or bE or bd or be or cd or $ce = aB$ or aC or bC or aE or b or Ad or Ae or bd or cd or ce . By section 326, rule (7), we may now omit all alternants in which b occurs as a determinant, and by rule (9), B may be omitted wherever it occurs as a determinant; accordingly our term is reduced to a or aC or aE or b or Ad or Ae or cd or ce . Since a is now an alternant, a further application of the same rules leaves us with a or b or d or e or cd or ce ; and this is immediately reducible to a or b or d or e .

329. Shew that BC or bD or CD is equivalent to BC or bD . [K.]

¹ These rules are equivalent to Boole's second Method of Abbreviation (*Laws of Thought*, p. 131).

² We might also proceed as follows: AD or $acD = AD$ or AcD or acD [by rule (7)] = AD or cD [by rule (5)].

330. Give the contradictories of the following terms in their simplest forms as series of alternants:—

AB or BC or CD ;

AB or bC or cD ;

ABC or aBc ;

$ABcD$ or $Abcde$ or $aBCDe$ or $BCde$.

[K.]

331. Simplify the following terms:

(1) Ab or aC or BCd or Bc or bD or CD ;

(2) ACD or Ac or Ad or aB or bCD ;

(3) aBC or aCD or aBe or aDe or AcD or abD or bcD or aDE or cDE ;

(4) $(A$ or $b)$ $(A$ or $c)$ $(a$ or $B)$ $(a$ or $C)$ $(b$ or $C)$.

[K.]

332. Prove the following equivalences :

(1) AB or AC or BC or abc or aB or $C = a$ or B or C ;

(2) aBC or aBd or acd or bed or ABd or $Ac d$ or abd or aCd or $BCd = Bd$ or cd or ad or aBC ;

(3) Pqr or pQs or prs or qrs or pq or pS or $qR = p$ or q .

[K.]

CHAPTER II.

COMPLEX PROPOSITIONS AND COMPOUND PROPOSITIONS.

333. *Complex Propositions.*—A complex proposition may be defined as a proposition which has a complex term either for its subject or its predicate. The ordinary distinctions of quantity and quality may be applied to complex propositions; thus, *All AB is C or D* is a universal affirmative complex proposition. *Some AB is not EF* is a particular negative complex proposition. In the following pages propositions written in the indefinite form will be interpreted as universal, so that *AB is CD* will be understood to mean that *all AB is CD*. It must be added that in dealing with complex propositions we definitely adopt the view that universals do not imply, while particulars do imply, the existence of their subjects in the universe of discourse.

334. *The Opposition of Complex Propositions.*—The opposition of complex terms has been already dealt with, and the opposition of complex propositions in itself presents no special difficulty. It must, however, be borne in mind that since in dealing with complex propositions we definitely adopt the view that particulars imply the existence of their subjects while universals do not, we have the following divergences from the ordinary doctrine of opposition: (1) we cannot infer **I** from **A**, or **O** from **E**; (2) **A** and **E** are not necessarily inconsistent with each other; (3) **I** and **O** may both be false at the same time. The ordinary doctrine of *contradictory opposition* remains unaffected. The following are examples of contradictory propositions: *All X is both A and B*, *Some X is not both A*

and B ; Some X is Y and at the same time either P or Q or R ,
No X is Y and at the same time either P or Q or R .

335. Compound Propositions.—A compound proposition may be defined as a proposition which consists in a combination of other propositions. The combination may be either conjunctive (*i.e.*, when two or more propositions are affirmed to be true together) or alternative (*i.e.*, when an alternative is given between two or more propositions); for example, *All AB is C and some P is not either Q or R* is a compound conjunctive proposition; *Either all AB is C or some P is not either Q or R* is a compound alternative proposition. Propositions conjunctively combined may be spoken of as *determinants* of the resulting compound proposition; and propositions alternatively combined may be spoken of as *alternants* of the resulting compound proposition.

Only two types of compound propositions are here recognised, the *conjunctive* and the *alternative*. Pure hypothetical propositions are compound, but they are, as we have already seen, equivalent to alternative propositions, and they may accordingly be regarded as constituting one mode of expressing an alternative synthesis. Thus (taking x and y as symbols representing propositions, and \bar{x} and \bar{y} as their contradictories) the hypothetical proposition *If x then y* expresses an alternative between \bar{x} and y and is, therefore, equivalent to the alternative proposition \bar{x} or y . Combinations of the true disjunctive type (for example, *not both x and y*) may also be regarded as a mode of expressing an alternative synthesis; thus, the true disjunctive proposition just given is equivalent to the alternative proposition \bar{x} or \bar{y} ¹.

Mr Johnson shews that any ordinary proposition with a

¹ The above may seem to imply that an alternative synthesis may be expressed in a greater number of ways than a conjunctive synthesis. This, however, is not the case. It has been shewn that an alternative synthesis may be expressed by a hypothetical or by the denial of a conjunctive (that is, a true disjunctive). But corresponding to this, a conjunctive synthesis may be expressed by the denial of a hypothetical or by the denial of an alternative. Thus, representing the denial of a proposition by a bar drawn across it, we have

$$\begin{aligned} xy &= \overline{\bar{x} \text{ or } \bar{y}} = \text{If } x, \bar{y}; \\ \overline{xy} &= \bar{x} \text{ or } \bar{y} = \text{If } x, \bar{y}. \end{aligned}$$

general term as subject may be regarded as a compound proposition resulting from the conjunctive or alternative combination of singular (molecular) propositions, with a common predication, but different subjects. Let $S_1, S_2, \dots S_\infty$ represent a number of different individual subjects; and let S represent the aggregate collection of individuals $S_1, S_2, \dots S_\infty$. Then

S_1 and S_2 and $S_3 \dots$ and $S_\infty = \text{Every } S$;

S_1 or S_2 or $S_3 \dots$ or $S_\infty = \text{Some } S$.

"Thus we arrive at the common logical forms, *Every S is P*, *Some S is P*. The former is an abbreviation for a *determinative*, the latter for an *alternative*, synthesis of molecular propositions."¹ In other words,

Every S is P = S_1 is P and S_2 is P and S_3 is $P \dots$ and S_∞ is P ;
Some S is P = S_1 is P or S_2 is P or S_3 is $P \dots$ or S_∞ is P .

336. The Opposition of Compound Propositions.—The rule for obtaining the contradictory of a complex term given in section 319 may be applied also to compound propositions. Thus, the contradictory of a compound proposition is obtained by replacing the constituent propositions by their contradictories and everywhere changing the manner of their combination, that is to say, substituting conjunctive combination for alternative and *vice versa*². The following are examples: *All A is B and some P is Q* has for its contradictory *Either some A is not B or no P is Q*; *Either some A is both B and C, or all B is either C or both D and E* has for its contradictory *No A is both B and C, and some B is not either C or both D and E*.

It follows, as in section 320, that there is a duality of formal equivalences in the case of compound propositions, each equivalence yielding a reciprocal equivalence in which conjunctive

¹ *Mind*, 1892, p. 25. Mr Johnson of course recognises that a quantified subject-term (*all S*) is not usually a mere enumeration of individuals first apprehended and named. But he points out that "however the aggregate of things, to which the universal name applies, is mentally reached, the propositional force for purposes of inference or synthesis in general is the same" (p. 28).

² It has been shewn in the preceding section that the words *all* and *some* are abbreviations of conjunctive and alternative synthesis respectively. Hence the rule that, in the ordinarily recognised propositional forms, contradictories differ in quantity as well as in quality is itself only a particular application of the general law here laid down.

combination is throughout substituted for alternative combination and *vice versa*.

337. Formal Equivalences of Compound Propositions.—

The laws relating to the conjunctive or alternative synthesis of propositions are practically identical with those relating to the conjunctive or alternative combination of terms; and we have accordingly the following propositional equivalences corresponding to the equivalences of terms given in section 326. The symbols here stand for *propositions*, not terms; and *negation* is represented by a bar over the proposition denied.

- | | |
|------------------------------------------------------------------------------|---------------------------------------------------|
| (1) $x(y \text{ or } z) = xy \text{ or } xz,$ | } <i>Laws of Distribution ;</i> |
| (2) $x \text{ or } yz = (x \text{ or } y)(x \text{ or } z),$ | |
| (3) $xx = x,$ | } <i>Laws of Tautology (Law of Simplicity and</i> |
| (4) $x \text{ or } x = x,$ | |
| (5) $x = x \text{ or } y\bar{y} = (x \text{ or } y)(x \text{ or } \bar{y}),$ | } <i>Laws of Development</i> |
| (6) $x = x(y \text{ or } \bar{y}) = xy \text{ or } x\bar{y},$ | |
| (7) $x \text{ or } xy = x,$ | } <i>Laws of Absorption ;</i> |
| (8) $x(x \text{ or } y) = x,$ | |
| (9) $x \text{ or } y = x \text{ or } \bar{x}y,$ | } <i>Law of Exclusion and Law of</i> |
| (10) $xy = x(\bar{x} \text{ or } y),$ | |
- } *Inclusion*¹.

¹ It is not maintained that all the above laws are ultimate or even independent of one another. The synthesis of propositions is admirably worked out by Mr Johnson in his articles on *the Logical Calculus* (*Mind*, 1892). He gives *five independent laws* which are necessary and sufficient for propositional synthesis. These laws are briefly enumerated below; for a more complete exposition the reader must be referred to Mr Johnson's own treatment of them.

(i) *The Commutative Law*: The order of pure synthesis is indifferent ($xy = yx$).

(ii) *The Associative Law*: The mode of grouping in pure synthesis is indifferent ($xy . z = x . yz$).

(iii) *The Law of Tautology*: The mere repetition of a proposition does not in any way add to or alter its force ($xx = x$).

(iv) *The Law of Reciprocity*: The denial of the denial of a proposition is equivalent to its affirmation ($\bar{\bar{x}} = x$). "In this principle are included the so-called Laws of Contradiction and Excluded Middle, *viz.*, 'If x , then not not- x ', and 'If not not- x , then x .'"

(v) *The Law of Dichotomy*: The denial of any proposition is equivalent to the denial of its conjunction with any other proposition together with the denial of its conjunction with the contradictory of that other proposition ($\bar{x} = \bar{x}y \bar{x}\bar{y}$). "This is a further extension of the Law of Excluded Middle, when applied to the combination of propositions with one another. The denial that x is conjoined with y combined with the denial that x is conjoined with *not-y* is equiva-

338. *The Simplification of Complex Propositions.*—The terms of a complex proposition may of course often be simplified by means of the rules given in the preceding chapter, and the force of the assertion will remain unaffected. For the further simplification of complex propositions the following rules may be added:

(1) *In a universal negative or a particular affirmative proposition any determinant of the subject may be indifferently introduced or omitted as a determinant of the predicate, and vice versâ.*

To say that *No AB is AC* is the same as to say that *No AB is C*, or that *No B is AC*. For to say that *No AB is AC* is the same thing as to deny that anything is *ABAC*; but, as shewn in section 322, the repetition of the determinant *A* is superfluous, and the statement may therefore be reduced to the denial that anything is *ABC*. And this may equally well be expressed by saying *No AB is C*, or *No B is AC*¹.

Similarly, *No AB is AC or AD* may be reduced to *No AB is C or D*, or to *No B is AC or AD*.

Again, *Some AB is AC* may be shewn to be equivalent to *Some AB is C*, or to *Some B is AC*; for it simply affirms that something is *ABAC*, and the proof follows as above.

(2) *In a universal affirmative or a particular negative proposition any determinant of the subject may be indifferently introduced or omitted as a determinant of any alternant of the predicate.*

All A is AB may obviously be resolved into the two propositions *All A is A*, *All A is B*². But the former of these is a merely identical proposition and gives no information. *All A is AB* is, therefore, equivalent to the simple proposition *All A is B*.

lent to the denial of *x* absolutely. For, if *x* were true, it must be conjoined either with *y* or with *not-y*. This law, which (it must be admitted) looks at first a little complicated, is the special instrument of the logical calculus. By its means we may always resolve a proposition into two determinants, or conversely we may compound certain pairs of determinants into a single proposition."

¹ See also the sections in chapter 3 relating to the conversion of propositions.

² The resolution of complex propositions into a combination of relatively simple ones will be considered further in the following section.

Similarly, *All AB is AC or DE* is equivalent to *All AB is C or DE*.

Again, *Some A is not AB* affirms that *Some A is a or b¹*; but by the law of contradiction *No A is a*; therefore, *Some A is not B*, and obviously we can also pass back from this proposition to the one from which we started.

Similarly, *Some AB is not either AC or DE* is equivalent to *Some AB is not either C or DE*.

(3) *In a universal affirmative or a particular negative proposition any alternant of the predicate may be indifferently introduced or omitted as an alternant of the subject.*

If *All A is B or C*, then by the law of identity it follows that *Whatever is A or B is B or C*; it is also obvious that we can pass back from this to the original proposition.

Again, if *Some A or B is not either B or C*, then since by the law of identity *All B is B* it follows that *Some A is not either B or C*; and it is also obvious that we can pass back from this to the original proposition².

(4) *In a universal affirmative or a particular negative proposition the contradictory of any determinant of the subject may be indifferently introduced or omitted as an alternant of the predicate, and vice versa.*

By this rule the three following propositions are affirmed to be equivalent to one another:

$$\begin{cases} \text{All } AB \text{ is } a \text{ or } C; \\ \text{All } B \text{ is } a \text{ or } C; \\ \text{All } AB \text{ is } C; \end{cases}$$

and also the three following:

$$\begin{cases} \text{Some } AB \text{ is not either } a \text{ or } C; \\ \text{Some } B \text{ is not either } a \text{ or } C; \\ \text{Some } AB \text{ is not } C. \end{cases}$$

The rule follows directly from rule (1) by aid of the process of obversion (see chapter 3).

(5) *In a universal negative or a particular affirmative*

¹ The process of obversion will be considered in detail in chapter 3.

² What follows to the end of the section may be omitted till the chapter on immediate inferences from complex propositions has been read.

proposition the contradictory of any determinant of the subject may be indifferently introduced or omitted as an alternant of the predicate.

By this rule the two following propositions are affirmed to be equivalent to one another:

$$\begin{cases} \text{No } AB \text{ is } a \text{ or } C; \\ \text{No } AB \text{ is } C; \end{cases}$$

and also the two following:

$$\begin{cases} \text{Some } AB \text{ is } a \text{ or } C; \\ \text{Some } AB \text{ is } C. \end{cases}$$

The rule follows directly from rule (2) by obversion.

(6) *In a universal negative or a particular affirmative proposition the contradictory of any determinant of the predicate may be indifferently introduced or omitted as an alternant of the subject.*

This rule follows from rule (3) by obversion.

339. *The Resolution of Universal Complex Propositions into Equivalent Compound Propositions.*—We may enquire how far complex propositions are immediately resolvable into a conjunctive or alternative combination of relatively simple propositions. Universal propositions will be considered in this section, and particulars in the next.

Universal Affirmatives. Universal affirmative complex propositions may be immediately resolved into a conjunction of relatively simple ones, so far as there is alternative combination in the subject or conjunctive combination in the predicate.

Thus, (1) *Whatever is P or Q is R* = *All P is R and all Q is R*;

(2) *All P is QR* = *All P is Q and all P is R.*

Universal Negatives. Universal negative complex propositions may be immediately resolved into a conjunction of relatively simple ones, so far as there is alternative combination either in the subject or in the predicate. Thus,

(3) *Nothing that is P or Q is R* = *No P is R and no Q is R*;

(4) *No P is either Q or R* = *No P is Q and no P is R.*

So far as there is conjunctive combination in the subject or alternative combination in the predicate of universal affirmative propositions, or conjunctive combination either in the

subject or in the predicate of universal negative propositions, they cannot be *immediately*¹ resolved into either a conjunctive or an alternative combination of simpler propositions. It may, however, be added that propositions falling into this latter category are immediately *implied by* certain compound alternatives. Thus,

- (i) *All PQ is R* is implied by *All P is R* or *all Q is R*;
- (ii) *All P is Q or R* is implied by *All P is Q* or *all P is R*;
- (iii) *No PQ is R* is implied by *No P is R* or *no Q is R*;
- (iv) *No P is QR* is implied by *No P is Q* or *no P is R*.

340. *The Resolution of Particular Complex Propositions into Equivalent Compound Propositions.*—Complex particular propositions cannot be resolved into compound conjunctives, but they may under certain conditions be immediately resolved into equivalent *compound alternative propositions* in which the alternants are relatively simple. This is the case so far as there is alternative combination in the subject or conjunctive combination in the predicate of a particular negative, or alternative combination either in the subject or in the predicate of a particular affirmative. Thus,

- (1) *Some P or Q is not R* = *Some P is not R* or *some Q is not R*;
- (2) *Some P is not QR* = *Some P is not Q* or *some P is not R*;
- (3) *Some P or Q is R* = *Some P is R* or *some Q is R*;
- (4) *Some P is Q or R* = *Some P is Q* or *some P is R*.

Particular complex propositions cannot be immediately resolved into compound propositions (either conjunctive or alternative) so far as there is conjunctive combination in the subject or alternative combination in the predicate if the proposition is negative, or so far as there is conjunctive combination either in the subject or in the predicate if the proposition is affirmative. In these cases, however, the complex proposition *implies* a compound conjunctive proposition, though we cannot pass back from the latter to the former.

¹ It will be shewn subsequently that even in these cases universal complex propositions may be resolved into a conjunction of relatively simpler ones by the aid of certain immediate inferences.

Thus, (i) *Some PQ is not R* implies *Some P is not R* and *some Q is not R*;

(ii) *Some P is not either Q or R* implies *Some P is not Q* and *some P is not R*;

(iii) *Some PQ is R* implies *Some P is R* and *some Q is R*;

(iv) *Some P is QR* implies *Some P is Q* and *some P is R*.

It must be particularly noticed that although in these cases the compound proposition can be inferred from the complex proposition, still the two are not equivalent. For example, from *Some P is Q* and *some P is R* it does not follow that *Some P is QR*, for we cannot be sure that the same *P*'s are referred to in the two cases.

All the results of this section follow from those of the preceding by applying the rule of contradiction to the propositions themselves and the rule of contraposition to the relations of implication between them.

341. *The Omission of Terms from a Complex Proposition.*—

From the two preceding sections we may obtain immediately the following rules for inferring from a given proposition another proposition in which certain terms contained in the original proposition are omitted:

(1) *A determinant may at any time be omitted from an undistributed term*¹;

(2) *An alternant may at any time be omitted from a distributed term*².

For example,—

Whatever is A or B is CD, therefore, *All A is C*;

Some AB is CD, therefore, *Some A is C*;

Nothing that is A or B is C or D, therefore, *No A is C*;

Some AB is not either C or D, therefore, *Some A is not C*.

The above rules may also be justified independently, as will be shewn in the following section. The results which they yield must be distinguished from those obtained in section 338. In the cases discussed in that section, the terms omitted were superfluous in the sense that their omission left us with propositions equivalent to our original propositions; but in the

¹ The subject of a particular or the predicate of an affirmative proposition.

² The subject of a universal or the predicate of a negative proposition.

above inferences we cannot pass back from conclusion to premiss. From *Some A is C*, for example, we cannot infer that *Some AB is C*.

342. *The Introduction of Terms into a Complex Proposition.*—Corresponding to the rules laid down in the preceding section we have also the following:

(1) *A determinant may at any time be introduced into a distributed term;*

(2) *An alternant may at any time be introduced into an undistributed term.*

These rules, and also the rules given in the preceding section, may be established by the aid of the following axioms: *What is true of all (distributively) is true of every part; What is true of part of a part is true of a part of the larger whole.*

When we add a determinant to a term, or remove an alternant, we potentially narrow the extension of the term; when, on the other hand, we add an alternant, or remove a determinant, we potentially widen its extension. Hence it follows that if a term is distributed we may add a determinant or remove an alternant, whilst if a term is undistributed we may add an alternant or remove a determinant.

Thus, *All A is CD*, therefore, *All AB is C*;

No A is C, therefore, *No AB is CD*;

Some AB is C, therefore, *Some A is C or D*;

Some AB is not either C or D, therefore, *Some A is not C*.

From the above rules taken in connexion with the rules given in section 338 we may obtain the following corollaries:

(3) *In universal affirmatives, any determinant may be introduced into the predicate, if it is also introduced into the subject; and any alternant may be introduced into the subject if it is also introduced into the predicate.*

Given *All A is C*, then *All AB is C* by rule (1) above; and from this we obtain *All AB is BC* by rule (2) of section 338.

Again, given *All A is C*, then *All A is B or C*; and therefore, by rule (3) of section 338, *Whatever is A or B is B or C*.

(4) *In universal negatives any alternant may be introduced into subject or predicate, if its contradictory is introduced into the other term as a determinant.*

Given *No A is C*, then *No AB is C*; and, therefore, by rule (5) of section 338, *No AB is b or C*.

Again, given *No A is C*, then *No A is BC*; and, therefore, by rule (6) of section 338, *No A or b is BC*.

In none of the inferences considered in this section is it possible to pass back from the conclusion to the original proposition.

343. Interpretation of Anomalous Forms.—It will be found that propositions which apparently involve a contradiction in terms and are thus in direct contravention of the fundamental laws of thought—for example, *No AB is B*, *All Ab is B*—sometimes result from the manipulation of complex propositions. In interpreting such propositions as these, a distinction must be drawn between universals and particulars, at any rate if it is held that particulars imply, while universals do not imply, the existence of their subjects.

It can be shewn that a universal proposition of the form *No AB is B* or *All Ab is B* must be interpreted as affirming the non-existence of the subject of the proposition. For a universal negative denies the existence of anything that comes under both its subject and its predicate; thus, *No AB is B* denies the existence of *ABB*, that is, it denies the existence of *AB*. Again, a universal affirmative denies the existence of anything that comes under its subject without also coming under its predicate; thus, *All Ab is B* denies the existence of anything that is *Ab* and at the same time *not-B*, that is, *b*; but *Ab* is *Ab* and also *b*, and hence the existence of *Ab* is denied.

Since the existence of its subject is held to be part of the implication of a particular proposition, the above interpretation is obviously inapplicable in the case of particulars. Hence if a proposition of the form *Some Ab is B* is obtained, we are thrown back on the alternative that there is some inconsistency in the premisses; either some one individual premiss is self-contradictory, or the premisses are inconsistent with one another.

EXERCISES.

344. Shew that if *No A is bc or Cd*, then *No A is bd*. [K.]

345. Give the contradictory of each of the following propositions:—(1) Flowering plants are either endogens or exogens, but not both; (2) Flowering plants are vascular, and either endogens or exogens, but not both. [M.]

346. Simplify the following propositions:—

(1) *All AB is BC or be or CD or cE or DE*;

(2) *Nothing that is either PQ or PR is Pqr or pQs or prs or qrs or pq or pS or qR*. [K.]

CHAPTER III.

IMMEDIATE INFERENCES FROM COMPLEX PROPOSITIONS.

347. The Obversion of Complex Propositions.—The doctrine of obversion is immediately applicable to complex propositions; and no modification of the definition of obversion already given is necessary. From any given proposition we may infer a new one by changing its quality and taking as a new predicate the contradictory of the original predicate. The proposition thus obtained is called the obverse of the original proposition.

The only difficulty connected with the obversion of complex propositions consists in finding the contradictory of a complex term; but a simple rule for performing this process has been given in section 319:—*Replace all the simple terms involved by their contradictories, and throughout substitute alternative combination for conjunctive and vice versa.*

Applying this rule to AB or ab , we have $(a$ or $b)$ and $(A$ or $B)$, that is, Aa or Ab or aB or Bb ; but since the alternants Aa and Bb involve self-contradiction, they may by rule (5) of section 326, be omitted. The obverse, therefore, of $All\ X\ is\ AB\ or\ ab$ is $No\ X\ is\ Ab\ or\ aB$.

As additional examples we may find the obverse of the following propositions: (1) $All\ A\ is\ BC\ or\ DE$; (2) $No\ A\ is\ BcE\ or\ BCF$; (3) $Some\ A\ is\ not\ either\ B\ or\ bcDEf\ or\ bcdEF$.

(1) $All\ A\ is\ BC\ or\ DE$ yields $No\ A\ is\ (b\ or\ c)\ and\ at\ the\ same\ time\ (d\ or\ e)$, or by the reduction of the predicate to a series of alternants, $No\ A\ is\ bd\ or\ be\ or\ cd\ or\ ce$.

(2) $No\ A\ is\ BcE\ or\ BCF$. Here the contradictory of the predicate is $(b\ or\ C\ or\ e)\ and\ (b\ or\ c\ or\ f)$, which yields $b\ or\ Cc$

or *Cf* or *ce* or *ef*. *Cc* may be omitted by rule (5) of section 326; also *ef* by rule (7), since *ef* is either *Cef* or *cef*. Hence the required obverse is *All A is b* or *Cf* or *ce*.

(3) *Some A is not either B* or *bcDEf* or *bcdEF*. The obverse is *Some A is b* and (*B* or *C* or *d* or *e* or *F*) and (*B* or *C* or *D* or *e* or *f*); and by the application of the rules summarised in section 326 this will be found to be equivalent to *Some A is bc* or *bDF* or *be* or *bdF*.

348. *The Conversion of Complex Propositions.*—Generalising, we may say that we have a process of conversion whenever from a given proposition we infer a new one in which any term that appeared in the predicate of the original proposition now appears in the subject, or *vice versa*.

Thus the inference from *No A is BC* to *No B is AC* is of the nature of conversion. The process may be simply analysed as follows:—

No A is both B and C,
therefore, *Nothing is at the same time A, B, and C,*
therefore, *No B is both A and C.*

The reasoning may also be resolved into a series of ordinary conversions:—

No A is BC,
therefore (by conversion), *No BC is A,*
that is, *within the sphere of C, no B is A,*
therefore (by conversion), *within the sphere of C, no A is B,*
that is, *No AC is B,*
therefore (by conversion), *No B is AC.*

Or, it may be treated thus,

No A is BC,
therefore, by section 338, rule (1), *No AC is BC,*
therefore, also by section 338, rule (1), *No AC is B,*
therefore (by conversion), *No B is AC.*

Similarly it may be shewn that from *Some A is BC* we may infer *Some B is AC*.

Hence we obtain the following rule: *In a universal negative or a particular affirmative proposition any determinant of the subject may be transferred to the predicate or vice versa without affecting the force of the assertion.*

We have just shewn how from

No A is BC,

we may obtain by conversion

No B is AC.

Similarly, we may infer

No C is AB,

No AB is C,

No AC is B,

No BC is A.

The proposition may also be written in the form

There is no ABC,

or, *Nothing is at the same time A, B, and C.*

The last of these is a specially useful form to which to bring universal negatives for the purpose of logical manipulation.

In the same way from *Some A is BC or BD* we may infer

Some AB is C or D,

Some AC or AD is B,

Some B is AC or AD,

Some C or D is AB,

Some BC or BD is A,

Something is ABC or ABD.

There is no inference by conversion from a universal affirmative or from a particular negative.

349. The Contraposition of Complex Propositions.—According to our original definition of contraposition, we contrapose a proposition when we infer from it a new proposition having the contradictory of the old predicate for its subject. Adopting this definition, the contrapositive of *All A is B or C* is *All bc is a*.

The process can be applied to universal affirmatives and to particular negatives. By obversion, conversion, and then again obversion, it is clear that in each of these cases we may obtain a legitimate contrapositive in its obverted form by taking as a new subject the contradictory of the old predicate, and as a new predicate the contradictory of the old subject, the proposition retaining its original quality. For example: *All A is BC*, therefore, *Whatever is b or c is a*; *Some A is not either B or C*, therefore, *Some bc is not a*.

The above may be called the full contrapositive of a complex proposition. It should be observed that any proposi-

tion and its full contrapositive are equivalent to each other; in other words, we can pass back from a contrapositive to the original proposition.

In dealing with complex propositions, however, it is convenient to give to the term contraposition an extended meaning. We may say that we have a *process of contraposition* when from a given proposition we infer a new one in which the contradictory of any term that appeared in the predicate of the original proposition now appears in the subject, or the contradictory of any term that appeared in the subject of the original proposition now appears in the predicate.

Three operations may be distinguished all of which are included under the above definition, and all of which leave us with a full equivalent of the original proposition, so that there is no loss of logical power.

(1) The operation of obtaining the full contrapositive of a given proposition, as above described and defined¹.

(2) An operation which may be described as *the generalisation of the subject of a proposition by the addition of one or more alternants in the predicate*. Thus, from *All AB is C* we may infer *All A is b or C*; from *Some AB is not either C or D* we may infer *Some A is not either b or C or D*.

For inferences of this type the following general rule may be given: *Any determinant may be dropped from the subject of a universal affirmative or a particular negative proposition, if its contradictory is at the same time added as an alternant in the predicate*.

This rule may be established as follows: Given *All AB is C* (or *Some AB is not C*)—and these may be taken, so far as the rule in question is concerned, as types of universal affirmatives and particular negatives respectively—we have by obversion *No AB is c* (or *Some AB is c*), and thence, by the rule for conversion given in section 348, *No A is Bc* (or *Some A is Bc*); then again obverting we have *All A is either b or C* (or *Some A is not either b or C*), the required result.

¹ In some cases we may desire to drop part of the information given by the complete contrapositive. Thus, from *All A is BC or E* we may infer *Whatever is be or ce is a*; but in a given application it may be sufficient for us to know that *All be is a*.

It will be observed that, as stated at the outset, these operations leave us with a proposition that is equivalent to our original proposition. There is, therefore, no loss of logical power.

By the application of the above rule with regard to all the explicit determinants of the subject any universal affirmative proposition may be brought to the form *Everything is X_1 or X_2 ...or X_n* ; and it will be found that by means of this transformation, complex inferences are in many cases simplified and rendered easy.

(3) An operation which may be described as *the omission of one or more of a series of alternants in the predicate by a further particularisation of the subject*. Thus, from *All A is B or C* we may infer *All Ab is C*; from *Some A is not either B or C* we may infer *Some Ab is not C*.

For inferences of this type the following general rule may be given: *Any alternant may be dropped from the predicate of a universal affirmative or a particular negative proposition, if its contradictory is at the same time introduced as a determinant of the subject*¹.

This rule is the converse of that given under the preceding head; and it follows from the fact that the application of that rule leaves us with an equivalent proposition.

The following may be taken as typical examples of the different operations included above under the name contra-

¹ The application of this rule again leaves us with a proposition equivalent to our original proposition. The following rule, which may be regarded as a corollary from the above rule, or which may be arrived at independently, does not necessarily leave us with an equivalent: *If a new determinant is introduced into the subject of a universal affirmative proposition (see section 342) every alternant in the predicate which contains the contradictory of this determinant may be omitted*. Thus, from *Whatever is A or B is C or DX or Ex*, we may infer *Whatever is AX or BX is C or D*.

The application of this rule may sometimes result in the disappearance of all the alternants from the predicate; and the meaning of such a result is that we now have a non-existent subject.

Thus, given *All P is ABCD or Abcd or aBCd*, if we particularise the subject by making it *PbC*, we find that all the alternants in the predicate disappear. The interpretation is that the class *PbC* is non-existent, that is, *No P is bC*; a conclusion which might of course have been obtained directly from the given proposition.

position :—

All AB is CD or de;

therefore, (1) *Anything that is either cD or dE is a or b¹;*

(2) *All A is b or CD or de;*

(3) *Whatever is ABD or ABE is CD.*

Combinations of the second and third operations give

Anything that is Ac or Ad is b or de;

Anything that is BD or BE is a or CD;

&c.

In all the above cases one or more terms disappear from the subject or the predicate of the original proposition, and are replaced by their contradictories in the predicate or the subject accordingly. Only in the full contrapositive, however, is every term thus transposed.

The importance of contraposition as we are now dealing with it in connexion with complex propositions is that by its means, *given a universal affirmative proposition of any complexity, we may obtain separate information with regard to any term that appears in the subject, or with regard to the contradictory of any term that appears in the predicate, or with regard to any combination of such terms.*

Thus, given *All AB is C or De*, by the process described as the generalisation of the subject we have *All A is b or C or De*, *All B is a or C or De*, *Everything is a or b or C or De*; the particularisation of the subject yields *All ABc is De*, *Whatever is ABd or ABE is C*, &c.; and by the combination of these processes we have *All Ac is b or De*, &c.

Again, the full contrapositive of the original proposition is *Whatever is cd or cE is a or b*; from which we have *All c is a or b or De*, *Whatever is d or E is a or b or C*, &c.

350. *Summary of the results obtainable by Obversion, Conversion, and Contraposition.*—The following is a summary of the results obtainable by the aid of the processes discussed in the three preceding sections:

(1) By *obversion* any proposition may be changed from the affirmative to the negative form, or *vice versa*.

For example, *All AB is CD or EF*, therefore, *No AB is*

¹ From this, the propositions, *All cD is a or b*, *All dE is a or b*, are immediately deducible.

ce or cf or de or df; *Some P is not QR*, therefore, *Some P is either q or r*.

(2) By the *conversion* of a universal negative proposition separate information may be obtained with regard to any term that appears either in the subject or in the predicate, or with regard to any combination of these terms.

For example, from *No AB is CD or EF* we may infer *No A is BCD or BEF*, *No C is ABD or ABEF*, *No BD is AC or AEF*, &c.

Also by conversion any universal negative proposition may be reduced to the form *Nothing is either X_1 or X_2 ... or X_n* .

For example, the above proposition is equivalent to the following: *Nothing is either ABCD or ABEF*.

(3) By the *conversion* of a particular affirmative proposition separate information may be obtained with regard to any determinant of the subject or of the predicate, or with regard to any combination of such determinants.

For example, from *Some AB or AC is DE or DF* we may infer *Some A is BDE or BDF or CDE or CDF*, *Some D is ABE or ABF or ACE or ACF*, *Some AD is BE or BF or CE or CF*, &c.

Also by conversion any particular affirmative proposition may be reduced to the form *Something is either X_1 or X_2 ... or X_n* .

For example, the above proposition is equivalent to the following: *Something is either ABDE or ABDF or ACDE or ACDF*.

(4) By the *contraposition* of a universal affirmative proposition separate information may be obtained with regard to any term that appears in the subject, or with regard to the contradictory of any term that appears in the predicate, or with regard to any combination of these terms.

For example, from *All AB is CD or EF* we may infer *All A is b or CD or EF*, *All c is a or b or EF*, *All Be is a or CD*, *All ce is a or b*, *All Adf is b*, &c.

Also by contraposition any universal affirmative proposition may be reduced to the form *Everything is either X_1 or X_2 ... or X_n* .

For example, the above proposition is equivalent to the following: *Everything is a or b or CD or EF.*

(5) By the contraposition of a particular negative proposition separate information may be obtained with regard to any determinant of the subject or with regard to the contradictory of any alternant of the predicate or with regard to any combination of these.

For example, from *Some AB or AC is not either D or EF* we may infer *Some A is not either bc or D or EF*, *Some d is not either a or bc or EF*, *Some Ae or Af is not either bc or D*, &c.

Also by contraposition any particular negative proposition may be reduced to the form *Something is not either X_1 or X_2 ... or X_n .*

For example, the above proposition is equivalent to the following: *Something is not either a or bc or D or EF.*

EXERCISES.

351. No citizen is at once a voter, a householder, and a lodger; nor is there any citizen who is neither of the three.

Every citizen is either a voter but not a householder, or a householder and not a lodger, or a lodger without a vote.

Are these statements precisely equivalent? [v.]

It may be shewn that each of these statements is the logical obverse of the other. They are, therefore, precisely equivalent.

Let V = voter,	v = not voter;
H = householder,	h = not householder;
L = lodger,	l = not lodger.

The first of the given statements is *No citizen is VHL or vhl* ; therefore (by obversion), *Every citizen is either v or h or l and is also either V or H or L* ; therefore (combining these possibilities), *Every citizen is either Hv or Lv or Vh or Lh or Vl or Hl .*

But (by the law of excluded middle), *Hv is either HLv or Hlv* ; therefore, *Hv is Lv or Hl* . Similarly, *Lh is Vh or Lv* ; and *Vl is Hl or Vh* .

Therefore, *Every citizen is Vh or Hl or Lv* , which is the second of the given statements.

Again, starting from this second statement, it follows (by obversion) that *No citizen is at the same time v or H, h or L, l or V*; therefore, *No citizen is vh or vL or HL, and at the same time l or V*; therefore, *No citizen is vhl or VHL*, which brings us back to the first of the given statements.

352. Given "All *D* that is either *B* or *C* is *A*," shew that "Everything that is not-*A* is either not-*B* and not-*C* or else it is not-*D*." [De Morgan.]

This example and those given in section 359 are adapted from De Morgan, *Syllabus*, p. 42. They are also given by Jevons, *Studies*, p. 241, in connexion with his Equational Logic. They are all simple exercises in contraposition.

We have *What is either BD or CD is A*; therefore, *All a is (b or d) and (c or d)*; therefore, *All a is bc or d*.

353. Infer all that you possibly can by way of contraposition or otherwise, from the assertion, *All A that is neither B nor C is X*. [R.]

The given proposition may be thrown into the form

Everything is either a or B or C or X;

and it is seen to be symmetrical with regard to the terms *a, B, C, X*, and therefore with regard to the terms *A, b, c, x*. We are sure then that anything that is true of *A* is true *mutatis mutandis* of *b, c*, and *x*, that anything that is true of *Ab* is true *mutatis mutandis* of any pair of the terms, and similarly for combinations three and three together.

We have at once the four symmetrical propositions:

All A is B or C or X; (1)

All b is a or C or X; (2)

All c is a or B or X; (3)

All x is a or B or C. (4)

Then from (1) by particularisation of the subject:

All Ab is C or X; (i)

with the five corresponding propositions:

All Ac is B or X; (ii)

All Ax is B or C; (iii)

All bc is a or X; (iv)

All bx is a or C; (v)

All cx is a or B. (vi)

By a repetition of the same process, we have *All Abc is X* (which is the original proposition over again); (a)
 and corresponding to this: *All Abx is C*; (β)
All Acx is B; (γ)
All bcx is a. (δ)

It will be observed that the following are pairs of full contrapositives:—(1) (δ), (2) (γ), (3) (β), (4) (a), (i) (vi), (ii) (v), (iii) (iv).

A further series of propositions may be obtained by obverting all the above; and as there has been no loss of logical power in any of the processes employed we have in all thirty propositions that are equivalent to one another.

354. If *AB* is either *Cd* or *cDe*, and also either *eF* or *H*, and if the same is true of *BH*, what do we know of that which is *E*? [K.]

Whatever is AB or BH is (Cd or cDe) and (eF or H);
 therefore, *Whatever is AB or BH is CdeF or cDeF or CdH or cDeH;*
 therefore, *Whatever is ABE or BHE is CdH;*
 therefore, *All E is CdH or b or ah.*

355. Given *A is BC or BDE or BDF*, infer descriptions of the terms *Ace*, *Acf*, *ABcD*. [Jevons, *Studies*, pp. 237, 238.]

In accordance with rules already laid down, we have immediately—

Ace is BDF;
Acf is BDE;
ABcD is E or F.

356. Find the obverse of each of the following propositions:—

- (1) *Nothing is A, B, or C;*
- (2) *All A is Bc or bD;*
- (3) *No Ab is CDEf or Cd or cDf or cdE;*
- (4) *No A is BCD or Bcd;*
- (5) *Some A is not either bcd or Cd or cD.* [K.]

357. Shew that the two following propositions are equivalent to each other:—*No A is B or CD or CE or EF; All A is bCde or bcEf or bce.* [K.]

358. Contraposit the proposition, *All A that is neither B nor C is both X and Y.* [L.]

359. Find the full contrapositive of each of the following propositions :

- (1) *All A is either BC or BD;*
- (2) *Whatever is B or CD or CE is A;*
- (3) *Whatever is either B or C and at the same time either D or E is A;*
- (4) *Whatever is A or BC and at the same time either D or EF is X.*

[De Morgan.]

360. Find the full contrapositive of each of the following propositions :—

All A is BCDe or bcDe;

Some AB is not either CD or cDE or de;

Whatever is AB or bC is aCd or Acd;

Where A is present along with either B or C, D is present and C absent or D and E are both absent;

Some ABC or abc is not either DEF or def. [K.]

361. What information can you obtain about *c, Be, Af, D*, from the proposition *All AB is CD or EF?* [M.]

362. Establish the following: Where *B* is absent, either *A* and *C* are both present or *A* and *D* are both absent; therefore, where *C* is absent, either *B* is present or *D* is absent. [K.]

363. Establish the following: Where *A* is present, either *B* and *C* are both present or *C* is present *D* being absent or *C* is present *F* being absent or *H* is present; therefore, where *C* is absent, *A* cannot be present *H* being absent. [K.]

364. Given that *Whatever is PQ or AP is bCD or abdE or aBCdE or Abcd*, shew that (1) *All abP is CD or dE or q;* (2) *All DP is bC or aq;* (3) *Whatever is B or Cd or cD is a or p;* (4) *All B is C or p or aq;* (5) *All Cd is a or p;* (6) *All AB is p;* (7) *If ae is c or d it is p or q;* (8) *If BP is c or D or e it is aq.* [K.]

365. Bring the following propositions to the form *Everything is either X₁ or X₂ ... or X_n*:—

Whatever is Ac or ab or aC is bdf or deF;

Nothing that is A and at the same time either B or C is D or dE. [K.]

366. Shew that the results in section 340 follow from those in section 339 by the rules of contradiction and contraposition. [K.]

CHAPTER IV.

THE COMBINATION OF COMPLEX PROPOSITIONS.

367. *The Problem of combining Complex Propositions.*—Two or more complex propositions given in simple combination, either conjunctive or alternative, constitute a compound proposition. Hence the problem of dealing with a combination of complex propositions so as to obtain from them a single equivalent complex proposition, which is the problem to be considered in the present chapter, is identical with that of passing from a compound proposition to an equivalent complex proposition; and it is, therefore, the converse of the problem which was partially discussed in sections 339, 340. The latter problem, namely, that of passing from a complex to an equivalent compound proposition, will be further discussed in chapter 6.

368. *The Conjunctive Combination of Universal Affirmatives.*—We may here distinguish two cases according as the propositions to be combined have or have not the same subject.

(1) *Universal affirmatives having the same subject.*

All X is P_1 or P_2 or P_m ,

All X is Q_1 or Q_2 or Q_n ,

may for our present purpose be taken as types of universal affirmative propositions having the same subject. By conjunctively combining their predicates, thus,

All X is (P_1 or P_2 ... or P_m) and also (Q_1 or Q_2 ... or Q_n),

that is, All X is P_1Q_1 or P_1Q_2 ... or P_1Q_n

or P_2Q_1 or P_2Q_2 ... or P_2Q_n

or

.....

or P_mQ_1 or P_mQ_2 ... or P_mQ_n ,

we may obtain a new proposition which is equivalent to the conjunctive combination of the two original propositions; it sums up all the information which they jointly contain, and we can pass back from it to them.

In almost all cases of the conjunctive combination of terms there are numerous opportunities of simplification; and after a little practice, the student will find it unnecessary to write out all the alternants of the new predicate in full. The following are examples:—

- (i) *All X is AB or bce,*
All X is aBC or DE;
 therefore, *All X is ABDE.*

It will be found that all the other combinations in the predicate contain contradictories.

- (ii) *All X is A or Bc or D,*
All X is aB or Bc or Cd;
 therefore, *All X is Bc or aBD or ACd.*

- (iii) *Everything is A or bd or cE,*
Everything is AC or aBe or d;
 therefore, *Everything is AC or bd or Ad or cdE.*

(2) *Universal affirmatives having different subjects.*

Given the conjunctive combination of two universal affirmative propositions with different subjects a new complex proposition may be obtained by conjunctively combining both their subjects and their predicates. Thus, if *All X is P₁ or P₂* and *All Y is Q₁ or Q₂*, it follows that *All XY is P₁Q₁ or P₁Q₂ or P₂Q₁ or P₂Q₂*. But in this case the new proposition thus obtained is not equivalent to the conjunctive combination of the original propositions; and we cannot pass back from it to them.

A single complex proposition which sums up all the information contained in the original propositions may, however, be obtained by first reducing each of them to the form *Everything is X₁ or X₂ ... or X_n*, and then conjunctively combining their predicates.

369. *The Conjunctive Combination of Universal Negatives.*—

Here again we may distinguish two cases according as the propositions to be combined have or have not the same subject.

(1) *Universal negatives having the same subject.**No X is P_1 or P_2 or P_m ,**No X is Q_1 or Q_2 or Q_n ,*

may for our present purpose be taken as types of universal negative propositions having the same subject. Given these two propositions in conjunctive combination, a new complex proposition may be obtained by alternatively combining their predicates. Thus,

No X is P_1 or P_2 or P_m or Q_1 or Q_2 or Q_n .

This new proposition is equivalent to the two original propositions taken together, so that we can pass back from it to them. The process of combining the predicates is again likely to give opportunities of simplification of which advantage should be taken. The following are examples :

- (i) *No A is bc,*
No A is Cd;
therefore, *No A is bc or Cd.*
- (ii) *No X is either aB or aC or bC or aE or bE,*
No X is either Ad or Ae or bd or be or cd or ce;
therefore, *No X is either a or b or d or e¹.*
- (iii) *Nothing is aBC or aCD or aBe or aDe,*
Nothing is AcD or abD or bcD or aDE or cDE;
therefore, *Nothing is aBC or aD or cD or aBe.*

(2) *Universal negatives having different subjects.*

Given the conjunctive combination of two universal negative propositions with different subjects a new complex proposition may be obtained by conjunctively combining their subjects and alternatively combining their predicates. Thus, if *No X is P_1 or P_2* and *No Y is Q_1 or Q_2* , it follows that *No XY is P_1 or P_2 or Q_1 or Q_2* . In this case, as in that considered in the preceding section, the inferred proposition is not equivalent to the premisses; and we cannot pass back from it to them.

A single complex proposition which sums up all the information contained in the original propositions may, however, be obtained by first reducing each of them to the form *Nothing is X_1 or X_2 ... or X_n* , and then alternatively combining their predicates.

¹ Compare section 323.

370. *The Conjunctive Combination of Universals with Particulars of the same Quality.*—We may here consider, first, affirmatives, and then, negatives.

(1) *Affirmatives.* From the conjunctive combination of a universal affirmative and a particular affirmative having the same subject, a new particular affirmative proposition may be obtained by conjunctively combining their predicates. If *All X is P_1 or P_2* and *Some X is Q_1 or Q_2* , it follows that *Some X is P_1Q_1 or P_1Q_2 or P_2Q_1 or P_2Q_2* . It will be observed that the particular premiss affirms the existence of either XQ_1 or XQ_2 , and therefore certainly guarantees the existence of X ; and the universal premiss implies that if X exists then either XP_1 or XP_2 exists.

We can pass back from the conclusion to the particular premiss, but not to the universal premiss. The conclusion is, therefore, not equivalent to the two premisses taken together.

A new complex proposition cannot be directly obtained from the conjunctive combination of a universal affirmative and a particular affirmative having different subjects. The propositions may, however, be reduced respectively to the forms *Everything is P_1 or $P_2 \dots$ or P_m* , *Something is Q_1 or $Q_2 \dots$ or Q_n* , and their predicates may then be conjunctively combined in accordance with the above rule.

(2) *Negatives.* From the conjunctive combination of a universal negative and a particular negative having the same subject, a new particular negative proposition may be obtained by the alternative combination of their predicates. If *No X is either P_1 or P_2* and *Some X is not either Q_1 or Q_2* , it follows that *Some X is not either P_1 or P_2 or Q_1 or Q_2* . The validity of this process is obvious since the particular premiss guarantees the existence of X . By obversion it can also be exhibited as a corollary from the rule given above in regard to affirmatives. We can again pass back from the conclusion to the particular premiss, but not to the universal premiss.

With regard to the conjunctive combination of universal negatives and particular negatives having different subjects, the remarks made concerning affirmatives apply *mutatis mutandis*.

371. *The Conjunctive Combination of Affirmatives with Negatives.*—By first obverting one of the propositions, the conjunctive combination of an affirmative with a negative may be made to yield a new complex proposition in accordance with the rules given in the preceding sections. For example,

- (1) *All X is A or B,*
 No X is aC,
 therefore, *All X is A or Bc;*
- (2) *Everything is P or Q,*
 Nothing is Pq or pR,
 therefore, *Nothing is pR or q;*
- (3) *All X is AB or bce,*
 Some X is not either aBC or DE,
 therefore, *Some X is ABd or ABe or bce.*

372. *The Conjunctive Combination of Particulars with Particulars.*—Particulars cannot to any purpose be conjunctively combined with particulars so as to yield a new complex proposition. It is true that from *Some X is P_1 or P_2 and some X is Q_1 or Q_2 ,* we can pass to *Some X is P_1 or P_2 or Q_1 or Q_2 .* But this is a mere weakening of the information given by either of the premisses singly; and by the rule that an alternant may at any time be introduced into an undistributed term (section 342), it could equally well be inferred from either premiss taken by itself. Again from *Some X is not either P_1 or P_2 and some X is not either Q_1 or Q_2 ,* we can pass to *Some X is not either P_1Q_1 or P_1Q_2 or P_2Q_1 or P_2Q_2 .* But similar remarks again apply, since we have already found that a determinant may at any time be introduced into a distributed term.

373. *The Alternative Combination of Universal Propositions.*—Given a number of universal propositions as alternants in a compound alternative proposition we cannot obtain a single equivalent complex proposition¹. From the compound proposition *Either all A is P_1 or P_2 or all A is Q_1 or Q_2* we can indeed infer *All A is P_1 or P_2 or Q_1 or Q_2 ;* but we cannot pass back from this to the original proposition.

374. *The Alternative Combination of Particular Propositions.*—It follows from the equivalences shewn in section 340

¹ Compare section 339.

that a compound alternative proposition in which all the alternants are particular can be reduced to the form of a single complex proposition. If all the alternants of the compound proposition have the same subject and are all affirmative, their predicates must be alternatively combined in the complex proposition; if they all have the same subject and are all negative, their predicates must be conjunctively combined in the complex proposition. If the alternants have different subjects, they must all be reduced to the form *Something is ...* before their predicates are combined; if they differ in quality, recourse must be had to the process of obversion. It is unnecessary to discuss these different cases in detail, but the following may be taken as examples:

- (i) *Some X is P or some X is Q = Some X is P or Q;*
- (ii) *Some X is not P or some X is not Q = Some X is not PQ;*
- (iii) *Some X is P or some Y is Q = Something is XP or YQ;*
- (iv) *Some X is P or some Y is not Q = Something is XP or Yq.*

375. *The Alternative Combination of Particulars with Universals.*—From a compound alternative proposition in which some of the alternants are particular and some universal, we can infer a particular complex proposition; but in this case we cannot pass back from the complex proposition to the compound proposition. The following are examples:

- (1) *All A is P or some A is Q, therefore, Something is a or P or Q¹;*
- (2) *All A is P or some B is not Q, therefore, Something is a or P or Bq.*

EXERCISES.

376. Reduce the propositions *All P is Q, No Q is R*, to such a form that the universe of discourse appears as the subject of each; and then combine them into a single complex proposition. How is your result related to the ordinary syllogistic conclusion *No P is R*?

[K.]

¹ We cannot infer *Some A is P or Q*, since this implies the existence of *A*, whereas the non-existence of *A* is compatible with the premiss.

377. Combine the following propositions into a single equivalent complex proposition: *All X is either A or b ; No X is either AC or acD or CD ; All a is B or x .* [K.]

378. Every voter is both a ratepayer and occupier, or not a ratepayer at all; If any voter who pays rates is an occupier, then he is on the list; No voter on the list is both a ratepayer and an occupier.

Examine the results of combining these three statements. [V.]

379. Every A is BC , except when it is D ; everything which is not A is D ; what is both C and D is B ; and every D is C . What can be determined from these premisses as to the contents of our universe of discourse? [M.]

CHAPTER V.

INFERENCES FROM COMBINATIONS OF COMPLEX PROPOSITIONS.

380. *Conditions under which a universal proposition affords information in regard to any given term.*—The problem to be solved in order to determine these conditions may be formulated as follows: *Given any universal proposition, and any term X, to discriminate between the cases in which the proposition does and those in which it does not afford information with regard to this term.*

In the first place, it is clear that if the proposition is to afford information in regard to any term whatever it must be non-formal. If it is negative, let it by obversion be made affirmative. Then it may be written in the form

Whatever is $A_1A_2\dots$ or $B_1B_2\dots$ or &c. is $P_1P_2\dots$ or $Q_1Q_2\dots$ or &c., where A_1, B_1, P_1, Q_1 , &c. are all simple terms¹.

As shewn in section 339, this may be resolved into the independent propositions:—

All $A_1A_2\dots$ is $P_1P_2\dots$ or $Q_1Q_2\dots$ or &c.;

All $B_1B_2\dots$ is $P_1P_2\dots$ or $Q_1Q_2\dots$ or &c.;

&c. &c. &c.;

in none of which is there any alternation in the subject.

These propositions may be dealt with separately, and if any one of them affords information with regard to X , then of course the original proposition does so.

We have then to consider a proposition of the form

All $A_1A_2\dots A_n$ is $P_1P_2\dots$ or $Q_1Q_2\dots$ or &c.;

¹ So that both subject and predicate consist of a series of alternants which themselves contain only simple determinants; that is, there is no alternant of the form (A or B) (C or D).

and this proposition may by contraposition be reduced to the form

Everything is a_1 or $a_2 \dots$ or a_n or $P_1P_2 \dots$ or $Q_1Q_2 \dots$ or &c.;
from which may be inferred

All X is a_1 or $a_2 \dots$ or a_n or $P_1P_2 \dots$ or $Q_1Q_2 \dots$ or &c.

Any alternant in the predicate of this proposition which contains x may clearly be omitted.

If all the alternants contain x , then the information afforded with regard to X is that it is non-existent.

If some alternants are left, then the proposition will afford information concerning X unless, when the predicate has been simplified to the fullest possible extent¹, one of the alternants is itself X uncombined with any other term, in which case it is clear that we are left with a merely formal proposition.

Now one of these alternants will be X in the following cases, and only in these cases:—

First, If one of the alternants in the predicate of the original proposition, when reduced to the affirmative form, is X .

Secondly, If any set of alternants in the predicate of the original proposition, when reduced to the affirmative form, constitutes a development of X , since any such development (for example, AX or aX , ABX or AbX or aBX or abX) is equivalent to X simply².

Thirdly, If one of the alternants in the predicate of the original proposition, when reduced to the affirmative form, contains X in combination solely with some determinant that is also a determinant of the subject or the contradictory of some other alternant of the predicate; since in either of these cases such alternant is equivalent to X simply³.

Fourthly, If one of the determinants of the subject is x ; since in that case we shall after contraposition have X as one of the alternants of the predicate.

The above may be summed up in the following proposi-

¹ All superfluous terms being omitted, but the predicate still consisting of a series of alternants which themselves contain only simple determinants.

² See section 323.

³ By section 338, rule (2), *All AB is AX or D* is equivalent to *All AB is X or D* ; and by the law of exclusion (section 325) *A or aX* is equivalent to *A or X* .

tion:—Any non-formal universal proposition will afford information with regard to any term X , unless, after it has been brought to the affirmative form, (1) one of the alternants of the predicate is X , or (2) any set of alternants in the predicate constitutes a development of X , or (3) any alternant of the predicate contains X in combination solely with some determinant that is also a determinant of the subject or the contradictory of some other alternant of the predicate, or (4) x is a determinant of the subject.

If, after the proposition has been reduced to the affirmative form, all superfluous terms are omitted in accordance with the rules given in chapters 1 and 2, then the criterion becomes more simple:—Any non-formal universal proposition will afford information with regard to any term X , unless (after it has been brought to the affirmative form and its predicate has been so simplified that it contains no superfluous terms) X is itself an alternant of the predicate or x is a determinant of the subject¹.

If instead of X we have a complex term XYZ , then no determinant of this term must appear by itself as an alternant of the predicate, and there must be at least one alternant in the subject which does not contain as a determinant the contradictory of any determinant of this complex term; i.e., no alternant in the predicate must be X , Y , or Z , or any combination of these, and some alternant of the subject must contain neither x , y , nor z .

The above criterion is of simple application.

381. *Information jointly afforded by a series of universal propositions with regard to any given term.*—The great majority of direct problems² involving complex propositions may be brought under the general form, *Given any number of universal propositions involving any number of terms, to determine what is all the information that they jointly afford with regard to any given term or combination of terms.* If the student turns to

¹ It may be added that every universal proposition, unless it be purely formal, will afford information either with regard to X or with regard to x . For if both X and x appear as alternants of the predicate, or as determinants of the subject of a universal affirmative proposition, then the proposition will necessarily be formal.

² Inverse problems will be discussed in the following chapter.

Boole, Jevons, or Venn, he will find that this problem is treated by them as the central problem of symbolic logic¹.

A general method of solution is as follows:—

Let X be the term concerning which information is desired. Find what information each proposition gives separately with regard to X , thus obtaining a new set of propositions of the form

$$\text{All } X \text{ is } P_1 \text{ or } P_2 \dots \text{ or } P_n.$$

This is always possible by the aid of the rules for obversion and contraposition given in chapter 3. By the rule given in the preceding section those propositions which do not afford any information at all with regard to X may at once be left out of account.

Next let the propositions thus obtained be combined in the manner indicated in section 368. This will give the desired solution.

If information is desired with regard to several terms, it will be convenient to bring all the propositions to the form

$$\text{Everything is } P_1 \text{ or } P_2 \dots \text{ or } P_n;$$

and to combine them at once, thus summing up in a single proposition all the information given by the separate propositions taken together. From this proposition all that is known concerning X may immediately be deduced by omitting every alternant that contains x , all that is known concerning Y by omitting every alternant that contains y , and so on.

The method may be varied by bringing the propositions to the form

$$\text{No } X \text{ is } Q_1 \text{ or } Q_2 \dots \text{ or } Q_n,$$

or to the form

$$\text{Nothing is } Q_1 \text{ or } Q_2 \dots \text{ or } Q_n,$$

then combining them as in section 369, and (if an affirmative

¹ "Boole," says Jevons, "first put forth the problem of Logical Science in its complete generality:—*Given certain logical premisses or conditions, to determine the description of any class of objects under those conditions.* Such was the general problem of which the ancient logic had solved but a few isolated cases—the nineteen moods of the syllogism, the sorites, the dilemma, the disjunctive syllogism, and a few other forms. Boole shewed incontestably that it was possible, by the aid of a system of mathematical signs, to deduce the conclusions of all these ancient modes of reasoning, and an indefinite number of other conclusions. Any conclusion, in short, that it was possible to deduce from any set of premisses or conditions, however numerous and complicated, could be calculated by his method" (*Philosophical Transactions*, 1870). Compare also *Principles of Science*, chapter 6, § 5.

solution is desired) finally obverting the result. It will depend on the form of the original propositions whether this variation is desirable¹.

In an equational system of symbolic logic, a solution with regard to any term X generally involves a partial solution with regard to x also. In employing the above methods, x must be found separately. It may be added that the complete solution for X and that for x sum up between them the whole of the information given by the original data; in other words, they are, taken together, equivalent to the given premisses².

The following may be taken as a simple example of the first of the above methods. It is adapted from Boole (*Laws of Thought*, p. 118).

"Given 1st, that wherever the properties A and B are combined, either the property C , or the property D , is present also, but they are not jointly present; 2nd, that wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent; 3rd, that wherever the properties A and B are both absent, the properties C and D are both absent also; and *vice versa*, where the properties C and D are both absent, A and B are both absent also. Find what can be inferred from the presence of A with regard to the presence or absence of B , C , and D ."

The premisses may be written as follows: (1) *All AB is Cd or cD*; (2) *All BC is AD or ad*; (3) *All ab is cd*; (4) *All cd is ab*.

Then, from (1), *All A is b or Cd or cD*;

and from (2), *All A is b or c or D*;

therefore (by combining these), *All A is b or cD*;

(3) gives no information regarding A (see the preceding section); but by (4), *All A is C or D*;

¹ This second method is analogous to that which is usually employed by Dr Venn in his *Symbolic Logic*. Both methods bear a certain resemblance to Jevons's Indirect Method; but neither of them is identical with that method.

² Having determined that *All X is P* and that *All x is q*, we may by contraposition bring the latter proposition to the form *All Q is X*, when it may be found that P and Q have some alternants in common. These alternants are the terms which (in Boole's system) are taken in their whole extent in the equation giving X ; and the solution thus obtained is closely analogous to that given by any equational system of symbolic logic.

therefore, *All A is bC or bD or cD*;
 and, since *bD* is by development either *bCD* or *bcD*, this becomes
*All A is bC or cD*¹.

This solves the problem as set. Proceeding also to determine *a*, we find that (1) gives no information with regard to this term; but by (2), *All a is b or c or d*; and by (3), *All a is B or cd*; therefore, *All a is Bc or Bd or cd*. Again by (4), *All a is b or C or D*. Therefore, *All a is BCd or BcD or bcd*; and by contraposition, *Whatever is Bcd or bC or bD or CD is A*².

382. The Problem of Elimination.—By *elimination* in logic is meant the omission of certain elements from a proposition or set of propositions with the object of expressing more directly and concisely the connexion between the elements which remain. An example of the process is afforded by the ordinary categorical syllogism, where the so-called *middle term* is eliminated. Thus, given the premisses *All M is P*, *All S is M*, we may infer *All S is MP*; but if we desire to know the relation between *S* and *P* independently of *M* we are content with the less precise but sufficient statement *All S is P*; in other words, we eliminate *M*.

Elimination has been considered by some writers to be absolutely essential to logical reasoning. It is not, however, necessarily involved either in the process of contraposition or in the process discussed in the preceding section; and from the point of view of the formal logician it is difficult to see how the name of inference can be denied to these processes. We must, therefore, refuse to regard elimination as of the essence of reasoning, although it may usually be involved therein³.

383. Elimination from Universal Affirmatives.—Any universal affirmative proposition (or, by combination, any set of universal affirmative propositions) involving the term *X* and

¹ We shall find that by the elimination of *D*, in accordance with the rule given in the next section but one, we have *All A is bC or c*. This is Boole's result.

² Taking into account the result arrived at above with regard to *A*, it will be seen that this may be resolved into *Whatever is bC or bD is A* and *Nothing is BCD or Bcd*. These two propositions taken together with the solution for *A* are equivalent to the original premisses.

³ Compare sections 155, 156.

its contradictory x may by contraposition be reduced to the form *Everything is* PX or Qx or R , where P , Q , R are themselves simple or complex terms not involving either X or x ; and since by the rule given in section 341 a determinant may at any time be omitted from an undistributed term, we may eliminate X (and x) from this proposition by simply omitting them, and reducing the proposition to the form *Everything is* P or Q or R ¹.

We must, however, here admit the possibility of P , Q , R being of the forms A or a , Aa . These are equivalent respectively to *the entire universe of discourse* and to *nothing*. Thus, if P is of the form A or a , and Q is of the form Aa , our proposition will before elimination more naturally be written *Everything is* X or R ; if Q is of the form A or a , and R of the form Aa , it will more naturally be written *Everything is* PX or x . It follows that if either P or Q is of the form A or a (that is, if either P or Q is equivalent to the entire universe of discourse), the proposition resulting from elimination will not afford any real information, since it is always true *a priori* that *Everything is* A or a or $\&c$. Thus we are practically unable to eliminate X from such a proposition as *All* A *is* X or BC .

The following may be given as an example of elimination from universal affirmatives.

Let it be required to eliminate X (together with x) from the propositions *All* P *is* XQ or xR , *Whatever is* X or R *is* p or XQR . Combining these propositions, we have *Everything is* XQR or p ; therefore, by elimination, *Everything is* QR or p , that is, *All* P *is* QR . It will be observed that P (together with p) cannot be eliminated from the above propositions.

384. Elimination from Universal Negatives.—Any universal negative proposition (or, by combination, any set of universal negative propositions) containing the term X and its contradictory x may by conversion be reduced to the form *Nothing is* PX or Qx or R , where P , Q , R are interpreted as in the preceding

¹ We might also proceed as follows: Solve for X and for x , as in section 381, so that we have *All* X *is* A , *All* x *is* B , where A and B are simple or complex terms not involving either X or x . Then, since *Everything is* X or x , we shall have *Everything is* A or B , and this will be a proposition containing neither X nor x .

section. Here we might, in accordance with the rule given in section 341, simply omit the alternants PX , Qx , leaving us with the proposition *Nothing is R*. This, however, is but part of the information obtainable by the elimination of X . We have also *No X is P*, and *No Q is x*, that is, *All Q is X*; whence by a syllogism in *Celarent* we may infer *No Q is P*. The full result of the elimination is, therefore, given by the proposition *Nothing is PQ or R*¹.

The following is an example: Let it be required to eliminate X from the propositions *No P is Xq or xr*, *No X or R is xP or Pq or Pr*. Combining these propositions we have *Nothing is XPq or XPr or xP or PqR*; therefore, by elimination in accordance with the above rule, *Nothing is Pq or Pr*, that is, *No P is q or r*.

385. Elimination from Particular Affirmatives.—Any particular affirmative proposition involving the term X may by conversion be reduced to the form *Something is either PX or Qx or R*, where P , Q , R are interpreted as in section 383. We may here immediately apply the rule given in section 341 that a determinant may at any time be omitted from an undistributed term; and the result of eliminating X is accordingly *Something is either P or Q or R*².

¹ Compare Mrs Ladd Franklin's Essay on *The Algebra of Logic* (*Studies in Logic by Members of the Johns Hopkins University*). The same conclusion may be deduced by obversion from the result obtained in the preceding section. *Nothing is PX or Qx or R* becomes by obversion *Everything is prX or qrx*. Therefore, by the elimination of X , *Everything is pr or qr*; and this proposition becomes by obversion *Nothing is PQ or R*.

Another method by which the same result may be obtained is as follows: By developing the first alternant with reference to Q and the second with reference to P , *Nothing is PX or Qx or R* becomes *Nothing is PQX or PqX or PQx or pQx or R*. But PQX or PQx is reducible to PQ , and omitting PqX and pQx , we have *Nothing is PQ or R*.

It is interesting to observe that the above rule for elimination from negatives is equivalent to Boole's famous rule for elimination. In order to eliminate X from the equation $F(X)=0$, he gives the formula $F(1) F(0)=0$. Now any equation containing X can be brought to the form $AX+Bx+C=0$, where A , B , C are independent of X . Applying Boole's rule we have $(A+C) (B+C)=0$, that is, $AB+C=0$; and this is precisely equivalent to the rule given in the text.

² Thus the rule for elimination from particular affirmatives is practically identical with the rule for elimination from universal affirmatives.

386. *Elimination from Particular Negatives.*—Any particular negative proposition involving the term X may by contraposition be reduced to the form *Something is not either* PX *or* Qx *or* R . By the development of the first alternant with reference to Q and that of the second alternant with reference to P , this proposition becomes *Something is not either* PQX *or* PqX *or* PQx *or* pQx *or* R . But PQX *or* PQx is reducible to PQ , and the alternants PqX , pQx may by the rule given in section 341 be omitted. Hence we get the proposition *Something is not either* PQ *or* R , from which X has been eliminated¹.

387. *Order of procedure in the process of elimination.*—Schröder (*Der Operationskreis des Logikkalküls*, p. 23) points out that first to eliminate and then combine is not the same thing as first to combine and then eliminate. For, as a rule, if a term X is eliminated from several isolated propositions the combined results give less information than is afforded by first combining the given propositions and then effecting the required elimination.

There are indeed many cases in which we cannot eliminate at all unless we first combine the given propositions. This is of course obvious in syllogisms; and we have a similar case if we take the premisses *Everything is* A *or* X , *Everything is* B *or* x . We cannot eliminate X from either of these propositions taken by itself, since in each of them X (or x) appears as an isolated alternant. But by combination we have *Everything is* Ax *or* BX ; and this by the elimination of X becomes *Everything is* A *or* B ².

There are other cases in which elimination from the separate propositions is possible, but where this order of procedure leads to a weakened conclusion. Take the propositions *Everything is*

¹ Thus the rule for elimination from particular negatives is practically identical with the rule for elimination from universal negatives. The same rule may be deduced by obversion from the result obtained in the preceding section. *Something is not either* PX *or* Qx *or* R ; therefore, *Something is either* prX *or* qrX ; therefore, *Something is either* pr *or* qr ; therefore, *Something is not either* PQ *or* R .

² Working with negatives we get the same result. Taking the propositions *Nothing is* ax , *Nothing is* bX , separately, we cannot eliminate X from either of them. But combining them in the proposition *Nothing is* ax *or* bX , we are able to infer *Nothing is* ab .

AX or Bx , *Everything is CX or Dx* . By first eliminating X and then combining, we have *Everything is AC or AD or BC or BD* . But by first combining and then eliminating X , our conclusion becomes *Everything is AC or BD* which gives more information than is afforded by the previous conclusion.

EXERCISES.

388. Suppose that an analysis of the properties of a particular class of substances has led to the following general conclusions, namely :

1st, That wherever the properties A and B are combined, either the property C , or the property D , is present also ; but they are not jointly present ;

2nd, That wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent ;

3rd, That wherever the properties A and B are both absent, the properties C and D are both absent also ; and *vice versa*, where the properties C and D are both absent, A and B are both absent also.

Shew that wherever the property A is present, the properties B and C are not both present ; also that wherever B is absent while C is present, A is present.

[Boole, *Laws of Thought*, pp. 118 to 120 ; compare also Venn, *Symbolic Logic*, pp. 276 to 278.]

A solution of this problem has already been given in section 381. We may also proceed as follows. The premisses are :

All AB is Cd or cd , (i)

All BC is AD or ad , (ii)

All ab is cd , (iii)

All cd is ab . (iv)

By (i), *No AB is CD* , therefore, *No A is BCD* . (1)

By (ii), *No BC is Ad* , therefore, *No A is BCd* . (2)

Combining (1) and (2), it follows immediately that *No A is BC* .

Boole also shews that *All bc is A* . This is a partial contrapositive of (iii). We have so far not required to make use of (iv) at all.

389. Given the same premisses as in the preceding section, prove that:—

(1) Wherever the property C is found, either the property A or the property B will be found with it, but not both of them together;

(2) If the property B is absent, either A and C will be jointly present, or C will be absent;

(3) If A and C are jointly present, B will be absent.

[Boole, *Laws of Thought*, p. 129.]

First, By (i), *All C is a or b or d* ; by (ii), *All C is a or b or D* ; therefore, *All C is a or b* .

Also, by (iii), *All C is A or B* ;

therefore, *All C is Ab or aB* . (1)

Secondly, By (iii), *All b is A or c* ,
therefore, by section 325, *All b is AC or c* . (2)

Thirdly, from (1) it follows immediately that

All AC is b . (3)

The given premisses may all be summed up in the proposition: *Everything is AbC or AbD or $aBCd$ or $abcd$ or BcD* . From this, the above special results and others follow immediately.

390. Given that *everything is either Q or R* , and that *all R is Q , unless it is not P* , prove that *all P is Q* . [K.]

The premisses may be written as follows: (1) *All r is Q* , (2) *All PR is Q* .

By (1), *All Pr is Q* , and by (2), *All PR is Q* ; but *All P is Pr or PR* ; therefore, *All P is Q* .

391. Where A is present, B and C are either both present at once or absent at once; and where C is present, A is present. Describe the class *not- B* under these conditions.

[Jevons, *Studies*, p. 204.]

The premisses are (1) *All A is BC or bc* , (2) *All C is A* .

By (1) *All b is a or c* , and by (2) *All b is A or c* ; therefore, *All b is c* .

392. It is known of certain things that (1) where the quality A is, B is not; (2) where B is, and only where B is, C and D are. Derive from these conditions a description of the class of things in which A is not present, but C is.

[Jevons, *Studies*, p. 200.]

The premisses are: (1) *All A is b*; (2) *All B is CD*; (3) *All CD is B*.

No information regarding *aC* is given by (1). But by (2), *All aC is b or D*; and by (3), *All aC is B or d*.

Therefore, *All aC is BD or bd*.

393. Taking the same premisses as in the previous section, draw descriptions of the classes *Ac*, *ab*, and *cD*.

[Jevons, *Studies*, p. 244.]

By (1), *Everything is a or b*; and by (2), *Everything is b or CD*. Therefore, *Everything is aCD or b*; and by (3), *Everything is B or c or d*. Therefore, *Everything is aBCD or bc or bd*.

Hence we infer immediately *All Ac is b*, *All ab is c or d*, *All cD is b*.

394. Every *A* is one only of the two *B* or *C*; *D* is both *B* and *C* except when *B* is *E* and then it is neither; therefore, no *A* is *D*.

[De Morgan, *Formal Logic*, p. 124.]

This example, originally given by De Morgan (using, however, different letters), and taken by Professor Jevons to illustrate his symbolic method (*Principles of Science*, chapter 6, § 10; *Studies in Deductive Logic*, p. 203), is chosen by Professor Croom Robertson to shew that "the most complex problems can, as special logical questions, be more easily and shortly dealt with upon the principles and with the recognised methods of the traditional logic" than by Jevons's system.

"The mention of *E as E* has no bearing on the special conclusion *A* is not *D* and may be dropt, while the implication is kept in view; otherwise, for simplification, let *BC* stand for 'both *B* and *C*,' and *bc* for 'neither *B* nor *C*.' The premisses then are,—

(1) *D* is either *BC* or *bc*,

(2) *A* is neither *BC* nor *bc*,

which is a well-recognised form of Dilemma with the conclusion *A* is not *I*. Or, by expressing (2) as *A* is not either *BC* or *bc*, the conclusion may be got in *Camestres*. If it be objected that we have here by the traditional processes got only a special conclusion, it is a sufficient reply that any conclusion by itself must be special. What other conclusion from these premisses is the common logic powerless to obtain?" (*Mind*, 1876, p. 222.)

The solution is also obtainable as follows: By the first premiss, *All A is Bc or bC*, and by the second, *All A is BC or bc or d*; therefore, *All A is Bcd or bCd*, therefore, *All A is d*.

395. There is a certain class of things from which A picks out the ' X that is Z , and the Y that is not Z ,' and B picks out from the remainder 'the Z which is Y and the X that is not Y .' It is then found that nothing is left but the class ' Z which is not X .' The whole of this class is however left. What can be determined about the class originally?

[Venn, *Symbolic Logic*, pp. 267, 8.]

The chief difficulty in this problem consists in the accurate statement of the premisses. Call the original class W . We then have

$All\ W\ is\ XZ\ or\ Yz\ or\ YZ\ or\ Xy\ or\ xZ,$
that is, $All\ W\ is\ X\ or\ Y\ or\ Z;$ (1)

$All\ xZ\ is\ W;$ (2)

$No\ xZ\ is\ WXZ\ or\ WYz\ or\ WYZ\ or\ WXY,$
that is, $No\ xZ\ is\ WYZ.$ (3)

We may now proceed as follows:—By (1), $All\ W\ is\ X\ or\ Y\ or\ Z;$ and by (3), $All\ W\ is\ X\ or\ y\ or\ z.$ Therefore, $All\ W\ is\ X\ or\ Yz\ or\ yZ.$ (2) affords no information regarding the class W , except that everything that is Z but not X is contained within it.

396. Shew what may be inferred as a possible description of warm-blooded vertebrates from the following, and state whether any of the information there given is superfluous for the purpose:—(1) All vertebrates may be divided into warm-blooded, and cold-blooded, and all produce their young in but one of two ways, *i.e.*, are either viviparous or oviparous; (2) No feathered vertebrate is both viviparous and warm-blooded; (3) No oviparous vertebrate that is cold-blooded has feathers; (4) Every viviparous vertebrate is either feathered or warm-blooded. [L.]

Taking vertebrate as the universe of discourse, let P =warm-blooded, Q =viviparous, R =feathered. Then by (1), p =cold-blooded, q =oviparous. The remaining premisses are as follows: (2) $No\ R\ is\ PQ,$ (3) $No\ pq\ is\ R,$ (4) $All\ Q\ is\ P\ or\ R.$ Required a description of P .

By the obversion of (2), $All\ R\ is\ p\ or\ q;$ therefore, $All\ P\ is\ q\ or\ r.$ (3) and (4) give no information with regard to P , and are therefore superfluous for our purpose. Hence the required description is that *All warm-blooded vertebrates are either oviparous or featherless.*

397. In a certain town the old buildings are either ecclesiastical and built entirely of stone, or, if not ecclesiastical are built entirely of brick; the brick-and-stone buildings are all modern as well as secular or they are neither; but there are no modern buildings at once secular and built entirely of stone. State what assumptions you make in interpreting the above, and determine (a) in what cases brick may be found in the buildings of this town and in what cases it cannot be, (b) what old buildings it would be useless to look for. [L.]

Let A = old, B = ecclesiastical, P = containing brick, Q = containing stone. Then assuming that old and modern, ecclesiastical and secular, are respectively contradictories, a = modern, b = secular.

The premisses are (1) *All A is BpQ or bPq* , (2) *All PQ is AB or ab* , (3) *No a is bpQ* .

We may interpret the question as follows: (a) after eliminating Q , determine P both positively and negatively; (b) determine A negatively.

(3) gives no information with regard to P ; but by (1) *All P is a or bq* , and by (2) *All P is AB or ab or q* ; therefore, *All P is ab or aq or bq* . Eliminating Q , *All P is a or b* . Therefore, *Brick is to be found only in secular or modern buildings*; and by obversion, *No brick is to be found in old ecclesiastical buildings*.

(3) gives no information with regard to A , and (2) adds no information to that contained in (1). Hence the second part of the question is answered by simply obverting (1). *No old buildings are ecclesiastical and built of brick, or ecclesiastical and not built of stone, or secular and built of stone, or secular and not built of brick*.

398. (1) If a nation has natural resources, and a good government, it will be prosperous. (2) If it has natural resources without a good government, or a good government without natural resources, it will be contented, but not prosperous. (3) If it has neither natural resources nor a good government it will be neither contented nor prosperous.

Shew that these statements may be reduced to two propositions of the form of Hamilton's U . [O's.]

Let a nation with natural resources be denoted by R , a nation with a good government by G , a prosperous nation by P , and a contented nation by C . Then the given statements may be expressed

as follows:—(1) *All RG is P*; (2) *All Rg or rG is Cp*; (3) *All rg is cp*.

By contraposition, (2) may be resolved into the two propositions, *All cp is RG or rg*, *All P is RG or rg*. But by (1) *No cp is RG*; and by (3) *No P is rg*. Hence the two propositions into which (2) was resolved may be reduced to the form, *All cp is rg*, *All P is RG*.

The three original statements are accordingly equivalent to the two U propositions *All RG is all P*, *All rg is all cp*.

399. Let the observation of a class of natural productions be supposed to have led to the following general results.

1st. That in whichsoever of these productions the properties *A* and *C* are missing, the property *E* is found, together with one of the properties *B* and *D*, but not with both.

2nd. That wherever the properties *A* and *D* are found while *E* is missing, the properties *B* and *C* will either both be found, or both be missing.

3rd. That wherever the property *A* is found in conjunction with either *B* or *E*, or both of them, there either the property *C* or the property *D* will be found, but not both of them. And conversely, wherever the property *C* or *D* is found singly, there the property *A* will be found in conjunction with either *B* or *E*, or both of them.

Shew that it follows that *In whatever substances the property A is found, there will also be found either the property C or the property D, but not both, or else the properties B, C, and D will all be wanting*. And conversely, *Where either the property C or the property D is found singly, or the properties B, C, and D are together missing, there the property A will be found*. Shew also that *If the property A is absent and C present, D is present*.

[Boole, *Laws of Thought*, pp. 146—148. Venn, *Symbolic Logic*, pp. 280, 281. *Johns Hopkins Studies in Logic*, pp. 57, 58, 82, 83.]

The premisses are as follows:—

- | | |
|------------------------------------------------|-------|
| 1st, <i>All ac is BdE or bDE</i> ; | (i) |
| 2nd, <i>All ADe is BC or bc</i> ; | (ii) |
| 3rd, <i>Whatever is AB or AE is Cd or cD</i> ; | (iii) |
| <i>Whatever is Cd or cD is AB or AE</i> ; | (iv) |

We are required to prove :—

- All A is Cd or cD or bcd ;* (α)
All Cd is A ; (β)
All cD is A ; (γ)
All bcd is A ; (δ)
All aC is D. (ε)

First, By (iii), *All A is Cd or cD or be*. But by (ii), *All Abe is c or d ;* and by (iv), *All Abe is CD or cd ;* therefore, *All Abe is cd*. Hence, *All A is Cd or cD or bcd*. (α)

Secondly, (β) and (γ) follow immediately from (iv).

Thirdly, from (i), we have directly, *No ac is bd ;* therefore (by conversion), *No bcd is a ;* therefore, *All bcd is A*. (δ)

Lastly, by (iv), *All Cd is A ;* therefore, by contraposition, *All aC is D*. (ε)

We may obtain a complete solution so far as *A* is concerned as follows :

- By (ii)¹, *All A is BC or bc or d or E ;*
 by (iii), *All A is be or Cd or cD ;*
 therefore, *All A is Cd or cDE or bcD or bce or bde ;*
 by (iv), *All A is B or E or CD or cd ;*
 therefore, *All A is cDE or bcde or BCd or Cde*.

This includes the partial solution with regard to *A*,—*All A is Cd or cD or bcd*. Boole contents himself with this because he has started with the intention of eliminating *E* from his conclusion.

We may now solve for *a*. (ii) and (iii) give no information with regard to this term. But by (i), *All a is BdE or bDE or C ;* and by (iv), *All a is CD or cd*. Therefore *All a is BcdE or CD*. And this yields by contraposition, *Whatever is bc or Cd or cD or ce is A*.

400. Given the same premisses as in the preceding section, shew that,—

1st. *If the property B be present in one of the productions, either the properties A, C, and D are all absent, or some one alone of them is absent. And conversely, if they are all absent it may be concluded that the property B is present.*

2nd. *If A and C are both present or both absent, D will be absent, quite independently of the presence or absence of B.*

[Boole, *Laws of Thought*, p. 149.]

¹ No information whatever with regard to *A* is given by (i), since *a* appears as a determinant of the subject. See section 380.

We may proceed here by combining all the given premisses in the manner indicated in section 368. From the result thus obtained the above conclusions as well as those contained in the preceding section will immediately follow.

By (iii), *Everything is a or be or Cd or cD*;
 and by (iv), *Everything is AB or AE or CD or cd*;
 therefore, *Everything is ABCd or ABcD*
 or *ACdE or AcDE or aCD or acd or bCDe or bedc*;
 therefore by (i), *Everything is ABCd or ABcD or Abcde*
 or *ACdE or AcDE or aBcdE or aCD or bCDe*;
 therefore by (ii), *Everything is ABCd or Abcde*
 or *ACdE or AcDE or aBcdE or aCD*. (v)

Hence, *All B is ACd or AcDE or acdE or aCD*;

All acd is BE;

All AC is Bd or dE;

All ac is BdE.

Eliminating *E* from each of the above we have the results arrived at by Boole.

Eliminating both *A* and *E* from (v) we have

Everything is BCd or bed or Cd or cD or Bcd or CD;

that is, *Everything is C or D or cd*, which is an identity. This is equivalent to Boole's conclusion that "there is no independent relation among the properties *B*, *C*, and *D*" (*Laws of Thought*, p. 148).

Any further results that may be desired are obtainable immediately from (v).

401. Given $XY = A$, $YZ = C$, find XZ in terms of *A* and *C*. [Venn, *Symbolic Logic*, pp. 279, 310—312. *Johns Hopkins Studies in Logic*, pp. 53, 54.]

The premisses may be written as follows:

Everything is AXY or ax or ay; (1)

Everything is CYZ or cy or cz. (2)

By (1), *All XZ is AY or ay*; and by (2), *All XZ is CY or cy*;
 therefore, *All XZ is ACY or acy*. Hence, eliminating *Y*, *All XZ is AC or ac*.

This solves the problem as set. But in order to get a complete solution equivalent to that which would be obtained by Boole, the following may be added: Solving as above for *x* or *z*, and eliminating *Y*, we have *All that is either x or z is ACx or ACz or ac*. Whence, by contraposition, *Whatever is AC or Ax or AZ or CX or*

Cz is XZ . In other words, *Whatever is AC or AZ or CX is XZ*; and *Nothing is Ax or Cz*.

402. Shew the equivalence between the three following systems of propositions: (1) *All Ab is cd; All aB is Ce; All D is E*; (2) *All A is B or c or D; All BE is A; All Be is Ad or Cd; All bD is aE*; (3) *Whatever is A or e is B or d; All a is bE or bd or BCE; All bC is a; All D is E*. [K.]

By obversion, the first set of propositions become *No Ab is C or D; No aB is c or E; No D is e*; and these propositions are combined in the statement *Nothing is either AbC or AbD or aBc or aBE or De*. (1)

By obverting and combining the second set of propositions, we have *Nothing is AbCd or aBE or aBce or BDe or AbD or bDe*. (2)

But *AbCd or AbD* is equivalent to *AbC or AbD*; *aBE or aBce* to *aBE or aBc*; *BDe or bDe* to *De*. Hence (1) and (2) are equivalent.

Again, by obverting and combining the third set of propositions, we have *Nothing is AbD or bDe or aBc or aBE or abDe or acDe or AbC or De*. (3)

But since *bDe, abDe, acDe* are all subdivisions of *De*, (3) immediately resolves itself into (1).

403. Given (i) *All Pqr is ST*; (ii) *Q and R are always present or absent together*; (iii) *All QRS is PT or pt*; (iv) *All QRs is Pt*; (v) *All pqrS is T*; then it follows that (1) *All Pg is rST*; (2) *All Ps is QRt*; (3) *All pQ is RSt*; (4) *All pT is qr*; (5) *All Qs is PRt*; (6) *All QT is PRS*; (7) *All qS is rT*; (8) *All qs is pr*; (9) *All qt is prs*; (10) *All sT is pqr*. [K.]

By (i), *Everything is p or Q or R or ST*;

by (ii), *Everything is QR or qr*;

therefore, *Everything is QR or pqr or qrST*;

by (iii), *Everything is q or r or s or PT or pt*;

therefore, *Everything is pqr or qrST or QRs or PQRT or pQRt*;

by (iv), *Everything is q or r or S or Pt*;

therefore, *Everything is pqr or qrST or PQRst or PQRST or pQRSt*;

by (v), *Everything is s or P or Q or R or T*;

therefore, *Everything is pqrs or pqrT or qrST or PQRst or PQRST or pQRSt*.

The desired results follow from this immediately.

404. From the premisses (1) *No Ax is cd or cy*, (2) *No*

BX is *cde* or *cey*, (3) *No ab* is *cdx* or *cEx*, (4) *No A* or *B* or *C* is *xy*, deduce a proposition containing neither *X* nor *Y*.

[*Johns Hopkins Studies*, p. 53.]

By (2), *No X* is *Bcde*, and by (1) and (3), *No x* is *Acd* or *abcd* or *abcE*; therefore, by section 384, *No Acd* or *abcd* or *abcE* is *Bcde*; therefore, *No Acd* is *Be*.

It will be observed that since *Y* does not appear in the premisses, *y* can be eliminated only by omitting all the terms containing it.

405. The members of a scientific society are divided into three sections, which are denoted by *A*, *B*, *C*. Every member must join one, at least, of these sections, subject to the following conditions: (1) any one who is a member of *A* but not of *B*, of *B* but not of *C*, or of *C* but not of *A*, may deliver a lecture to the members if he has paid his subscription, but otherwise not; (2) one who is a member of *A* but not of *C*, of *C* but not of *A*, or of *B* but not of *A*, may exhibit an experiment to the members if he has paid his subscription, but otherwise not; but (3) every member must either deliver a lecture or perform an experiment annually before the other members. Find the least addition to these rules which will compel every member to pay his subscription or forfeit his membership.

[*Johns Hopkins Studies*, p. 54.]

Let *A* = member of section *A*, &c.; *X* = one who gives a lecture; *Y* = one who performs an experiment; *Z* = one who has paid his subscription.

The premisses are

- (1) *All Ab* or *aC* or *Bc* is *x* or *Z*;
- (2) *All Ac* or *aB* or *aC* is *y* or *Z*;
- (3) *Every member* is *X* or *Y*;
- (4) *Every member* is *A* or *B* or *C*.

The problem is to find what is the least addition to these rules which will result in the conclusion that *Every member* is *Z*.

By (1), *All z* is either *x* or else (*a* or *B*) (*A* or *c*) (*b* or *C*); therefore, *All z* is *x* or *ABC* or *abc*.

Similarly, by (2), *All z* is *y* or *AC* or *abc*; therefore, *All z* is *xy* or *xAC* or *ABC* or *abc*.

By (3), *All z* is *X* or *Y*;

therefore, *All z* is *XABC* or *Xabc* or *xYAC* or *YABC* or *Yabc*.

By (4), *All z is A or B or C* ;
 therefore, *All z is $XABC$ or $xYAC$ or $YABC$* ;
 but *All $YABC$ is either $XYABC$ or $xYABC$* ;
 therefore, *All z is $XABC$ or $xYAC$* .

Hence, we gain the desired result if we add to the premisses *No z is $XABC$ or $xYAC$* . The required rule is therefore as follows: *No one who has not paid his subscription may join all three sections and deliver a lecture, nor may he join A and C and exhibit an experiment without delivering a lecture.*

406. What may be inferred independently of X and Y from the premisses: (1) *Either some A that is X is not Y , or all D is both X and Y* ; (2) *Either some Y is both B and X , or all X is either not Y or C and not B ?*

[*Johns Hopkins Studies*, p. 85.]

The premisses may be written as follows: (1) *Either something is AXy , or everything is XY or d* ; (2) *Either something is BXY , or everything is x or y or bC* .

By combining these premisses as in chapter 4, *Either something is AXy and something is BXY , or something is AXy and everything is x or y or bC , or something is BXY and everything is XY or d , or everything is $bCXY$ or bCd or dx or dy ¹.*

Therefore, eliminating X and Y (see sections 383 and 385), *Either something is A and something is B , or something is A , or something is B , or everything is bC or d* ; and by combining the first three alternants as in section 374, this becomes

Either something is A or B or everything is bC or d .

This conclusion may also be expressed in the form

If everything is ab , then every c is d .

407. Six children, A, B, C, D, E, F are required to obey the following rules: (1) on Monday and Tuesday no four can go out together; (2) on Thursday, Friday, and Saturday no three can stay in together; (3) on Tuesday, Wednesday, and Saturday, if B and C are together, then A, B, E , and F must be together; (4) on Monday and Saturday B cannot go out unless either D , or A, C , and E stay at home. A and B are

¹ We cannot, if we are to be left with an equivalent proposition, express the first three of these alternants in a non-compound form. See sections 370, 372.

first to decide what they will do, and C makes his decision before D , E , and F . Find (α) when C must go out, (β) when he must stay in, and (γ) when he may do as he pleases.

[*Johns Hopkins Studies*, p. 58.]

Let A = case in which A goes out, a = that in which he stays in, &c. Then the premisses are as follows:

- (1) On Monday and Tuesday,—*three at least must stay in*;
- (2) On Thursday, Friday, and Saturday,—*no three can stay in together*;
- (3) On Tuesday, Wednesday, and Saturday,—*Every case is $ABEF$ or $abef$ or Bc or bC* ;
- (4) On Monday and Saturday,—*Every case is ace or b or d* .

In order to solve the problem, we must combine the possibilities for each day, then eliminate D , E , and F , and find in what ways the movements of A and B determine those of C .

(i) On Monday,—we have *Every case is ace or b or d* , combined with the condition that three at least must stay in. One alternant therefore is def without further condition, and it follows that we can determine no independent relation between A , B , and C .

Hence on Monday C may do as he pleases.

(ii) On Tuesday,—we have *Every case is $ABEF$ or $abef$ or Bc or bC* , combined with the condition that three at least must stay in. Therefore, *Every case is $abef$ or Bc or bC ¹*; and eliminating D , E , and F , *Every case is ab or Bc or bC* .

Hence it follows that on Tuesday (α) if A goes out while B stays in, C must go out, and (β) if B goes out, C must stay in.

(iii) On Wednesday,—*Every case is $ABEF$ or $abef$ or Bc or bC* ; or eliminating D , E , and F , *Every case is AB or ab or Bc or bC* . Therefore, *All Ab is C and All aB is c* .

Hence on Wednesday (α) if A goes out while B stays in, C must go out, and (β) if A stays in while B goes out, C must stay in.

(iv) On Thursday and Friday,—the only condition is that no three can stay in together.

Hence on Thursday and Friday if A and B both stay in, C must go out.

¹ The two alternants Bc and bC might here be made more determinate, thus, $aBed$ or $aBee$ or $aBef$ or $Bede$ or $Bedf$ or $Bcef$ and $abCd$ or $abCe$ or $abCf$ or $bCde$ or $bCdf$ or $bCef$. But since we know that we are going on immediately to eliminate d , e , and f , it is obvious, even without writing them out in full, that these more determinate expressions will at once be reduced again to Bc and bC simply.

(v) On Saturday,—*Every case is AB \bar{E} F or ab \bar{e} f or Bc or bC ; also Every case is ace or b or d.* Combining these premisses, *Every case is AB \bar{d} E \bar{F} or ab \bar{e} f or aB \bar{c} e or Bcd or bC.* But we have the further condition that no three can stay in together. Therefore, *Every case is AB \bar{d} E \bar{F} or ABcdE \bar{F} or AbCDE or AbCDF or AbCE \bar{F} or bCDEF.* Therefore, eliminating D, E, and F, *Every case is AB or bC.*

Hence on Saturday if B stays in C must go out.

408. Given (1) *All P is QR*, (2) *All p is qr* ; shew that (3) *All Q is PR*, (4) *All R is PQ*. [K.]

409. Eliminate R from the propositions *All R is P or pq*, *All q is Pr or R*, *All qR is P*. [K.]

410. Shew the equivalence between the following sets of propositions :—(1) *a is BC ; b is AC ; C is Ab or aB* ; (2) *a is BC ; B is Ac or aC ; c is AB* ; (3) *A is Bc or bC ; b is AC ; c is AB*. [K.]

411. Say by inspection, stating your reasons, which of the following propositions give information concerning A, aB, b, bCd, respectively : *All Ab is bCd or c* ; *All bd is A or bC or abc* ; *Whatever is a or B is c or D* ; *Whatever is Ab or bc is bD or cD or e* ; *Everything is A or ab or Bc or Cd*. [K.]

412. Determine the conditions under which a particular proposition affords information in regard to any given term. [K.]

413. It is known of certain things that the quality A is always accompanied by C and D, but never by B ; and further, that the qualities C and D never occur together, except in conjunction with A. What can we infer about C ? [M.]

414. Given that everything that is Q but not S is either both P and R or neither P nor R and that neither R nor S is both P and Q, shew that no P is Q. [K.]

415. Where C is present, A, B, and D are all present ; where D is present, A, B, and C are either all three present or all three absent. Shew that when either A or B is present, C and D are either both present or both absent. How much of the given information is superfluous so far as the desired conclusion is concerned ? [K.]

416. Given that no X that is Z is either Y or W , and that no Y that is W is either X or Z ; what are the least additional data required in order to secure that no X is Y and that no Z is W ? [v.]

417. If thriftlessness and poverty are inseparable, and virtue and misery are incompatible, and if thrift be a virtue, can any relation be proved to exist between misery and poverty? If more-over all thriftless people are either virtuous or not miserable, what follows? [v.]

418. At a certain examination, all the candidates who were entered for Latin were also entered for either Greek, French, or German, but not for more than one of these languages; all the candidates who were not entered for German were entered for two at least of the other languages; no candidate who was entered for both Greek and French was entered for German, but all candidates who were entered for neither Greek nor French were entered for Latin. Shew that all the candidates were entered for two of the four languages, but none for more than two. [k.]

419. (1) Wherever there is smoke there is also fire or light; (2) Wherever there is light and smoke there is also fire; (3) There is no fire without either smoke or light.

Given the truth of the above propositions, what is all that you can infer with regard to (i) circumstances where there is smoke; (ii) circumstances where there is not smoke; (iii) circumstances where there is not light? [w.]

420. In a certain warehouse, when the articles offered are antique, they are costly, and at the same time either beautiful or grotesque, but not both. When they are both modern and grotesque, they are neither beautiful nor costly. Everything which is not beautiful is offered at a low price, and nothing cheap is beautiful. What can we assert (1) about the antique, and (2) about the grotesque articles? [m.]

421. Shew that the following sets of propositions are equivalent to one another:—

(1) *All a is b or c; All b is aC; All c is aB; All D is c.*

(2) *All A is BC; All b is aC; All C is ABd or abd.*

(3) *All A is B; All B is A or c; All c is aB; All D is c.*

(4) *All b is aC; All A is C; All C is d; All aC is b.*

(5) *All c is aB; All D is aB; All A is B; All aB is c.*

(6) *All A is BC; All BC is A; All D is Bc; All b is C.* [k.]

422. Shew that a certain set of four properties must be found somewhere together, if the following facts are known: "Everything that has the first property or is without the last has the two others; and if everything that has both the first and last has one or other but not both of the two others, then something that has the first two must be without the last two." [J.]

423. Given the propositions: (i) all material goods are external; (ii) no internal (= non-external) goods are dispropriable; (iii) all dispropriable goods are appropriable; (iv) no collective goods are appropriable or immaterial (= non-material); what is all that we can infer about (a) appropriable goods, (b) immaterial goods? [J.]

424. Eliminate X and Y from the following propositions: *All aX is BcY or bcy ; No AX is BY ; All AB is Y ; No $ABCD$ is xy .* Shew also that it follows from these propositions that *All XY is Ab or aBc .* [K.]

425. Given (1) *All A is Bc or bC* , (2) *All B is DE or de* , (3) *All C is De* ; shew that (i) *All A is $BcDE$ or $Bcde$ or $bCDe$* , (ii) *All BcD is E* , (iii) *All abd is c* , (iv) *All cd is ab or Be* , (v) *All bCD is e .* [Jevons, *Pure Logic*, § 160.]

426. Given (1) *All aB is c or D* , (2) *All BE is DF or cdF* , (3) *All C is aB or BE or D* , (4) *All bd is e or F* , (5) *All bf is a or C or DE* , (6) *All $bcdE$ is Af or aF* , (7) *All A is B or $CDEf$ or cDf or cdE* ; shew that (i) *All A is B* , (ii) *All C is D* , (iii) *All E is F .* [K.]

427. Shew the equivalence between the two following sets of propositions:—

(1) *All A is BC or BE or CE or D ;
 All B is $ACDE$ or $ACde$ or cdE ;
 All C is AB or AE or aD ;
 All D is $ABCE$ or Ace or aC ;
 All E is AC or aCD or Bc .*

(2) *All a is $BcdE$ or $bcdE$ or bD ;
 All b is a or ce or dE ;
 All c is $AbDe$ or $abde$ or BdE ;
 All d is $abce$ or BcE or Be or bE ;
 All e is ab or bc or d .*

[K.]

428. Given
- (1) *All bc is DE or Df or hc,*
 - (2) *All C is aB or DEFG or BFH,*
 - (3) *All Bcd is eL or hk,*
 - (4) *All Acf is d,*
 - (5) *All k is BC or Cd or Cf or H,*
 - (6) *All ABCDEFG is H or K,*
 - (7) *All DEFGH is B,*
 - (8) *All ABl is f or h,*
 - (9) *All ADFKl is H,*
 - (10) *All ADEFH is B or C or G or L ;*

shew that *All A is L.*

[K.]

CHAPTER VI.

THE INVERSE PROBLEM.

429. *Nature of the Inverse Problem.*—By the *inverse problem* is here meant a certain problem so-called by Jevons. Its nature will be indicated by the following extracts, which are from the *Principles of Science* and the *Studies in Deductive Logic* respectively.

“In the Indirect process of Inference we found that from certain propositions we could infallibly determine the combinations of terms agreeing with those premisses. The inductive problem is just the inverse. Having given certain combinations of terms, we need to ascertain the propositions with which they are consistent, and from which they may have proceeded. Now if the reader contemplates the following combinations,—

ABC	abC
aBC	abc

he will probably remember at once that they belong to the premisses $A = AB$, $B = BC$. If not, he will require a few trials before he meets with the right answer, and every trial will consist in assuming certain laws and observing whether the deduced results agree with the data. To test the facility with which he can solve this inductive problem, let him casually strike out any of the possible combinations involving three terms, and say what laws the remaining combinations obey. Let him say, for instance, what laws are embodied in the combinations,—

ABC	aBC
Abc	abC

“The difficulty becomes much greater when more terms enter into the combinations. It would be no easy matter

to point out the complete conditions fulfilled in the combinations,—

ACe
 $aBCe$
 $aBcdE$
 $abCe$
 $abcE.$

After some trouble the reader may discover that the principal laws are $C = e$, and $A = Ae$; but he would hardly discover the remaining law, namely that $BD = BDe$ " (*Principles of Science*, 1st ed., vol. I., p. 144; 2nd ed., p. 125).

"The inverse problem is always tentative, and consists in inventing laws, and trying whether their results agree with those before us" (*Studies in Deductive Logic*, p. 252).

The problem may preferably be stated as follows:—

Given a complex proposition of the form

Everything is $P_1P_2\dots$ or $Q_1Q_2\dots$ or....,

to find a set of propositions not involving any alternative combination of terms, which shall together be equivalent to it¹.

The inverse problem is in a sense indeterminate, for we may find a number of sets of propositions, not involving any alternative combination of terms, which are precisely equivalent in logical force, and hence any inverse problem may admit of a number of solutions. But it is not necessary to have recourse to a series of guesses in order to solve any inverse problem, nor need the method of solution be described as wholly tentative. Several systematic methods of solution applicable to any inverse problem are formulated in the following sections. Since, however, a number of solutions are possible, some of which are simpler than others, the process may be regarded as more or less tentative in so far as we seek to obtain the most satisfactory solution.

¹ The problem may also be stated as follows:—*Given a universal affirmative complex proposition containing alternative terms to find an equivalent compound conjunctive proposition all the determinants of which are affirmative and free from alternative terms.*

It may be observed that Jevons does not definitely exclude alternative terms in his solutions of inverse problems, though he generally seeks to avoid them. The problem cannot, however, be defined with scientific accuracy unless such terms are explicitly excluded.

The following may be taken as our criterion of simplicity. Comparing two equivalent sets of propositions, not involving any alternative combination of terms, that set may be regarded as the simpler which contains the smaller number of propositions. If the number of propositions is equal, then we may count the number of terms involved in their subjects and predicates taken together, and regard that one as the simpler which involves the fewer terms.

430. *A General Solution of the Inverse Problem.*—Let us suppose, then, that we are given a complex proposition involving alternative combination, and that we are to find a set of propositions, not involving alternative combination, which shall together be equivalent to it.

The data may be written in the form

Everything is P or Q or S or T or &c.,

where *P*, *Q*, &c., are themselves complex terms involving conjunctive, but not alternative, combination¹.

By contraposition one or more of these complex terms may be brought over from the predicate into the subject, so that we have

Whatever is not either P or S or &c. is Q or T or &c.

The selection of certain terms for transposition in this way is arbitrary (and it is here that the indeterminateness of the problem becomes apparent); but it will generally be found best to take two or three which have as many common determinants as possible.

What is not either P or S &c. is Q or T or &c.

will, when the subject is written in the affirmative form, immediately be resolvable into a series of propositions, which taken together give all the information originally given². Any of these propositions which still involve alternative combination may be dealt with in the same way, until no alternative combination remains.

We shall now be left with a set of propositions which will

¹ The proposition in its original form may admit of simplification in accordance with the rules laid down in chapter 1. It will generally speaking be found advantageous to have recourse to such simplification before proceeding further with the solution.

² See section 339.

satisfy the required conditions. The possibility of various simplifications has, however, to be considered. Thus, it will be necessary to make sure that each of the propositions is itself expressed in its simplest form¹; and to observe whether any two or more of the propositions admit of a simple recombination². It may also be found that some of the propositions can be altogether omitted, inasmuch as they add nothing to the information jointly afforded by the remainder; or that, considered in their relation to the remaining propositions, they may, at any rate, be simplified by the omission of one or more of the terms which they contain³. When these simplifications have been carried as far as is possible we shall have our final solution⁴.

The solution may, if we wish, be verified by recombining into a single complex proposition the propositions that have been obtained, an operation by which we shall arrive again at a series of alternants substantially identical with those originally given us. Such verification is, however, not essential to the validity of our process, which, if it has been correctly performed, contains no possible source of error.

The following examples will serve to illustrate the above method.

I. For our first example we may take one of those chosen by Jevons in the extract quoted in the preceding section.

Given the proposition, *Everything is either ABC or Abc or aBC or abC*, we are to find a set of propositions not involving alternative combination which shall be equivalent to it.

By the reduction of dual terms and contraposition we have

¹ For example, *All AB is BC* may be reduced to *All AB is C*.

² For example, *All ac is d* and *All Bc is d* may be combined into *All cD is Ab*.

³ Thus, for the propositions *All AB is CD* and *All Ab is C* we may substitute the propositions *All AB is D* and *All A is C*.

⁴ It may be observed that it is no part of our object to obtain a set of propositions which are mutually independent. As a matter of fact, it will generally be found that the maximum simplification involves the repetition of some items of information. Thus, in the example given in the preceding note the propositions *All AB is CD* and *All Ab is C* are quite independent of one another; but the proposition *All A is C* repeats part of the information given by the proposition *All AB is D*.

What is neither ABC nor Abc is aC ; therefore, What is a or Bc or bC is aC ; and this may be resolved into the three propositions:—

$$\begin{cases} All\ a\ is\ C, \\ Bc\ is\ non-existent, \\ All\ bC\ is\ a. \end{cases}$$

$Bc\ is\ non-existent$ is reducible to $All\ B\ is\ C$; and this proposition and $All\ a\ is\ C$ may be combined into $All\ c\ is\ Ab$.

Hence we have for our solution the two propositions:—

$$\begin{cases} All\ c\ is\ Ab, \\ All\ bC\ is\ a. \end{cases}$$

It will be found that by the recombination of these propositions we regain the original proposition.

II. We may next take the more complex example contained in the same extract from Jevons.

The given alternants are ACe , $aBCe$, $aBcdE$, $abCe$, $abcE$; and by the reduction of dual terms, they become $aBcdE$, $abcE$, Ce . Therefore, What is not $aBcdE$ or $abcE$ is Ce ; and this proposition may be resolved into the four propositions:—

$$\begin{cases} All\ A\ is\ Ce; & (1) \\ All\ BD\ is\ Ce; & (2) \\ All\ C\ is\ e; & (3) \\ All\ e\ is\ C. & (4) \end{cases}$$

But since by (3) $All\ C\ is\ e$, (1) may be reduced to $All\ A\ is\ C$; and this proposition may be combined with (4) yielding $All\ c\ is\ aE$. Also by (3), (2) may be reduced to $All\ BD\ is\ C$.

Hence our solution becomes

$$\begin{cases} All\ BD\ is\ C; \\ All\ C\ is\ e; \\ All\ c\ is\ aE. \end{cases}$$

III. The following problem is from Jevons, *Principles of Science*, 2nd ed., p. 127 (Problem v.).

The given alternants are $ABCD$, $ABCd$, $ABcd$, $AbCD$, $AbcD$, $aBCD$, $aBcD$, $aBcd$, $abCd$.

By the reduction of duals these alternants may be written as follows: ABC or $ABcd$ or AbD or $aBCD$ or aBc or $abCd$.

Therefore by contraposition, Whatever is not ABC or AbD or aBc is $ABcd$ or $aBCD$ or $abCd$.

But *Whatever is not ABC or AbD or aBc* is equivalent to *Whatever is ABc or aBC or ab or bd*. Hence we have for our solution the following set of propositions:

- (1) *All ABc is d,*
- (2) *All aBC is D,*
- (3) *All ab is Cd,*
- (4) *All bd is a¹.*

This is equivalent to the solution given in Jevons, *Studies*, p. 256.

If we wish to verify our solution we may proceed as follows:

By (3), *Everything is A or B or Cd;*

By (4), *Everything is a or B or D;*
therefore, *Everything is AD or aCd or B.*

By (1), *Everything is a or b or C or d;*
therefore, *Everything is AbD or ACD or aB or aCd or BC or Bd;*

By (2), *Everything is A or b or c or D;*
therefore, *Everything is ABC or ABd or AbD or ACD or aBc or aBD or abCd or BCD or Bcd;*

But, *AbD is AbCD or AbcD;* and expanding all the terms similarly, we have *Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcD or aBcd or abCd.* These are the alternants originally given.

IV. The following example is also from Jevons, *Principles of Science*, 2nd edition, p. 127 (Problem viii). In his *Studies*, p. 256, he speaks of the solution as *unknown*. A fairly simple solution may, however, be obtained by the application of the general rule formulated in this section.

The given alternants are *ABCDE, ABCDe, ABCde, ABcde, AbCDE, AbcdE, Abcde, aBCDe, aBCde, aBcDe, abCDE, abCdE, abcDe, abcdE.*

By the reduction of duals these alternants may be written: *ABCe or ABcde or Abcd or ACDE or aBCde or abdE or aDe.*

Therefore by contraposition, *Whatever is not either ABCE or ABcde or Abcd or abdE or aDe is ACDE or aBCde.*

But it will be found that, by the application of the ordinary

¹ We first obtain *All bd is aC*; but since by (3) *All abd is C*, this may be reduced to *All bd is a*.

rule for obtaining the contradictory of a given term, *Whatever is not either $ABCe$ or $ABcde$ or $Abcd$ or $abdE$ or aDe* is equivalent to *Whatever is AbC or ade or BE or AcD or DE .*

Hence our proposition is resolvable into the following:

- (i) *All AbC is DE ;*
- (ii) *All ade is BC ;*
- (iii) *All BE is ACD ;*
- (iv) *AcD is non-existent;*
- (v) *All DE is AC .*

But by (v) *All BE is AC or d* ; therefore, (iii) may be reduced to *All BE is D* . Again by (iv), *All DE is a or C* ; therefore, (v) may be reduced to *All DE is A* .

Hence we have the following as our final solution:—

- (1) *All AbC is DE ;*
- (2) *All ade is BC ;*
- (3) *All BE is D ;*
- (4) *All cD is a ;*
- (5) *All DE is A .*

431. *Another Method of Solution of the Inverse Problem.*—

Another method of solving the inverse problem, suggested to me (in a slightly different form) by Dr Venn, is to write down the original complex proposition in the negative form, *i.e.*, to obvert it, before resolving it. It has been already shewn that a negative proposition with an alternative predicate may be immediately broken up into a set of simpler propositions.

In some cases, especially where the number of destroyed combinations as compared with those that are saved is small, this plan is of easier application than that given in the preceding section.

To illustrate this method we may take two or three of the examples already discussed.

I. *Everything is ABC or Abc or aBC or abc ;*

therefore, by obversion, *Nothing is AbC or Bc or ac* ;
and this proposition is at once resolvable into

$$\begin{cases} \text{All } Ab \text{ is } c. \\ \text{All } c \text{ is } Ab^1. \end{cases}$$

¹ The equivalence between this and our former solution is immediately obvious. Equationally it would be written $Ab=c$.

II. *Everything is ACe or aBCe or aBcdE or abCe or abcE*; therefore, by obversion, *Nothing is CE or Ac or BcD or ce*.

This proposition may be successively resolved as follows :

$$\begin{cases} \text{No } E \text{ is } C; \\ \text{No } c \text{ is } A \text{ or } e; \\ \text{No } BD \text{ is } c. \\ \text{All } E \text{ is } c; \\ \text{All } c \text{ is } aE; \\ \text{All } BD \text{ is } C. \end{cases}$$

III. *Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcD or aBcd or abCd*; therefore, by obversion, *Nothing is Abd or ABcD or abc or abD or aBCd*; and this proposition may be successively resolved as follows :

$$\begin{cases} \text{No } bd \text{ is } A; \\ \text{No } ABc \text{ is } D; \\ \text{No } ab \text{ is } c \text{ or } D; \\ \text{No } aBC \text{ is } d. \\ \text{All } bd \text{ is } a; \\ \text{All } ABc \text{ is } d; \\ \text{All } ab \text{ is } Cd; \\ \text{All } aBC \text{ is } D. \end{cases}$$

It is rather interesting to find that notwithstanding the indeterminateness of the problem we obtain by independent methods the same result in each of the above cases.

432. A Third Method of Solution of the Inverse Problem.—The following is a third independent method of solution of the inverse problem, and it is in some cases easier of application than either of the two preceding methods.

Any proposition of the form

Everything is

may be resolved into the two propositions :

$$\begin{cases} \text{All } A \text{ is } \\ \text{All } a \text{ is } \end{cases}$$

which taken together are equivalent to it; similarly *All A is* may be resolved into the two *All AB is*, *All Ab is*; and it is clear that by taking pairs of contradictories in this way we may resolve any given complex proposition into a set

of propositions containing no alternative terms. Redundancies must of course as before be as far as possible avoided.

To illustrate this method we may again take the first three examples given in section 430.

I. *Everything is ABC or Abc or aBC or abC* may be resolved successively as follows:

$$\begin{cases} \text{All } C \text{ is } AB \text{ or } aB \text{ or } ab; \\ \text{All } c \text{ is } Ab. \\ \text{All } bC \text{ is } a^1; \\ \text{All } c \text{ is } Ab. \end{cases}$$

II. *Everything is ACe or aBCe or aBcdE or abCe or abcE* may be resolved successively as follows:

$$\begin{cases} \text{All } C \text{ is } Ae \text{ or } aBe \text{ or } abe; \\ \text{All } c \text{ is } aBdE \text{ or } abE. \\ \text{All } C \text{ is } e; \\ \text{All } c \text{ is } aE; \\ \text{All } c \text{ is } Bd \text{ or } b. \\ \text{All } C \text{ is } e; \\ \text{All } c \text{ is } aE; \\ \text{All } Bc \text{ is } d. \end{cases}$$

III. *Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcd or aBcd or abCd* may be resolved successively as follows:

$$\begin{cases} \text{All } B \text{ is } ACD \text{ or } ACd \text{ or } Acd \text{ or } aCD \text{ or } acD \text{ or } acd; \\ \text{All } b \text{ is } ACD \text{ or } AcD \text{ or } aCd. \\ \text{All } B \text{ is } AC \text{ or } aD \text{ or } cd; \\ \text{All } b \text{ is } AD \text{ or } aCd. \\ \text{All } BC \text{ is } A \text{ or } aD; \\ \text{All } Bc \text{ is } aD \text{ or } d; \\ \text{All } Ab \text{ is } D; \\ \text{All } ab \text{ is } Cd. \\ \text{All } BCd \text{ is } A; \\ \text{All } ABc \text{ is } d; \\ \text{All } Ab \text{ is } D; \\ \text{All } ab \text{ is } Cd. \end{cases}$$

The above solutions are practically the same as those obtained in the two preceding sections.

¹ Taking *BC* as our subject we have *All BC is A or a*, and since this is a merely formal proposition, it may be omitted.

433. *Mr Johnson's Notation for the Solution of Logical Problems.*—In his articles on the *Logical Calculus* Mr Johnson proposes a notation by the aid of which the solution of inverse problems may be facilitated. It consists in representing *conjunctive* combination by *horizontal* juxtaposition, and *alternative* combination by *vertical* juxtaposition. A bar—drawn horizontally or vertically—serves the purpose of a bracket where necessary. Thus, $\frac{AB}{CD}$ represents AB or CD ; $\frac{A}{C} \Big| \frac{B}{D}$ represents $(A \text{ or } C) \text{ and } (B \text{ or } D)$. These two forms are of course not equivalent to each other. But if *contradictories* are placed in a pair of diagonally opposite corners, then the combination is the same in whichever way we read it. Thus, $\frac{AB}{Ca}$ represents

AB or aC ; $\frac{A}{C} \Big| \frac{B}{a}$ represents $(A \text{ or } C) \text{ and } (a \text{ or } B)$. But these are equivalent to each other; for $(A \text{ or } C) \text{ and } (a \text{ or } B)$ is equivalent to AB or aC or BC , and—since BC by development is ABC or aBC —this is equivalent to AB or aC . Mr Johnson continues as follows:—"By adopting the plan of placing successive letter-symbols in opposite corners we may solve the *inverse problem* with surprising ease. The method of solution closely resembles the third of those adopted by Dr Keynes, and it was this that suggested mine. I will, therefore, illustrate by taking Dr Keynes's three examples which are the following:—

$$\text{I. } \frac{\frac{ABC}{Abc}}{\frac{aBC}{abc}} = \frac{BC}{Abc} = \frac{C}{Ab} \Big| \frac{B}{a}$$

Here the columns or determinants may be read off:—
 $(C \text{ or } Ab) \text{ and } (B \text{ or } a \text{ or } c) = (\text{If } c, \text{ then } Ab) \text{ and } (\text{If } AC, \text{ then } B)$.

$$\text{II. } \frac{\frac{ACe}{aBCe}}{\frac{aBcdE}{abcE}} = \frac{Ce}{acdE} = \frac{C}{aE} \Big| \frac{e}{d} \Big| c$$

This is read: $(\text{If } c, \text{ then } aE) \text{ and } (\text{If } BD, \text{ then } C) \text{ and } (\text{If } C, \text{ then } e)$.

$$\text{III. } \frac{\frac{ABC}{BCD}}{\frac{aBc}{Bcd}} = \frac{\frac{A}{D}}{\frac{Cd}{a}}$$

That is: (If ab , then Cd) and (If bd , then a) and (If ABD , then C) and (If BCd , then A). In this last problem, we first place B and b opposite; then for the B alternants, we place C and c opposite, and for the b alternants A and a . To get the simplest result, we should aim at dividing the columns into as equal divisions as possible.

The notation thus explained enables us to solve any problems in a simple manner. The expression in its final form may be read equally well in columns or in rows, i.e., as a determinative or as an alternative synthesis. Of course, a precisely similar process may be used, if we started with determinatively given or mixed data" (*Mind*, 1892, p. 351).

434. The Inverse Problem and Schröder's Law of Reciprocal Equivalences.—The inverse problem may also be solved, though somewhat laboriously, by the aid of the reciprocal relation between the laws of distribution given in section 321, this reciprocal relation depending upon the law that to every equivalence there corresponds another equivalence in which conjunctive combination is throughout substituted for alternative combination and *vice versa*. Thus, by the first law of distribution, (A or B) and (C or D) = AC or AD or BC or BD , and hence follows the corresponding equivalence AB or CD = (A or C) and (A or D) and (B or C) and (B or D). In this way any inverse problem may be practically resolved into the more familiar problem of conjunctively combining a series of alternative terms¹.

Taking as an example the first problem given in section 430, we may proceed as follows: (A or B or C) and (A or b

¹ It will be observed that the inverse problem involves the transformation of a logical expression consisting of a series of alternants into an equivalent

or c) and $(a$ or B or $C)$ and $(a$ or b or $C) = (A$ or Bc or $bC)$ and $(a$ or $C) = AC$ or aBc or bC . Therefore, we have the corresponding equivalence ABC or Abc or aBC or $abC = (A$ or $C)$ and $(a$ or B or $c)$ and $(b$ or $C)$. Hence the proposition *Everything is ABC or Abc or aBC or abC* may be resolved into the three propositions, *Everything is A or C* , *Everything is a or B or c* , *Everything is b or C* ; and we have for our solution of the inverse problem: *All c is A* , *All bC is a* , *All c is b* ; or, combining the first and last of these propositions, *All c is Ab* , *All bC is a* .

Similarly, the second problem in section 430 may be solved as follows:— $(A$ or C or $e)$ $(a$ or B or C or $e)$ $(a$ or B or c or d or $E)$ $(a$ or b or C or $e)$ $(a$ or b or c or $E) = aC$ or bCd or CE or ce . Hence the corresponding equivalence ACe or $aBcE$ or $aBcdE$ or $abCe$ or $abcE = (a$ or $C)$ $(b$ or C or $d)$ $(C$ or $E)$ $(c$ or $e)$; and we have for our solution of the inverse problem, *All A is C* , *All BD is C* , *All c is E* , *All C is e* ; or, combining the first and third of these propositions, *All c is aE* , *All BD is C* , *All C is e* .

EXERCISES.

435. Find propositions that leave only the following combinations, $ABCD$, $ABcD$, $AbCd$, $aBCd$, $abcd$.

[Jevons, *Studies*, p. 254.]

Jevons gives this as the most difficult of his series of inverse problems involving four terms. It may be solved as follows:—

Everything is $ABCD$ or $ABcD$ or $AbCd$ or $aBCd$ or $abcd$; therefore, by contraposition and the reduction of dual terms, *Whatever is not either $AbCd$ or $aBCd$ is ABD or abd* .

expression consisting of a series of determinants. Schröder's Law of Reciprocity shews that the process required for this transformation is practically the same as that by which an expression consisting of a series of determinants is transformed into an equivalent expression consisting of a series of alternants.

Therefore, *Whatever is AB or ab or c or D is ABD or abcd*; and this is resolvable into the four following propositions:

$$\begin{cases} \text{All } AB \text{ is } D, & (1) \\ \text{All } ab \text{ is } cd, & (2) \\ \text{All } c \text{ is } ABD \text{ or } abd, & (3) \\ \text{All } D \text{ is } AB. & (4) \end{cases}$$

Since by (4) *All D is AB*, and by (2) *All ab is cd*, (3) may be reduced to *All c is D or ab*, and therefore to *All cd is ab*. Also, by (4) *All ab is d*, and hence (2) may be reduced to *All ab is c*.

Our set of propositions may therefore be expressed as follows:—

$$\begin{cases} \text{All } AB \text{ is } D, \\ \text{All } ab \text{ is } c, \\ \text{All } cd \text{ is } ab, \\ \text{All } D \text{ is } AB^1. \end{cases}$$

436. Resolve the proposition *Everything is ABCDeF or ABcDEf or AbCDEF or AbCDeF or AbcDeF or aBCDEf or aBcDEf or abCDeF or abCdeF or abcDef or abcd ef* into a conjunction of relatively simple propositions.

[Jevons, *Principles of Science*, 2nd ed., p. 127 (Problem x.).]

The following is a solution:—

- (1) *All A is D*;
- (2) *All ABC is e*;
- (3) *All aF is bCe*;
- (4) *All Bf is DE*;
- (5) *All bf is ace*;
- (6) *All cF is be*.

This is somewhat less complex than the solution by Dr John Hopkinson given in Jevons, *Studies in Deductive Logic*, p. 256, namely:—

- (i) *All d is ab*;
- (ii) *All b is AF or ae*;
- (iii) *All Af is BcDE*;
- (iv) *All E is Bf or AbCDF*;
- (v) *All Be is ACDF*;
- (vi) *All abc is ef*;
- (vii) *All abef is c*.

¹ Restoring the second of these propositions to the form *All ab is cd*, and writing the propositions equationally, the solution may be expressed in a still simpler form, namely,

$$\begin{aligned} AB &= D, \\ ab &= cd. \end{aligned}$$

437. How many and what non-disjunctive propositions are equivalent to the statement that "What is either Ab or bC is Cd or cD , and *vice versa*?" [Jevons, *Studies*, p. 246.]

The given statement is at once resolvable into the four following propositions:

$$\begin{cases} \text{All } Ab \text{ is } Cd \text{ or } cD, & (i) \\ \text{All } bC \text{ is } Cd \text{ or } cD, & (ii) \\ \text{All } Cd \text{ is } Ab \text{ or } bC, & (iii) \\ \text{All } cD \text{ is } Ab \text{ or } bC. & (iv) \end{cases}$$

(i) may be resolved into $\begin{cases} \text{All } Abc \text{ is } D, & (v) \\ \text{All } AbD \text{ is } c. & (vi) \end{cases}$

But (vi) is inferable from (ii); and observing some other obvious simplifications we obtain immediately the following solution:

- (1) *All Abc is D ;*
- (2) *All bC is d ;*
- (3) *All Cd is b ;*
- (4) *All cD is Ab .*

438. Shew the equivalence between the two sets of propositions given in section 436. [K.]

439. Find which of the following propositions may be omitted without affecting the information given by the propositions as a whole: *All Ab is cDE ; All Ac is bDE ; All Ad is BCE ; All Ae is BCd ; No aE is B or C ; No B is c ; All Bd is ACe ; No bD is C or e ; No bE is Ad or C ; All C is B ; All Cd is ABe ; All cD is bE ; All cE is AbD or ab ; All de is ABC or abc .* [K.]

440. Resolve each of the following complex propositions into a conjunction of propositions not containing any alternative combination of terms:

- (1) *Everything is $ABCD$ or $AbCd$ or $aBcD$ or $abcd$;*
- (2) *Everything is $AbCD$ or $AbCd$ or $abCd$ or $aBcd$ or $abCD$ or $abCd$ or $abcd$;*
- (3) *Everything is $AbcDE$ or $aBCd$ or $aBCE$ or $aBcd$ or $aBde$ or $abCe$ or $abce$ or $abDe$ or $abde$ or $BcdE$ or $bCDe$;*
- (4) *Everything is $ABCE$ or $ABcd$ or $ABcE$ or $ABde$ or $Abcd$ or $abCE$ or $abcE$ or $abDE$ or $abde$ or $BCde$;*
- (5) *Everything is $ABCDE$ or $ABCdE$ or $ABcDE$ or $ABcDe$ or $ABcde$ or $AbCdE$ or $Abcde$ or $aBCDE$ or $aBCde$ or $abCDE$ or $abcDe$;*

(6) *Everything is* $ABDe$ or $ABDF$ or $AcDe$ or $Acef$ or $aBDe$ or $aBDF$ or $abCD$ or $abCd$ or $abcD$ or $abcd$ or $aCDE$ or $aCDe$ or $aCdE$ or $aCde$ or $acDe$ or $aDEF$ or $aDEf$ or $aDeF$ or $aDef$ or $BcDF$ or $bceF$ or $bcef$;

(7) *Everything is* $AbdE$ or $Abef$ or AbF or $Acdef$ or $aBDF$ or $abCF$ or $aCdE$ or ade or $bCDe$ or $bCdf$ or $bDEF$;

(8) *Everything is* $ABCEf$ or Abe or $aBCdf$ or $aBcdE$ or $aBcdeF$ or $abef$ or $bceF$. [K.]

441. Express the following proposition in as small a number as you can of propositions in which no alternative combination of terms occurs: *Everything is* $ABCDe$ or $ABCdE$ or $ABcDe$ or $AbCde$ or $AbCde$ or $aBCdE$ or $aBcDE$ or $aBcde$ or $aBcdE$ or $abCde$ or $abCdE$. [J.]

442. Solve the fourth problem given in section 430, (a) by the method described in section 431, (β) by that described in section 432. [K.]

443. Solve the problem given in section 435 and also the fourth problem given in section 430 by aid of the notation described in section 433. [K.]

444. Solve the third and fourth problems given in section 430 by the method described in section 434. [K.]

445. Shew that any universal complex proposition may be resolved into a set of propositions in which no conjunctive combination of terms occurs. [K.]

APPENDIX.

ON THE DOCTRINE OF DIVISION.

446. *Logical Division*.—The term *division*, as technically used in logic, may be defined as the setting forth of the smaller groups which are contained under the extension of a given term. It is also defined as the separation of a genus into its constituent species. These two definitions are practically equivalent to each other. *Division* is to be distinguished from the setting forth of the individual objects contained under a species, which is technically described as *enumeration*.

In logical division the larger class which is divided is called the *totum divisum*, the smaller classes into which it is divided being the *membra dividenda* (dividing members). By the ground or principle of division (*fundamentum sive principium divisionis*) is meant that attribute or characteristic of the *totum divisum* upon whose modifications the division is based. A given class may of course be divided in different ways according to the particular attribute or attributes whose variations are selected as differentiating its various species. Thus, having regard to the equality or inequality of the sides, triangles may be divided into equilateral, isosceles, and scalene; or, having regard to the size of the largest angle, into obtuse-angled, right-angled, and acute-angled. Again, propositions are divisible according to their truth or falsity, or according to their quantity, or their quality, and so on.

It is sometimes said that the principle of division must be present throughout the dividing members, though constantly varied. On the other hand, it is said that in division we invariably try to think of some attribute which is predicable of certain members of the group, but not of others. The former of these statements does not very well apply when we simply divide a class according to the

presence or absence of some attribute (for example, candidates for the Civil Service into successful and unsuccessful) or when the attribute in question may be entirely wanting in some instances whilst present in varying degrees in other instances. In other words, given the attribute whose variations constitute our principle of division, we may have to recognise a limiting case in which it is altogether absent; thus, in dividing undergraduates according to their colleges we may have to recognise a class of non-collegiate students. The second statement is always true when we simply contrast any given species with all the remaining species, and it may be considered adequate where we have division by contradictories. In other cases, however, it is inadequate; as, for instance, when we divide candidates who are successful at the Indian Civil Service Open Competition according to the province to which they are assigned.

447. *Physical Division, Metaphysical Division, and Verbal Division.*—Following the older logicians, we may distinguish division as defined in the preceding paragraph, that is, *logical division* in the strict sense, from other senses in which the term is used.

The division of an individual thing into its separate parts is called *physical division* or *physical definition* (Whately, *Logic*, p. 143) or *partition*; as, for example, if we divide a watch into case, hands, face, and works; or a book into leaves and binding. We have, on the other hand, a logical division if we divide watches into gold, silver, &c., or into English, Swiss, American, &c.; or if we divide books into folios, quartos, &c. Professor Bain (*Logic*, vol. II. p. 197) gives the analysis of a chemical compound as an instance of logical division. It is rather an instance of physical division. In logical division the *totum divisum* is always predicable of each of the *membra dividenda*; for example, All men are animals, All squares are rectangles. But this is not the case in chemical analysis. We cannot say that oxygen is water, or that sulphur is vitriol, or that sodium is salt.

Distinct both from logical division and from physical division is the mental division of a thing into its separate qualities. This is called *metaphysical division*. We have an example when we enumerate the separate qualities of a watch, its size, accuracy, the material of which its case is composed, &c.; or when we specify the size of a book, its thickness, colour, the material of its binding, the quality of the paper of which its leaves are composed, and so on. A

physical division can be actually made; a watch, for example, can be taken to pieces. A metaphysical division, on the other hand, is only possible mentally. It should be added that the metaphysical division of individual objects may be made the basis of a logical division of the class to which they belong.

One further kind of division may be noticed, namely, the division of an ambiguous or equivocal term into its several significations. This is called *verbal division* (Clarke, *Logic*, p. 231) or *distinction* (Mansel's *Aldrich*, p. 37). For example, we have to distinguish between a watch in the sense of a vigil, in the sense of a guard, and in the sense of a time-piece.

448. *Rules of Logical Division.*—The rules of logical division may be formulated as follows:—

- (1) Each distinct act of division must proceed throughout upon one and the same basis or principle;
- (2) The members of the division must together be exactly co-extensive with the class which is divided;
- (3) If the division involves more than one step, it should proceed gradually and continuously from the highest genus to the lowest species, that is to say, it should not pass suddenly from a high genus to a low species.

The first of these rules is of course not intended to condemn the processes of *sub-division* and *co-division*. Having made a division upon one principle, we may always proceed to subdivide the classes thus obtained in accordance with another principle, and so on indefinitely. A scientific classification will always consist of a hierarchy of classes thus obtained. There is again no reason why the same class should not for different purposes be divided in accordance with two or more different principles, so long as these are kept quite distinct from one another, and the members of the different resulting divisions in no way confused together.

A corollary from the first rule is that the members of a valid division will be mutually exclusive. For there cannot be over-lapping classes so long as a division proceeds correctly upon a single principle. In other words, over-lapping classes imply a cross-division. But the converse of this does not hold, since two principles of division may happen to yield coincident results. For example, there is a cross-division, but no over-lapping of classes, if we divide triangles into scalene, isosceles, and equiangular; or if we divide plants into acotyledons, monocotyledons, and exogens.

Rule 2 and the above corollary from rule 1 are summed up in the statement that the members of a division must be mutually exclusive and collectively exhaustive. Symbolically, if the division of X into XA , XB , XC is logically valid, we must have *No XA is B or C , No XB is C or A , No XC is A or B , All X is A or B or C* . The division of triangles into equiangular, right-angled, obtuse-angled, and scalene is an example of a thoroughly illogical division. It proceeds upon more than one principle, and the members of the division are neither mutually exclusive nor collectively exhaustive.

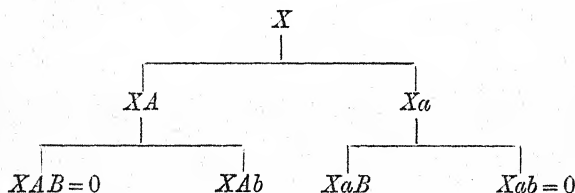
A distinction may be drawn between the first two of the above rules and the third. A breach of the third rule does not make a division absolutely invalid; but a division which proceeds *per saltum* will be far less effective for its purpose than if the intermediate steps were filled in. The worst violation of this rule occurs when the division is *disparate*, that is, when "one of the classes into which we divide is an immediate and proximate class, while others are mediate and remote" (Clarke, *Logic*, p. 242); as, for example, if we divide animals into invertebrates, fishes, amphibians, reptiles, birds, elephants, horses, dogs, &c.

Another rule of division is sometimes added, namely, that "none of the dividing members must be equal in extent to the divided whole" (Clarke, *Logic*, p. 236). When this rule is broken, the division is said to become null and void, because one of the subdivisions contains no members. From the strictly formal point of view, however, this rule must be rejected. Logically we need not regard a division as necessarily implying the actual occurrence of all its members in the universe of discourse; and the rule in question would deprive the logician of the right to employ the powerful method of division by contradictories. It is of course a different matter when we regard scientific classification from the material standpoint.

449. *Division by Dichotomy.*—Division by dichotomy or, as it is sometimes called more distinctively, *dichotomy by contradiction* is the division of a class simply with reference to the presence or absence of a given attribute or set of attributes; as, for example, when X is divided into XA and Xa (where $a = \text{not-}A$). An illustration is afforded by the Tree of Porphyry or Ramean Tree, in which Substances are first divided into Corporeal Substances (Bodies) and Incorporeal Substances, Bodies being then divided into Animate Bodies (Living Beings) and Inanimate Bodies, Living Beings being

next divided into Sensitive Living Beings (Animals) and Insensitive Living Beings, and Animals being in their turn divided into Rational Animals (Men) and Irrational Animals. At each step in this scheme we proceed by taking contradictories. It was in praise of dichotomal division that Jeremy Bentham, who is here quoted with approval by Jevons (*Principles of Science*, p. 704), spoke of "the matchless beauty of the Ramean Tree." When this method is employed we ensure formally that the members of our division shall be mutually exclusive and collectively exhaustive. For, by the law of contradiction, *No X is both A and a*; and, by the law of excluded middle, *Every X is either A or a*.

It is pointed out by Spalding (*Logic*, p. 146) and by Jevons (*Principles of Science*, p. 695) that all logically perfect division is ultimately reducible to dichotomy, usually with the implication that some of the sub-classes which are *a priori* possible are not as a matter of fact to be found in the universe of discourse. Thus, if we take the class X and divide it into XA and Xa , we imply that in the class X , A and B are never found either both present or both absent. Hence the division is equivalent to the following dichotomal division:—



Any other division, however complicated in its character, may be reduced to dichotomy in a similar way. This is interesting and important and brings out the value of dichotomy as a method of testing divisions. It must be understood, however, that in speaking of all division as ultimately reducible to dichotomy, it is not intended to imply that dichotomy usually represents our actual procedure in making divisions. Each sub-class is usually arrived at immediately by reference to some positive modification of the *fundamentum divisionis*; and the different sub-classes are co-ordinate with one another. Consider, for example, the division of conic sections into parabolas, hyperbolas, ellipses, circles, and pairs of straight lines. It must be added that from the material standpoint, pure division by dichotomy is of little scientific value, because

of the indefinite character of the sub-classes which are determined negatively.

450. *The place of the Doctrine of Division in Logic.*—The doctrine of division, as treated by the older logicians, receives little recognition by some modern writers on two very different grounds: (1) by Mill, taking the material standpoint, it is regarded as too purely formal, and hence is merged in the doctrine of scientific classification; (2) by some writers belonging to the conceptualist school, *e.g.*, Mansel, it is rejected as not being sufficiently formal.

(1) It is true that the rules of logical division lead us a very little way in practical science. They give certain conditions which must be complied with; but they neither help us towards making good divisions, nor provide us with a test which is capable of being formally applied. Leaving dichotomy on one side, we cannot, without the aid of material knowledge, even determine whether the members of a given division are mutually exclusive and collectively exhaustive. When, however, we avowedly pass beyond purely formal considerations and take up a material standpoint, then the doctrine of division should rightly give place to a doctrine of classification, which is not content with such rules as those laid down above, but which seeks to indicate the principles which should serve as a guide in classifying objects scientifically.

In regard to the use of the terms *division* and *classification*, Miss Jones draws a distinction which is of value and to which it might be well systematically to adhere. "Division and classification are the same thing looked at from different points of view; any table presenting a division presents also a classification. A division starts with unity and differentiates it; a classification starts with multiplicity, and reduces it to unity, or at least to system" (*Elements of Logic*, p. 123).

(2) It remains to be considered how far any treatment of division whatever can properly fall under the consideration of a purely formal logic. From this point of view division is usually contrasted with definition. The latter of these—using the phraseology of the conceptualist logicians—expounds the intension of a concept; the former expounds its extension. But the intension of a concept is said to be far more intrinsic to it than its extension. Given a concept its intension is necessarily given; but knowledge of its extension, such as may serve to determine its division, will require a fresh appeal to the subject-matter. "Division," says Mansel,

"is not, like definition, a mental analysis of *given* materials: the specific difference must be *added* to the given attributes of the genus; and to gain this additional material, it is necessary to go out of the act of thought, to seek for new empirical data" (*Prolegomena Logica*, p. 192). For example, the division of members of the University of Cambridge into those *in statu pupillari* and members of the Senate could not be obtained without something more being given than the mere conception of a member of the University. Moreover, unless we proceed by contradictories, we cannot, when we have got our division, formally determine whether it complies with our rules or not.

The above position may be accepted, if an exception is made for division by dichotomy. Mansel, however, and some other logicians will not even allow that division by dichotomy is a formal process; and here they lay themselves open to criticism. The grounds on which their view is based are twofold:—(i) It is not sufficient that the genus to be divided be given; the principle of division must be given also. "Even in the case of dichotomy by contradiction the principle of division must be given, as an addition to the attributes comprehended in the concept, before the logician can take a single step" (*Prolegomena Logica*, p. 207). "The division of *A* into *B* and *not-B* is not strictly formal; for *B*, the dividing attribute, not being part of the comprehension of *A*, has to be sought for out of the mere act of thought, after *A* has been given" (Mansel's *Aldrich*, p. 38). (ii) We cannot tell *à priori* that both the subclasses obtained by dichotomy really exist. How, for example, can we divide *A* into *B* and *not-B* when for anything we know to the contrary all *A* may be *B*? "Logically, the division of animal into mortal and immortal is as good as that into rational and irrational" (Mansel's *Aldrich*, p. 38). Both these arguments are summed up in the following quotation from Mr Monck: "It is alleged indeed that Logic enables us to divide all the *B*'s into the *B*'s which are *C*'s and the *B*'s which are not *C*'s.....But Logic does not supply us with the term *C*, and after we have obtained this term there are two cases in which the proposed division fails, namely, where all the *B*'s are *C*'s and where none of them are so. In either of these events the class *B* remains as whole and undivided as before; and whether they have occurred or not cannot be ascertained by Logic. This Division by Dichotomy, as it is called, is as much outside the province of Logic as any other kind of division" (*Logic*, p. 174).

As regards the first of the above arguments, there is no reason why the principle of division (A) should not be assumed given as well as the *totum divisum* (X). The question is whether we can then formally divide X into XA and Xa . The fact that A must be given as well as X does not prevent the possibility of formal division by dichotomy, any more than the fact that the conclusion of a syllogism is not contained in one premiss alone prevents the syllogism from being a formal process.

The force of the second argument depends entirely upon the implication that all the sub-classes obtained as the result of a division necessarily exist in the universe of discourse. If this implication is granted, then dichotomy is certainly not a formal process; but why need we assume the existence of all the sub-classes obtained by dichotomy? Without such an assumption, our division may not have much practical utility, but its formal validity will remain unaffected. We have only to make it clear that we are dividing the *extension* of a term, not its *denotation*, in the sense in which extension and denotation have been already distinguished. This is in keeping with the general standpoint of formal logic, which can deal with classes without regarding their existence as necessarily guaranteed in any assigned universe of discourse. If we are not allowed to apply the principle of excluded middle in formal logic and say *Every X is A or a* , until we know that there actually exist both XA 's and Xa 's, we shall be exceedingly hampered, and can make but little progress, especially in the treatment of complex inferences. Some schemes of symbolic logic (e.g., Jevons's) depend essentially and explicitly upon an antecedent scheme of dichotomical division.

We may then regard division by dichotomy as a formal process, but only on the understanding (1) that the principle of division is given as well as the genus to be divided; (2) that the division is regarded as hypothetical so far as concerns the existence of the resulting sub-classes in any assigned universe of discourse.

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